

Unit 5

BRep queries

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Volume

$$Vol = \iiint_V dV$$

Divergence theorem



$$Vol = \iint_S xn_x dS = \iint_S yn_y dS = \iint_S zn_z dS$$



Center of mass

$$G = \frac{1}{Vol} \left(\iiint_V x dV, \iiint_V y dV, \iiint_V z dV \right)$$

Div.
theorem

$$G = \frac{1}{2Vol} \left(\iint_S x^2 n_x dS, \iint_S y^2 n_y dS, \iint_S z^2 n_z dS \right)$$



Inertia tensor

$$I'_{xx} = \rho \iiint_V (y^2 + z^2) dV$$

Divergence theorem



$$I'_{xx} = \rho \frac{1}{3} \iint_S (y^3 n_y + z^3 n_z) dS$$

Inertia tensor

$$I'_{xx} = \sum_{i=1}^N \rho_i \int_{B_i} (y^2 + z^2) dV$$

$$I'_{yy} = \sum_{i=1}^N \rho_i \int_{B_i} (z^2 + x^2) dV$$

$$I'_{zz} = \sum_{i=1}^N \rho_i \int_{B_i} (x^2 + y^2) dV$$

$$I'_{xy} = I'_{yx} = \sum_{i=1}^N \rho_i \int_{B_i} xy dV$$

$$I'_{yz} = I'_{zy} = \sum_{i=1}^N \rho_i \int_{B_i} yz dV$$

$$I'_{zx} = I'_{xz} = \sum_{i=1}^N \rho_i \int_{B_i} zx dV$$



Inertia tensor

$$I_{xx} = I'_{xx} - m(r_y^2 + r_z^2)$$

$$I_{yy} = I'_{yy} - m(r_z^2 + r_x^2)$$

$$I_{zz} = I'_{zz} - m(r_x^2 + r_y^2)$$

$$I_{xy} = I'_{xy} - mr_x r_y$$

$$I_{yz} = I'_{yz} - mr_y r_z$$

$$I_{zx} = I'_{zx} - mr_z r_x.$$

$$(r_x, r_y, r_z) = (G_x, G_y, G_z)$$

Volume and center of mass

```
Vol = 0.; Gx = 0.; Gy = 0.; Gz = 0.;
```

```
per cada cara c fer
```

```
per cada poligon p de la cara fer
```

```
Copiar els vèrtexs del poligon a la taula V[1]..V[p.np]
```

```
i = 1
```

```
j = 2
```

```
/* Tractem el triangle V[i], V[j], V[k] */
```

```
per k en [3..p.np] fer
```

```
nx = (V[j].y - V[i].y) * (V[k].z - V[j].z)
```

```
      -(V[k].y - V[j].y) * (V[j].z - V[i].z);
```

```
ny = (V[k].x - V[j].x) * (V[j].z - V[i].z)
```

```
      -(V[j].x - V[i].x) * (V[k].z - V[j].z);
```

```
nz = (V[j].x - V[i].x) * (V[k].y - V[j].y)
```

```
      -(V[k].x - V[j].x) * (V[j].y - V[i].y);
```

```
/* bucle d'integració gaussiana en el triangle */
```

```
per l en [1..4] fer
```

```
si l = 1 llavors w = -27. / 96.;
```

```
sino w = 25. / 96.;
```

```
fsi
```

```
x = (M[1,1]*V[i].x + M[1,2]*V[j].x + M[1,3]*V[k].x) / 15.;
```

```
y = (M[1,1]*V[i].y + M[1,2]*V[j].y + M[1,3]*V[k].y) / 15.;
```

```
z = (M[1,1]*V[i].z + M[1,2]*V[j].z + M[1,3]*V[k].z) / 15.;
```

```
Vol = Vol + w * ( x*nx + y*ny + z*nz ) / 3.;
```

```
Gx = Gx + w * ( x*x*nx );
```

```
Gy = Gy + w * ( y*y*ny );
```

```
Gz = Gz + w * ( z*z*nz );
```

```
Ix = Ix + w * ( y*y*y*ny + z*z*z*nz ) / 3.;
```

```
fiper
```

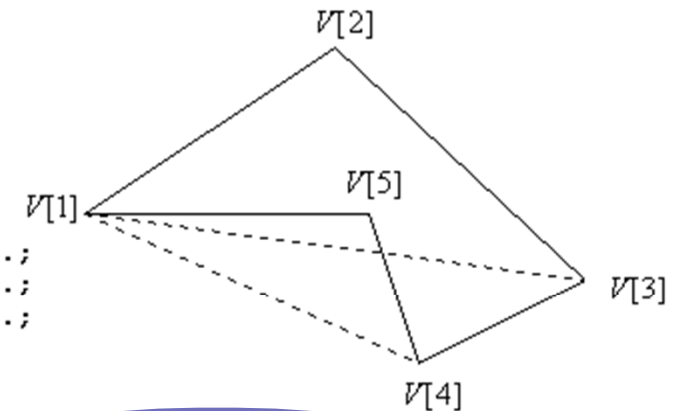
```
j = k;
```

```
fiper
```

```
fiper
```

```
fiper
```

$$M = \begin{pmatrix} 5 & 5 & 5 \\ 11 & 2 & 2 \\ 2 & 11 & 2 \\ 2 & 2 & 11 \end{pmatrix}$$



Final division of G by
2Vol is missing; Ixx
assumes constant
density



References

Brian Mirtich, *Fast and accurate computation of polyhedral mass properties*, Journal of Graphics Tools, 1(2), 31-50, 1996.

David Eberly, *Polyhedral Mass Properties (Revisited)*, Technical Report, Geometric Tools, LLC, www.geometrictools.com/ (2009)