CSG

Carlos Andujar

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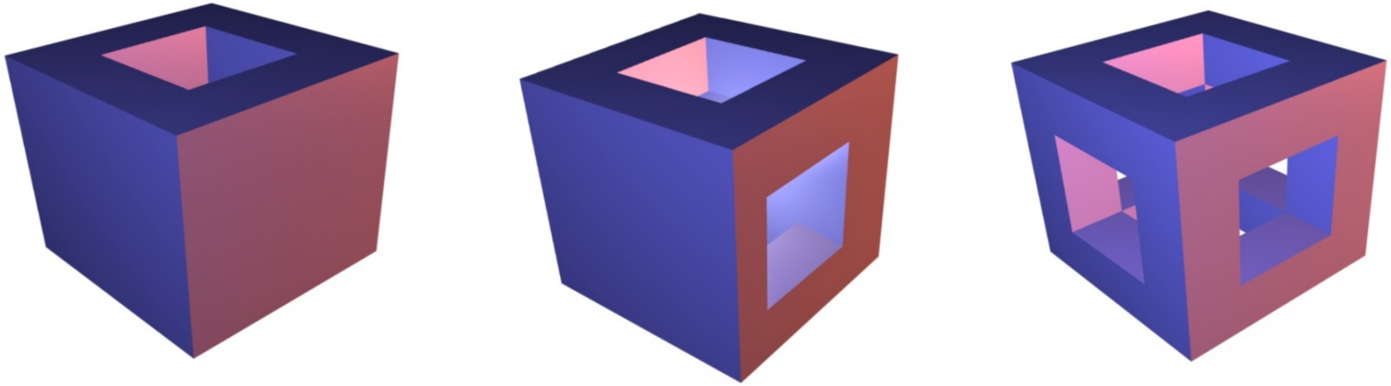
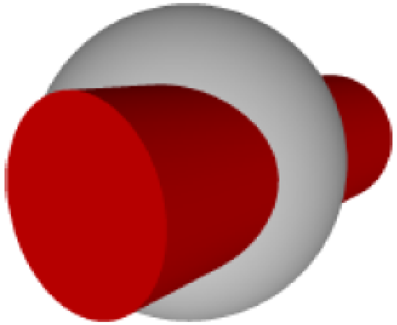
Some images by Florian Kirsch, HPI, University of Potsdam

CSG

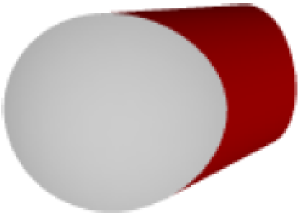


CSG

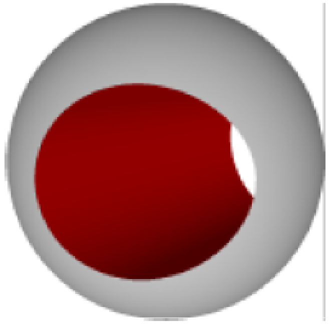
- Union (∪)



