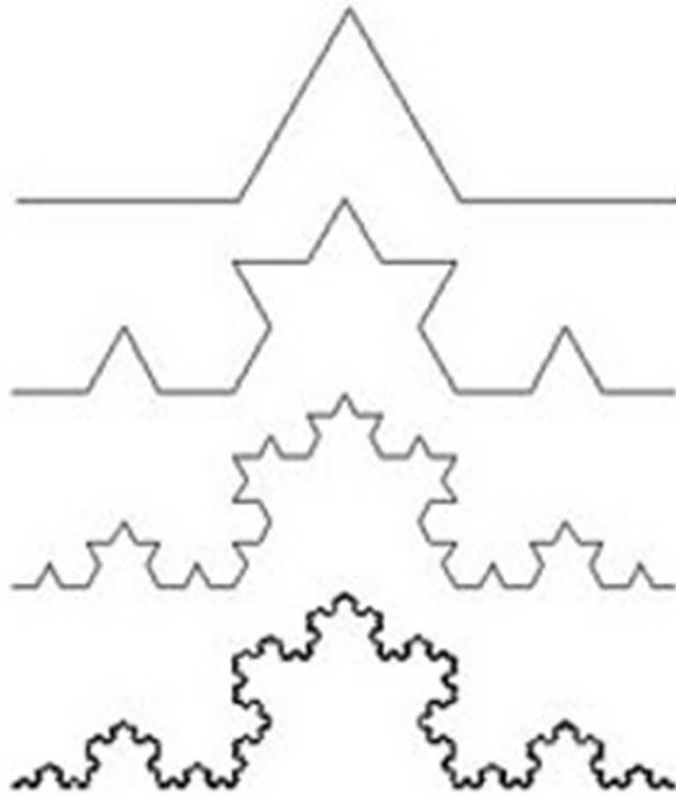


# Fractals

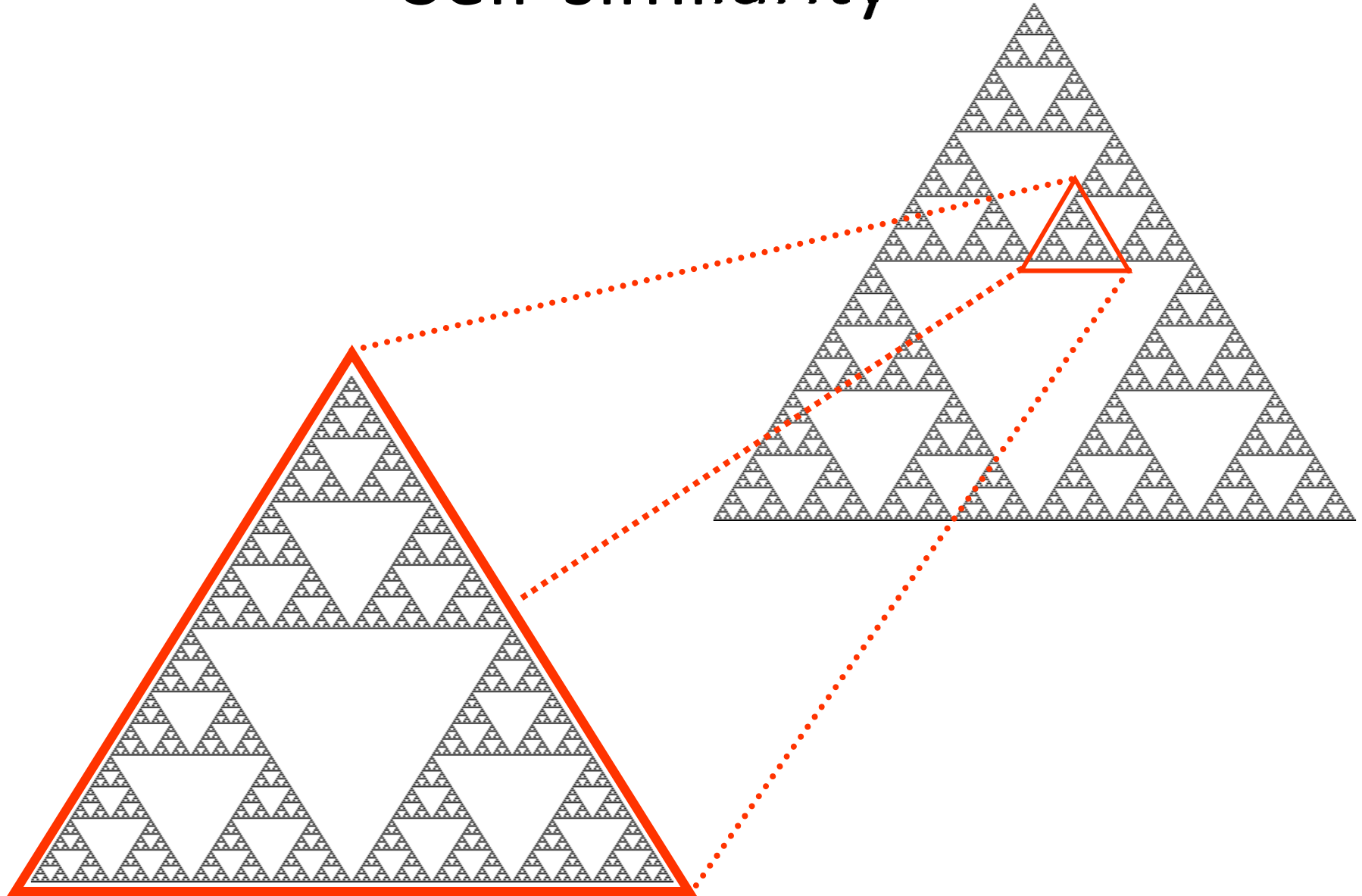
C. Andújar

June 2014

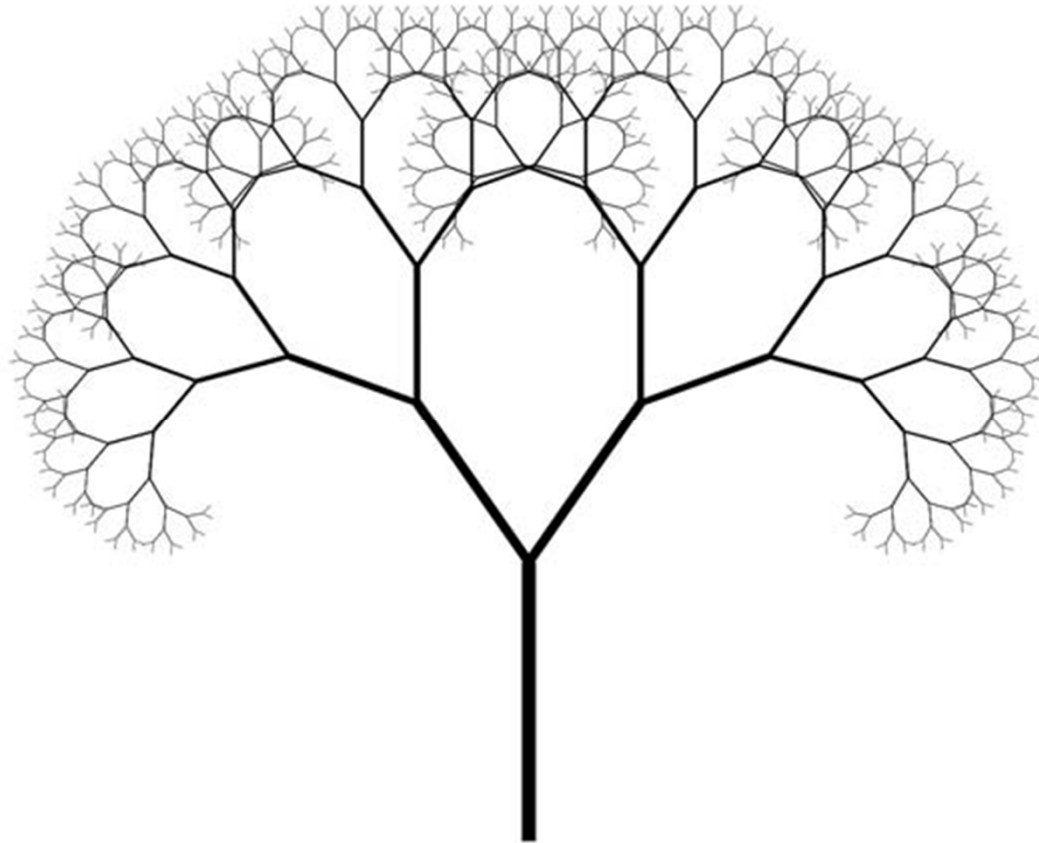
# Self-similarity



# Self-similarity



# Self-similarity



# Self-similarity