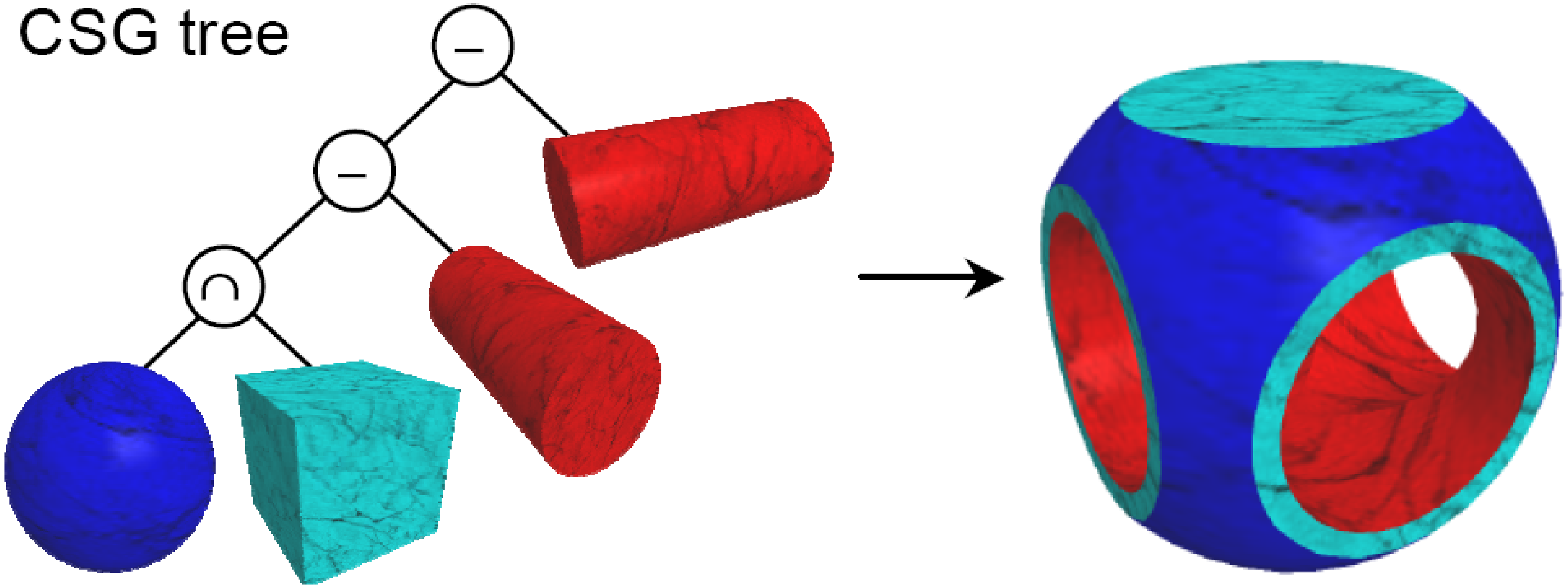
- Intersection (∩)



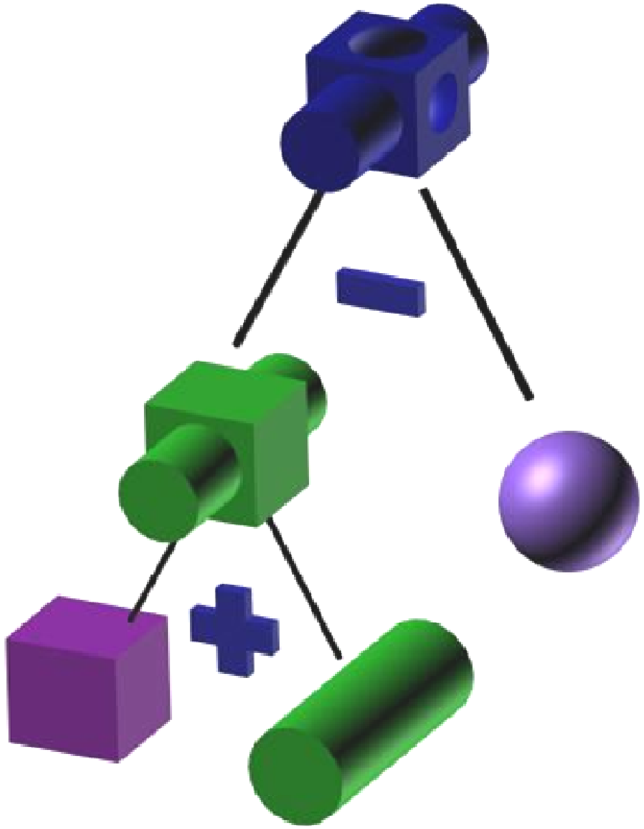
- Subtraction (-)

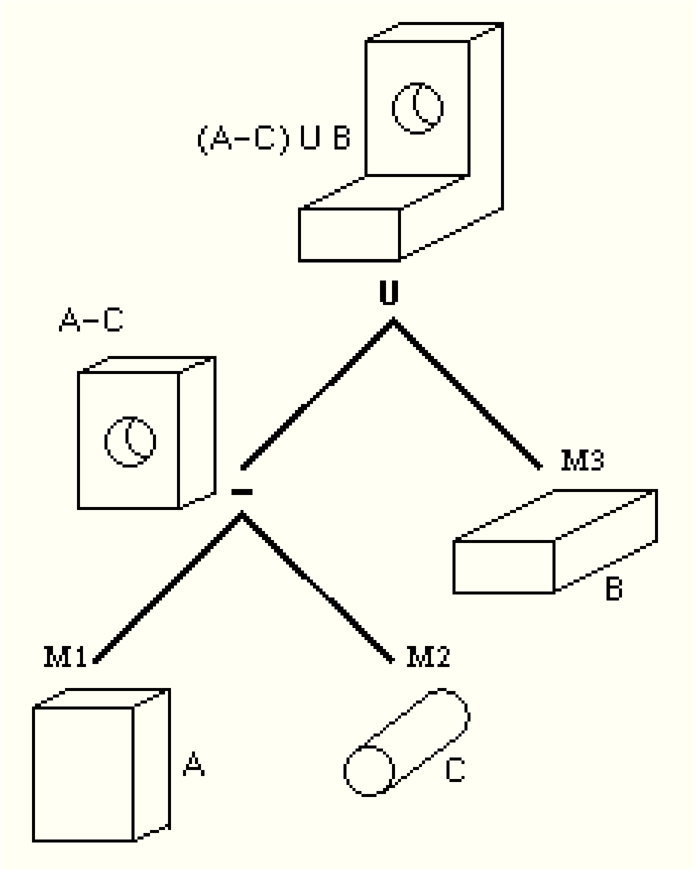


- CSG tree

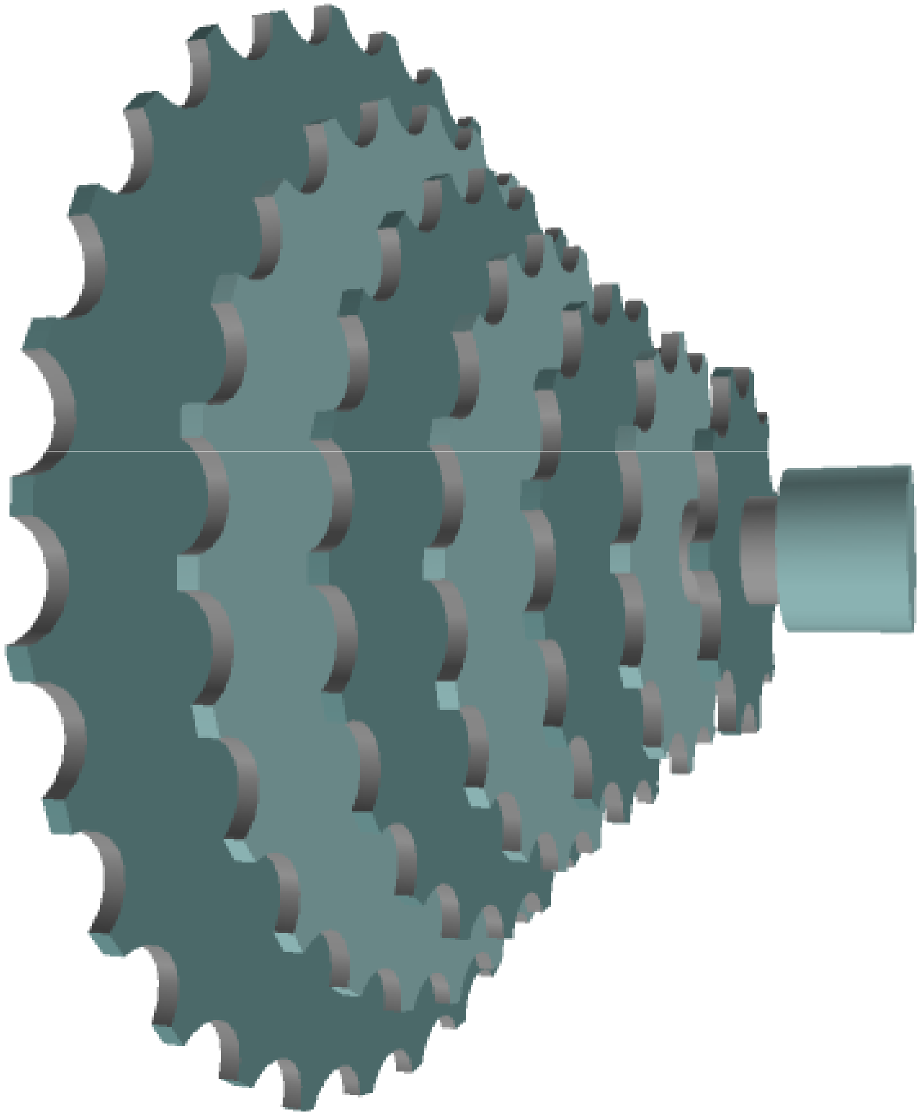


CSG tree



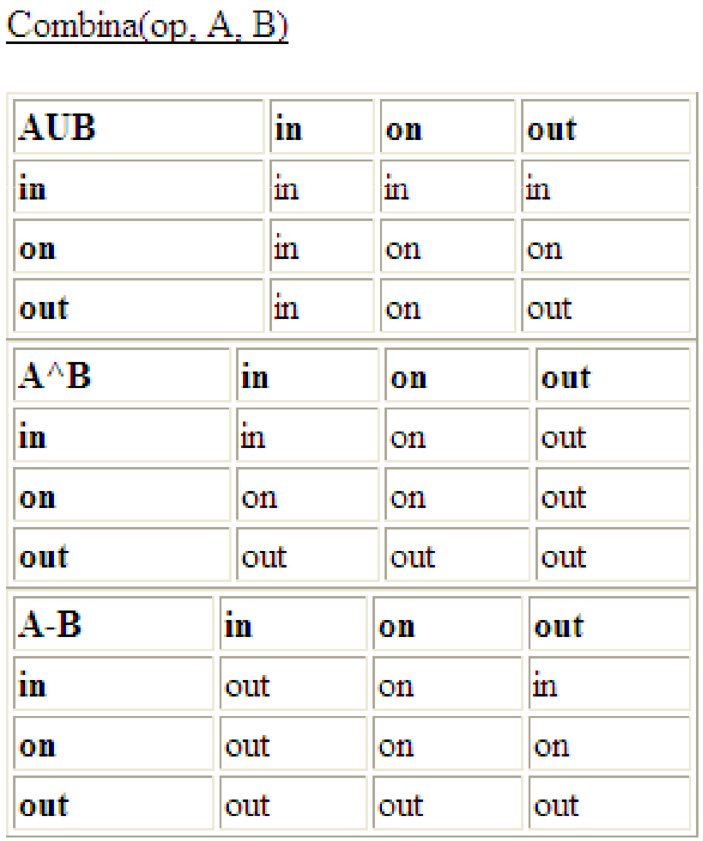


Compact



Point-inside-solid test (for CSG)

```
function classify(P:point, n:nodeCSG) return InOnOut
    si isLeaf(n) then
      case (n.type)
            Box: r:=classifyBox(P,n)
            Cylinder: r:=classifyCylinder(P,n)
            Sphere: r:=classifySphere(P,n)