# Self-similarity



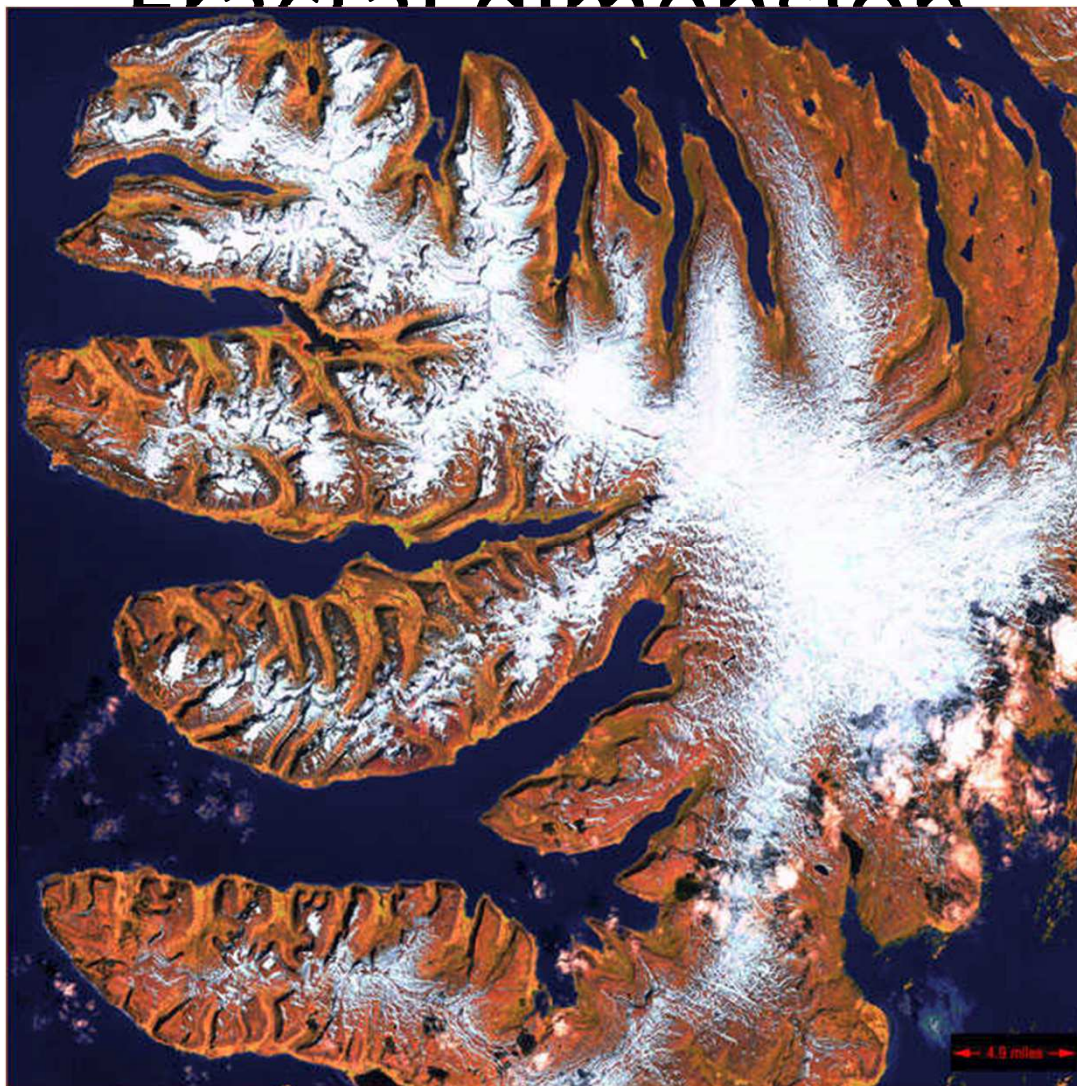
# Self-similarity



# Fractal dimension



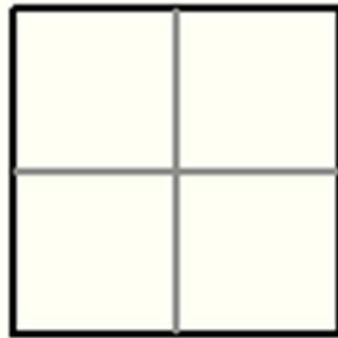
# Fractal dimension



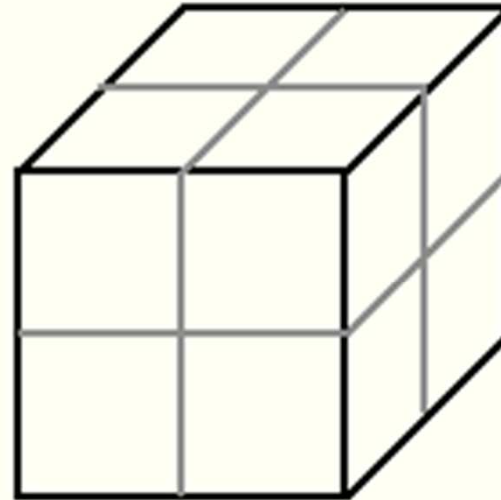
# Fractal dimension



$N=2, \alpha=1/2$