            ...
    else
            rA:= classify (P, n.left)
            rB:= classify (P, n.right)
            r:= combine(n.operation, rA, rB)
    end
    return r
end
```



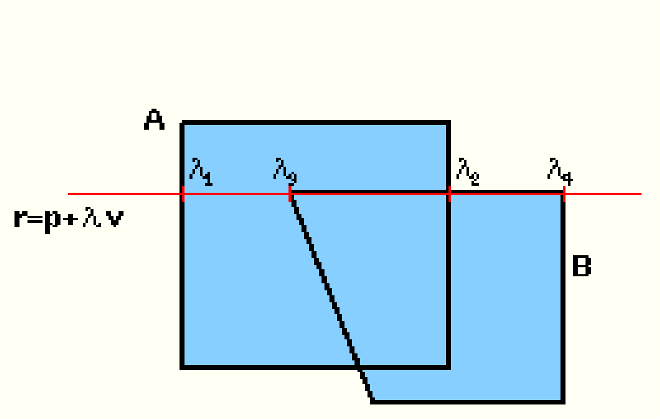
Combina(op. A, B)

AUB	**in**	**on**	**out**
in	in	in	in
on	in	on	on
out	in	on	out

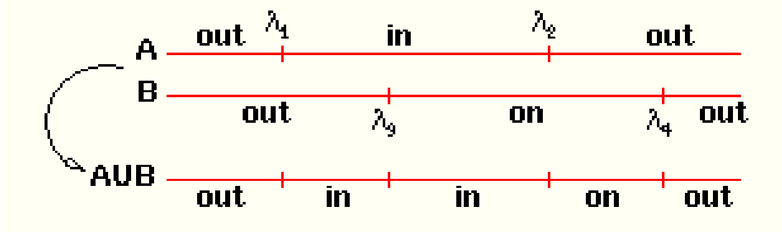
A^B	**in**	**on**	**out**
in	in	on	out
on	on	on	out
out	out	out	out

A-B	**in**	**on**	**out**
in	out	on	in
on	out	on	on
out	out	out	out

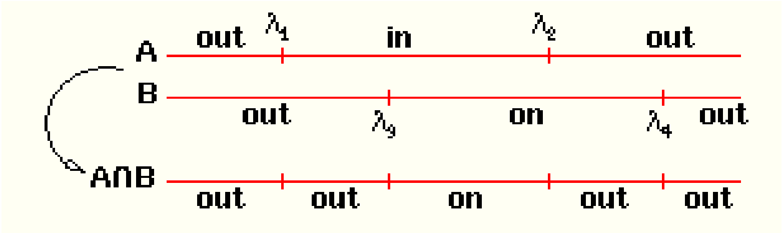
Line-solid classification



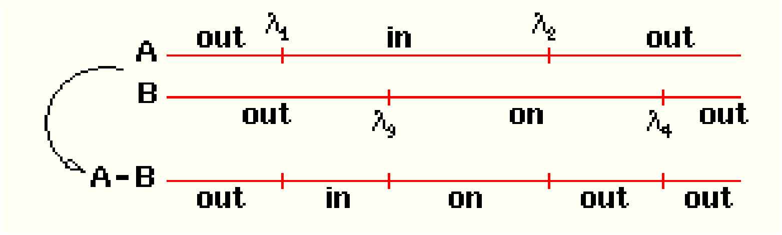
Sòlid A : [l1,in] [l2,out]
Sòlid B : [l3,on] [l4,out]



Resultat de la unió : [l1,in] [l2,on] [l4,out]
(s'han hagut de compactar dos intèrvals "in")

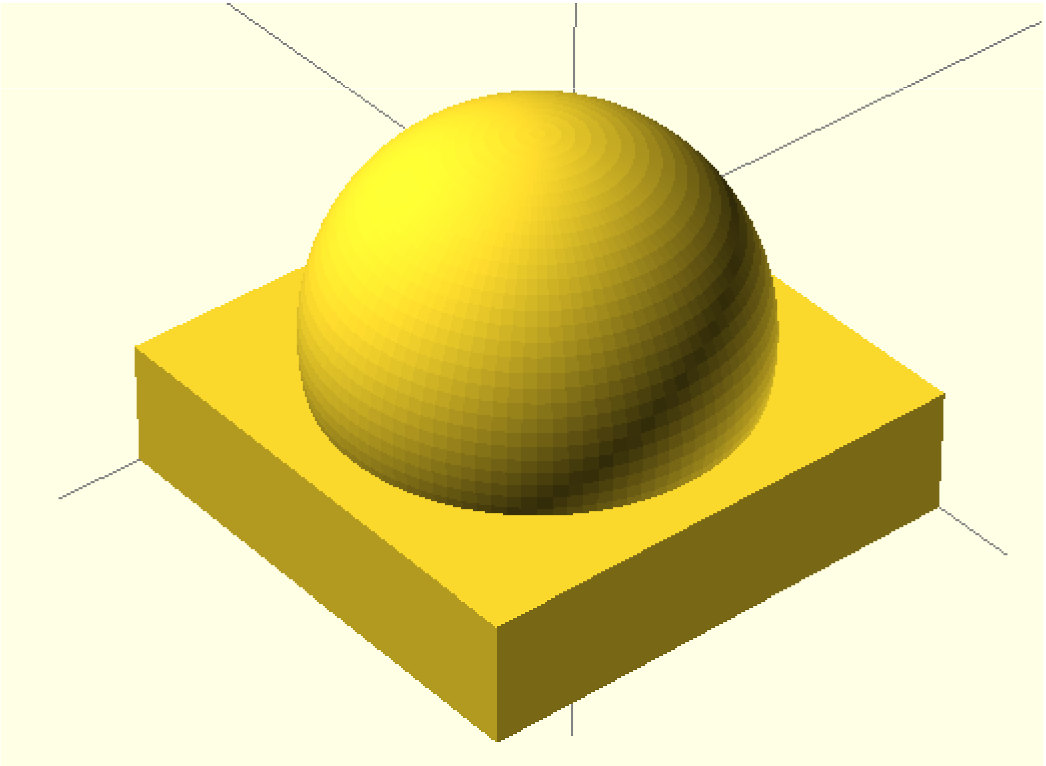


Resultat de la intersecció : [l1,out] [l3,on] [l2,out]
(s'han hagut de compactar dos intèrvals "out")