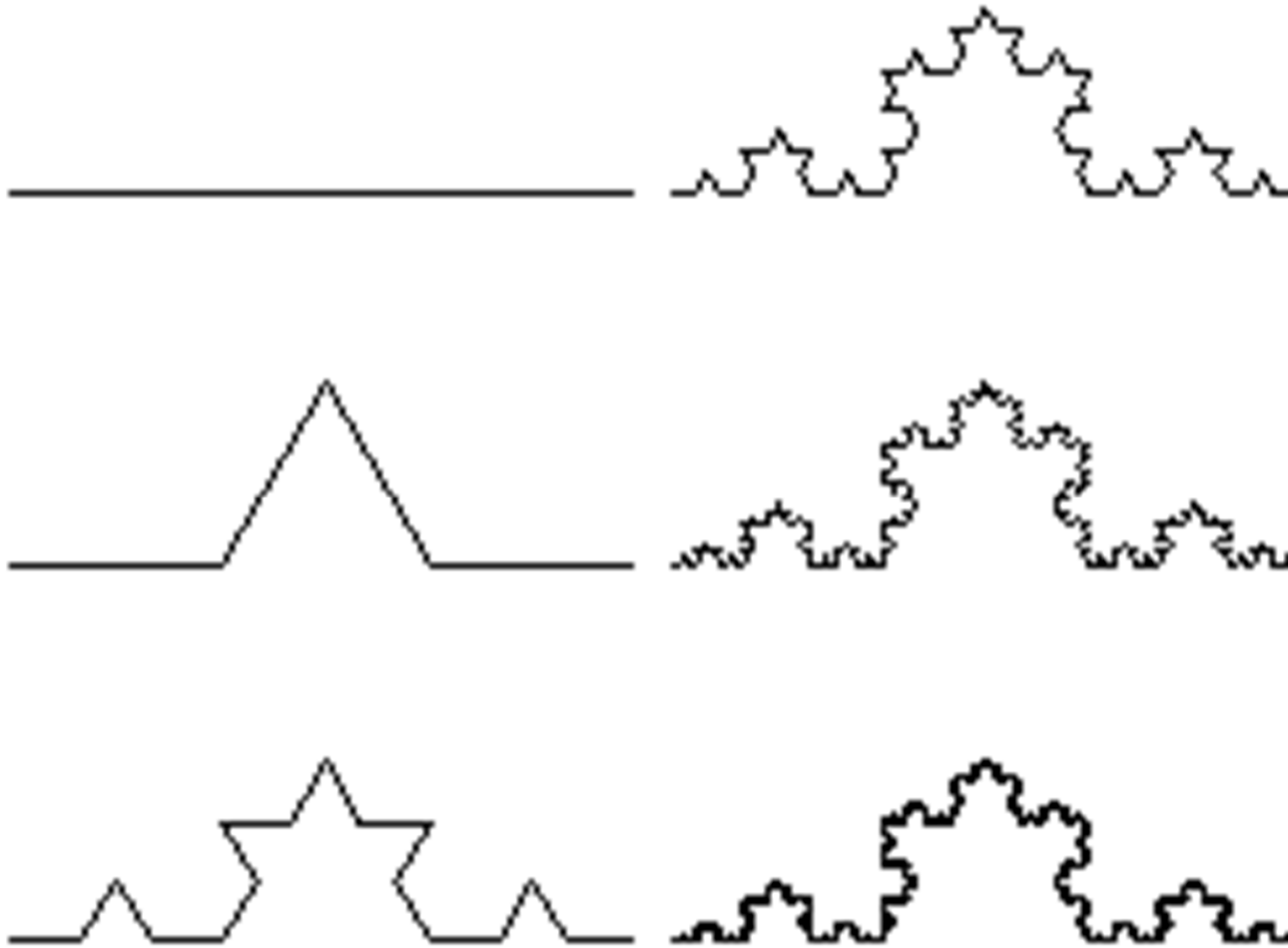


$N=4, \alpha=1/2$

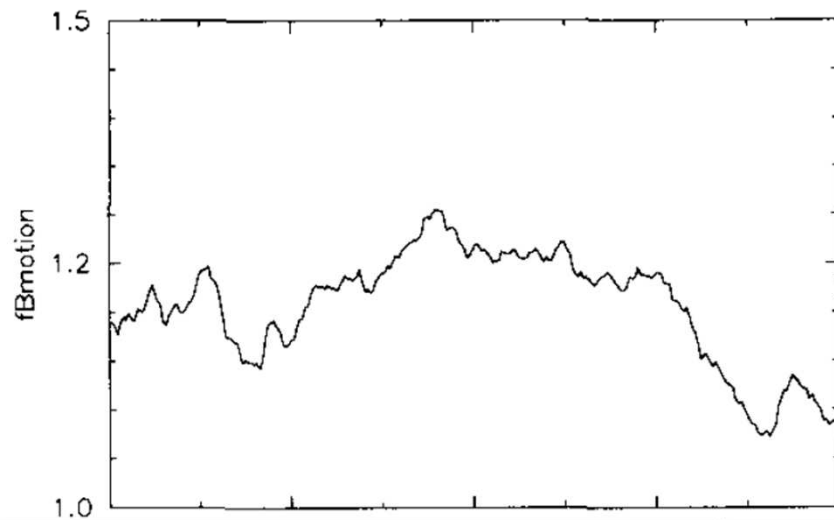
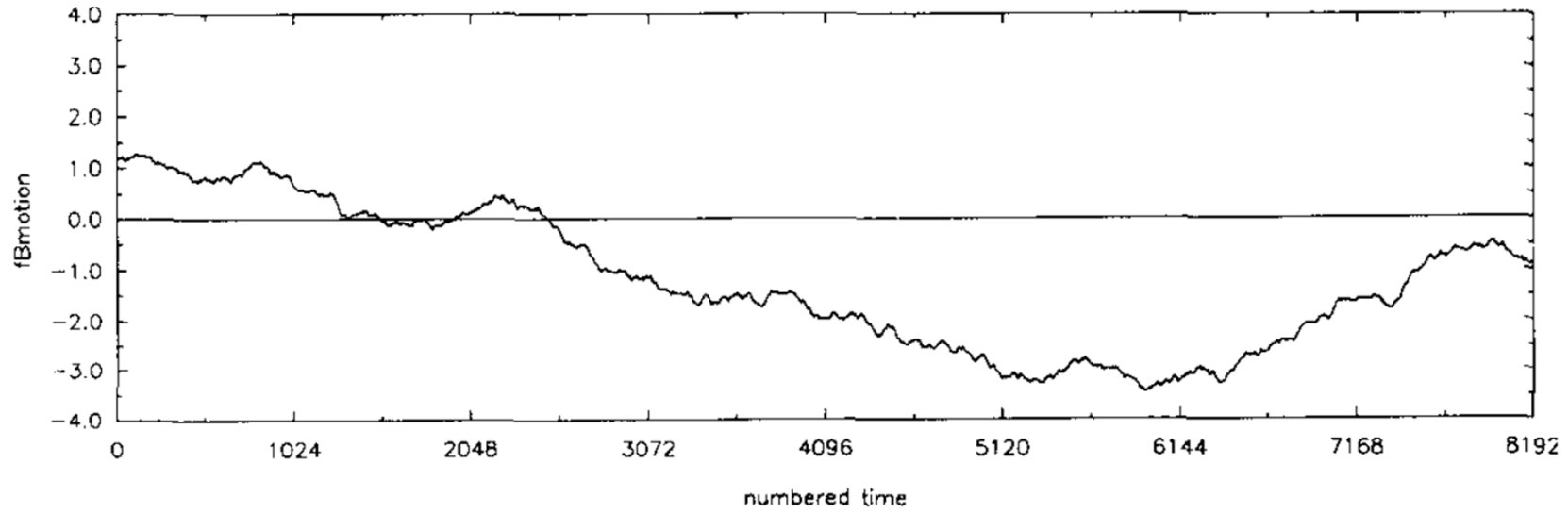


$N=8, \alpha=1/2$

# Fractal dimension



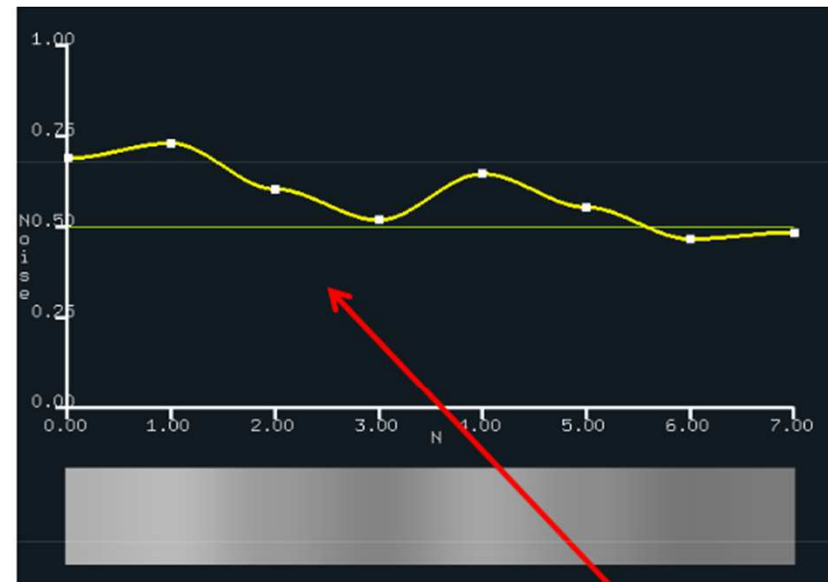
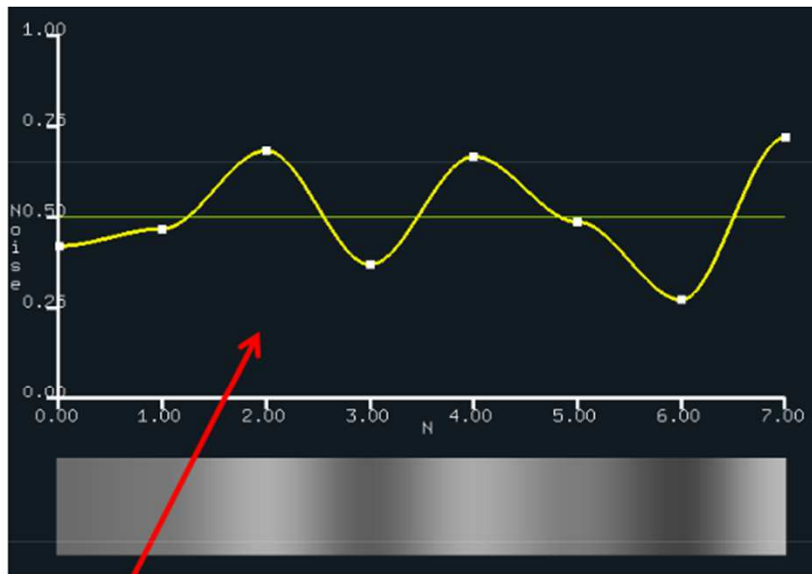
# Fractal analysis



# Fractal synthesis

## Positional Noise

**Idea:** Pick a random number at the whole-number input values and then fit a piecewise smooth curve through those points.

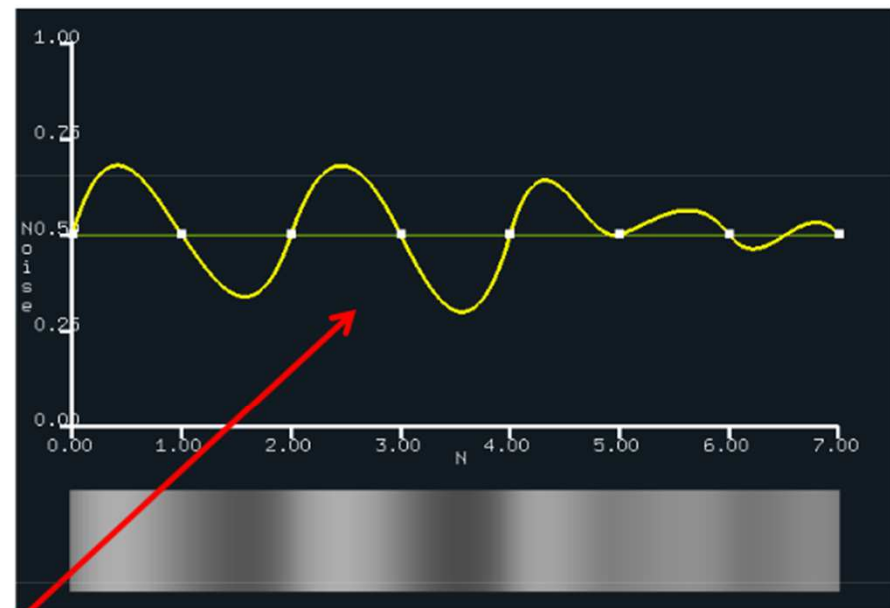
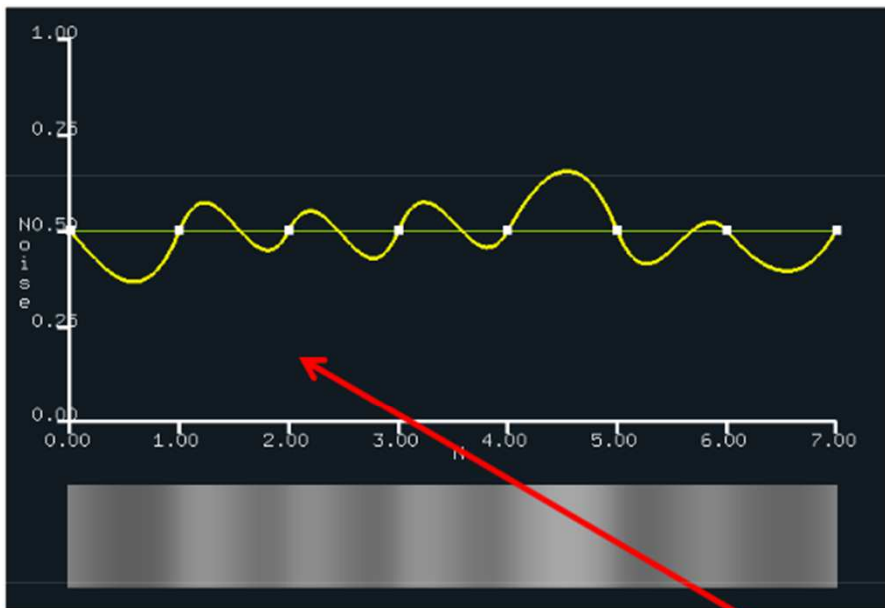


The problem is that, due to the uncertainty of random numbers, you might get a good plus-or-minus distribution, or you might not.

# Fractal synthesis

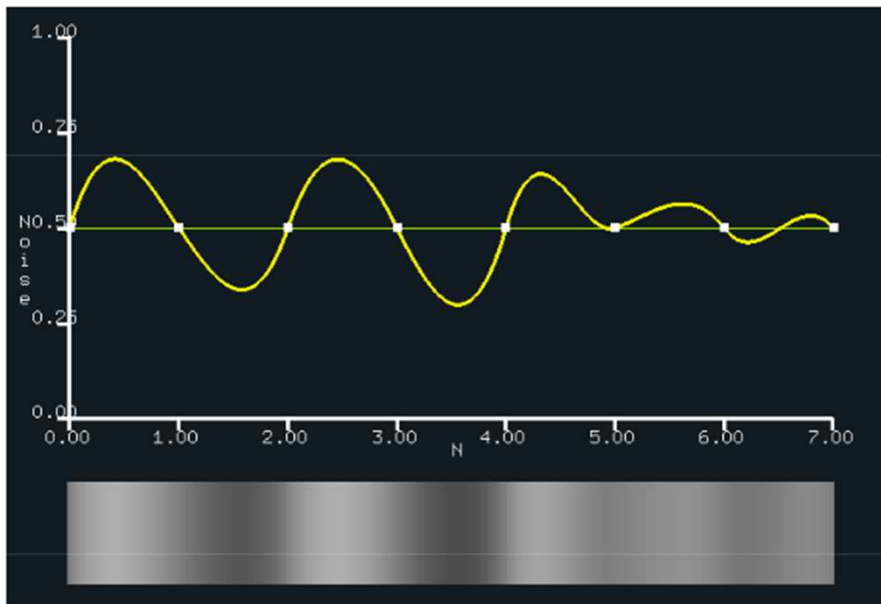
## Gradient Noise

**Idea:** Place points at the mid-line at the whole-number input values use random numbers to pick gradients (slopes) there, and then fit a piecewise smooth curve through those points with those slopes.

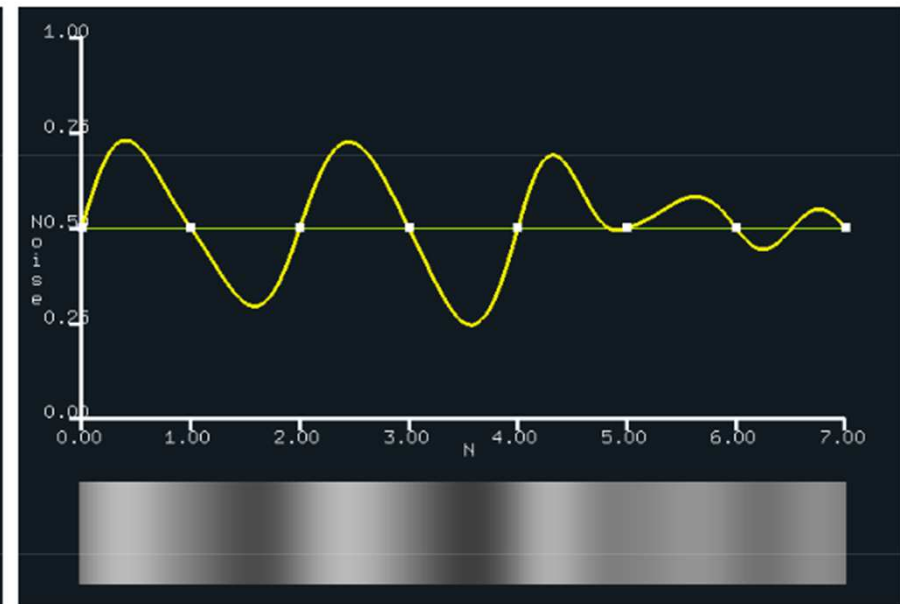


No matter what, you will get a good plus-or-minus distribution.

# Fractal synthesis

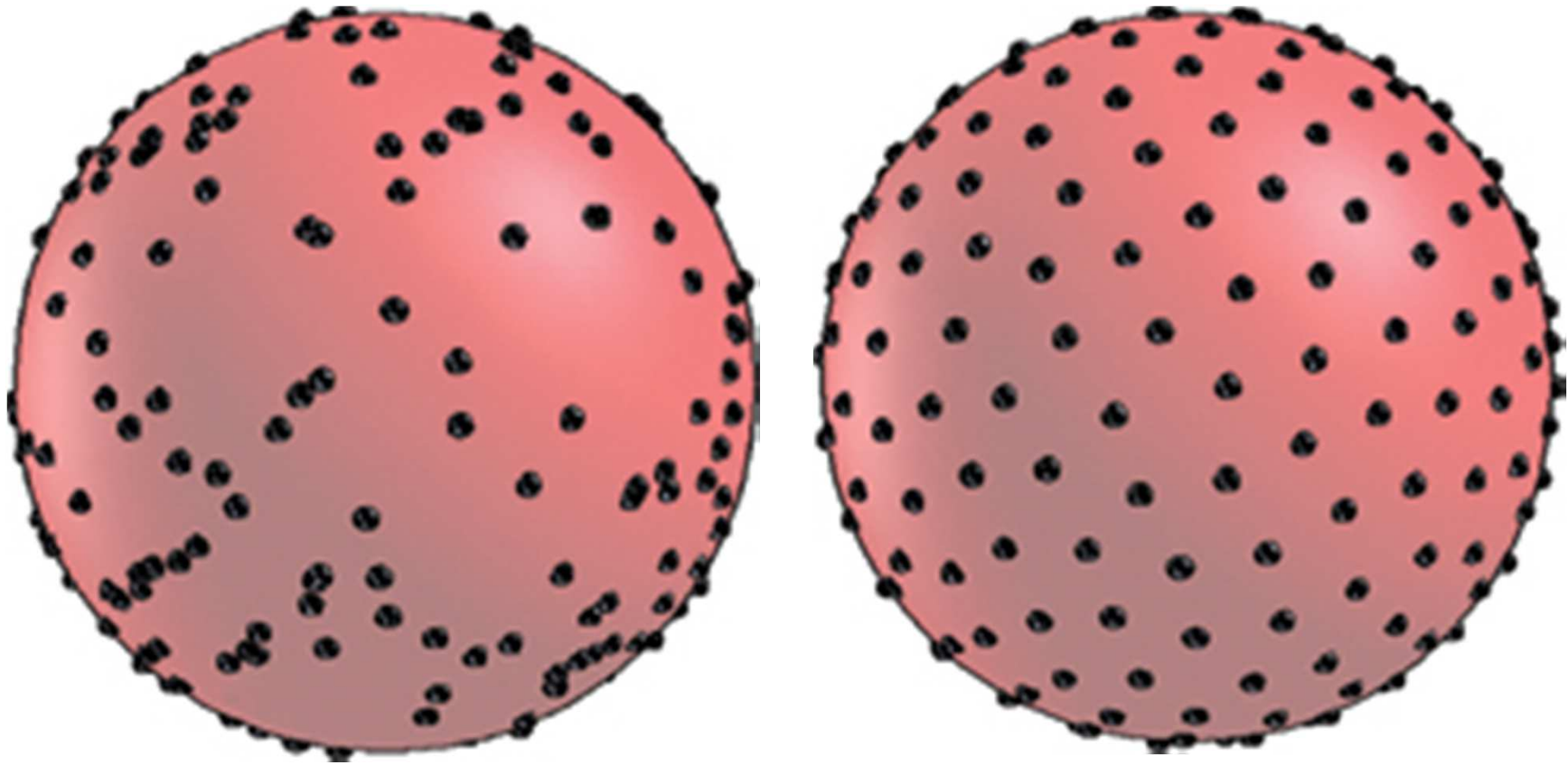


Cubic:  $C^1$  continuity at the whole-number values



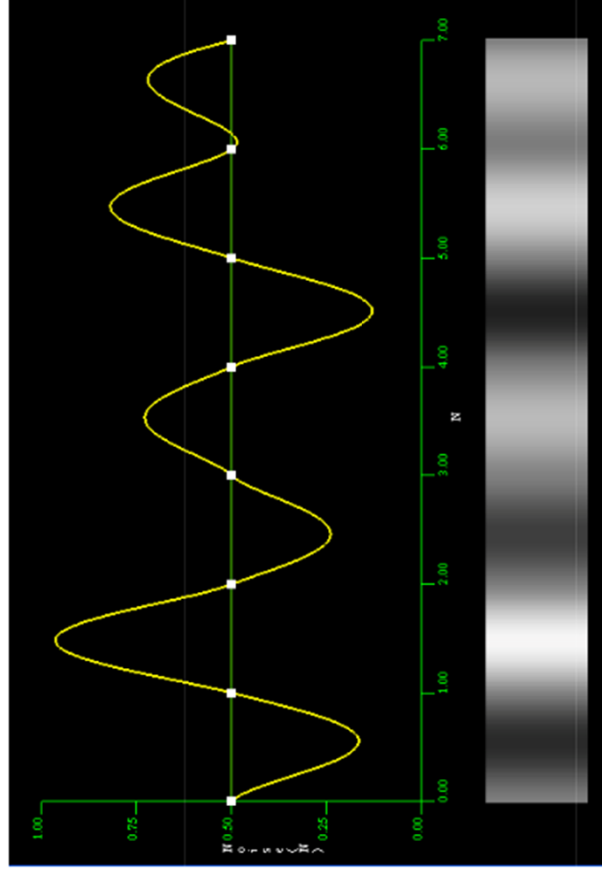
Quintic:  $C^2$  continuity at the whole-number values

# Fractal synthesis

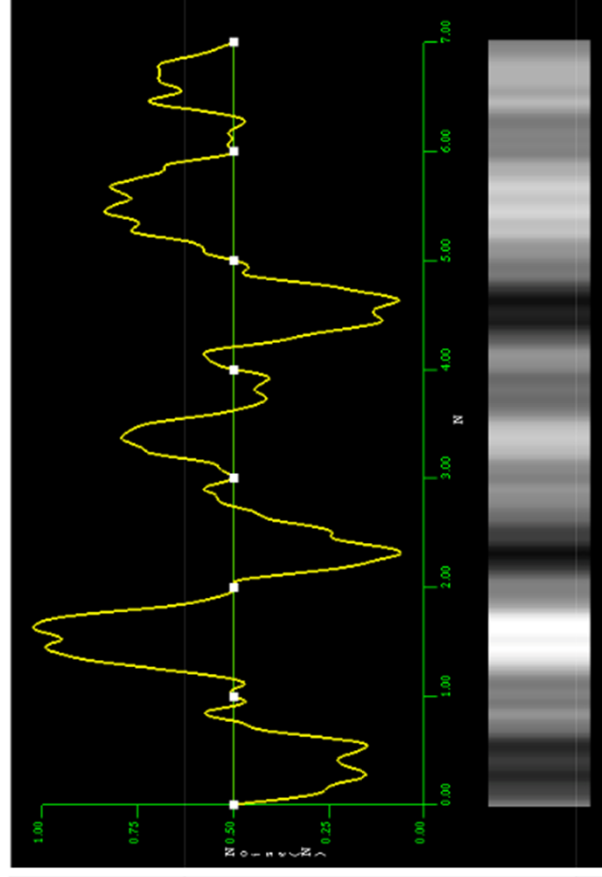


## Noise Octaves

**Idea:** Add multiple noise waves, each one twice the frequency and half the amplitude of the previous one



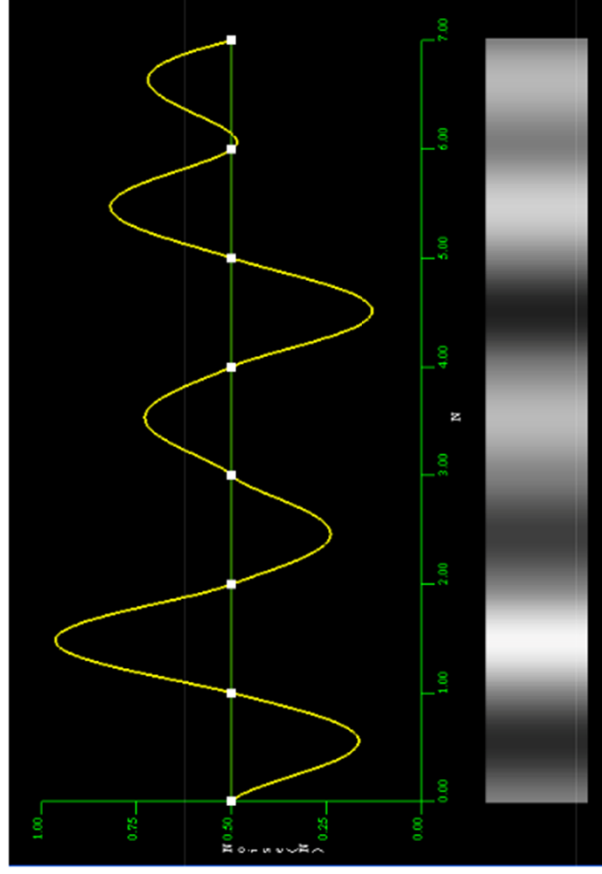
1 Octave



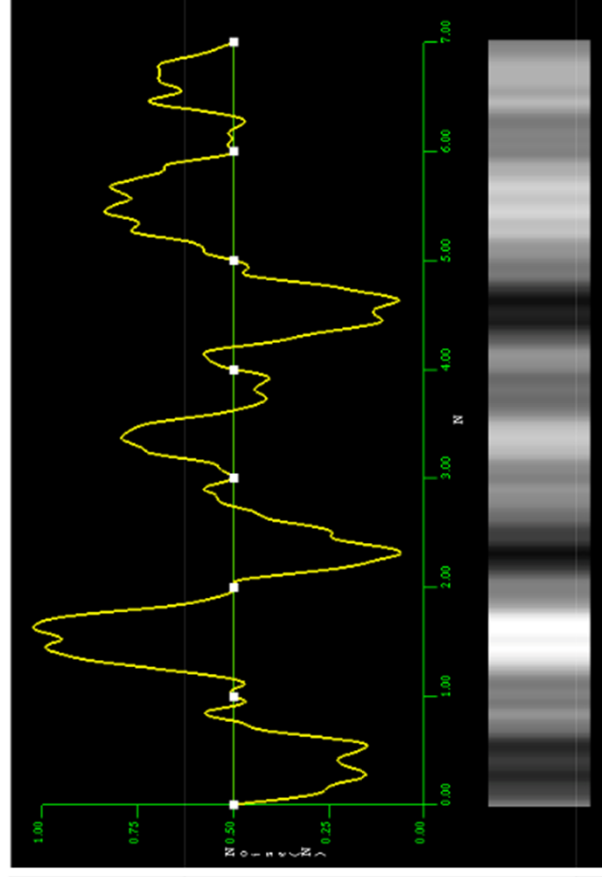
4 Octaves

## Noise Octaves

**Idea:** Add multiple noise waves, each one twice the frequency and half the amplitude of the previous one

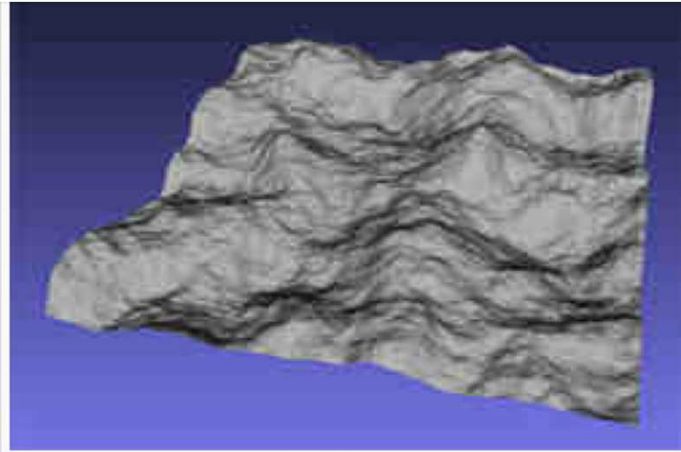


1 Octave

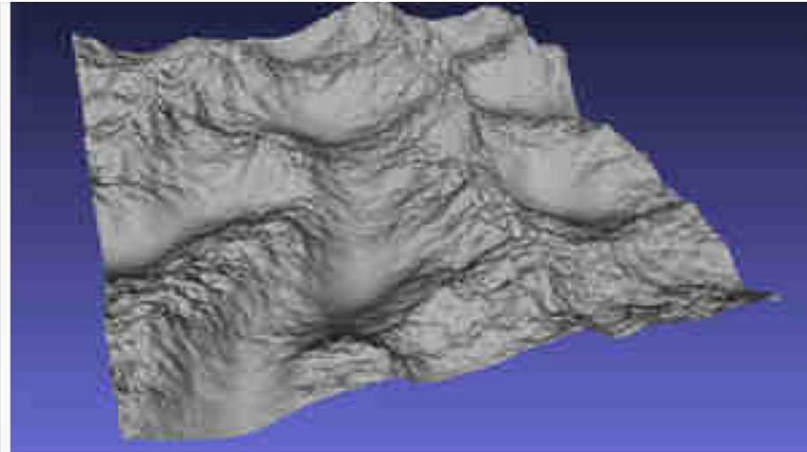


4 Octaves

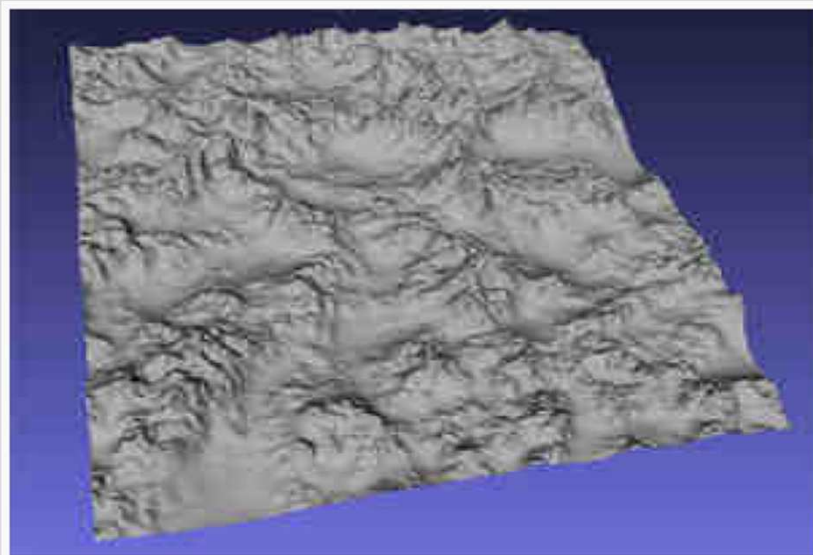
# More fractal functions



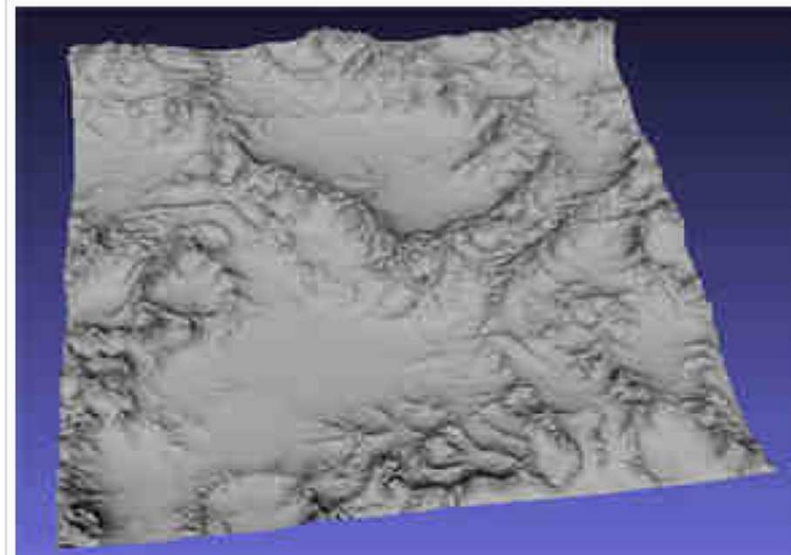
FBM terrain



Heterogeneous multifractal terrain



Hybrid multifractal terrain



Ridged multifractal terrain