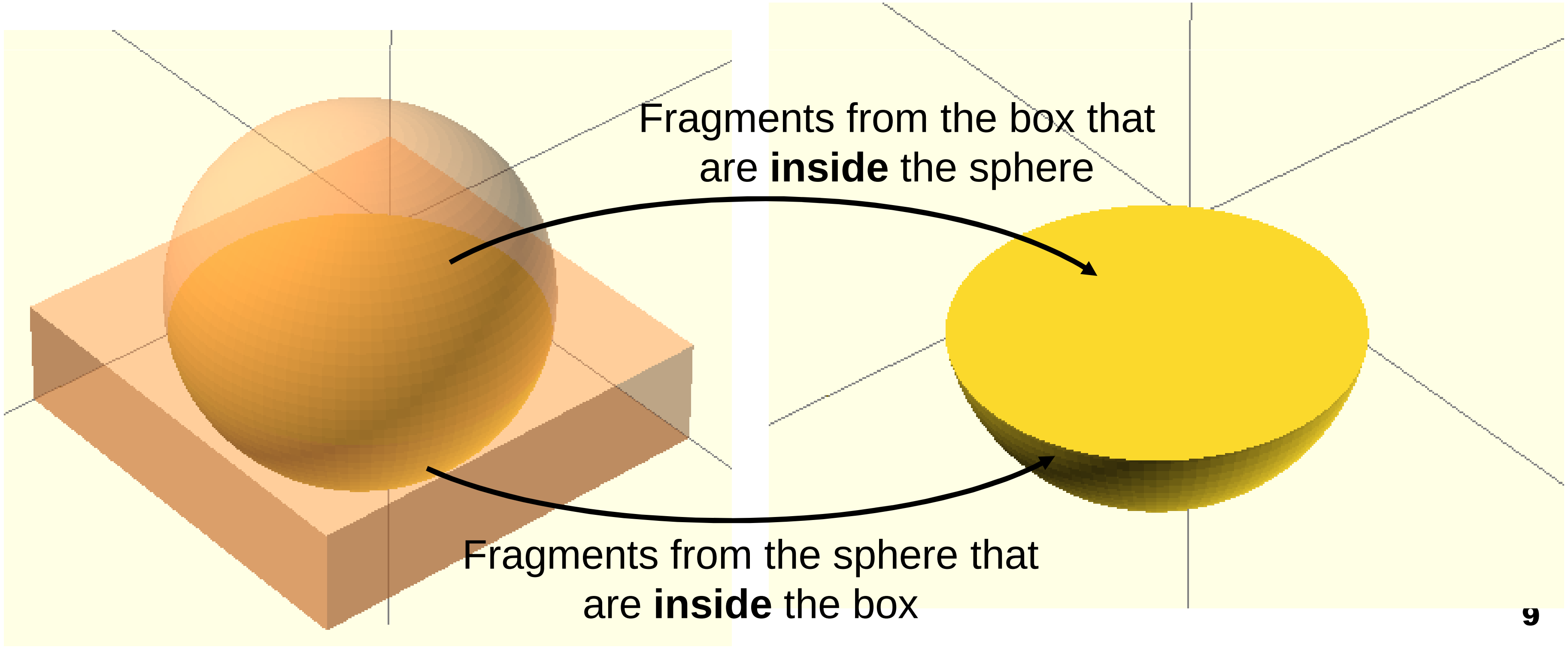
CSG rendering - union

- Visible fragments of A∪B = visible fragments produced by rendering A and B (using depth test)
- Rendering (an image equivalent to) A∪B is straightforward



CSG rendering - intersection

- Visible fragments of A∩B = **subset** of the fragments produced by **front faces** of A and **front faces** of B
- A ∩ B can be rendered using e.g. parity count.

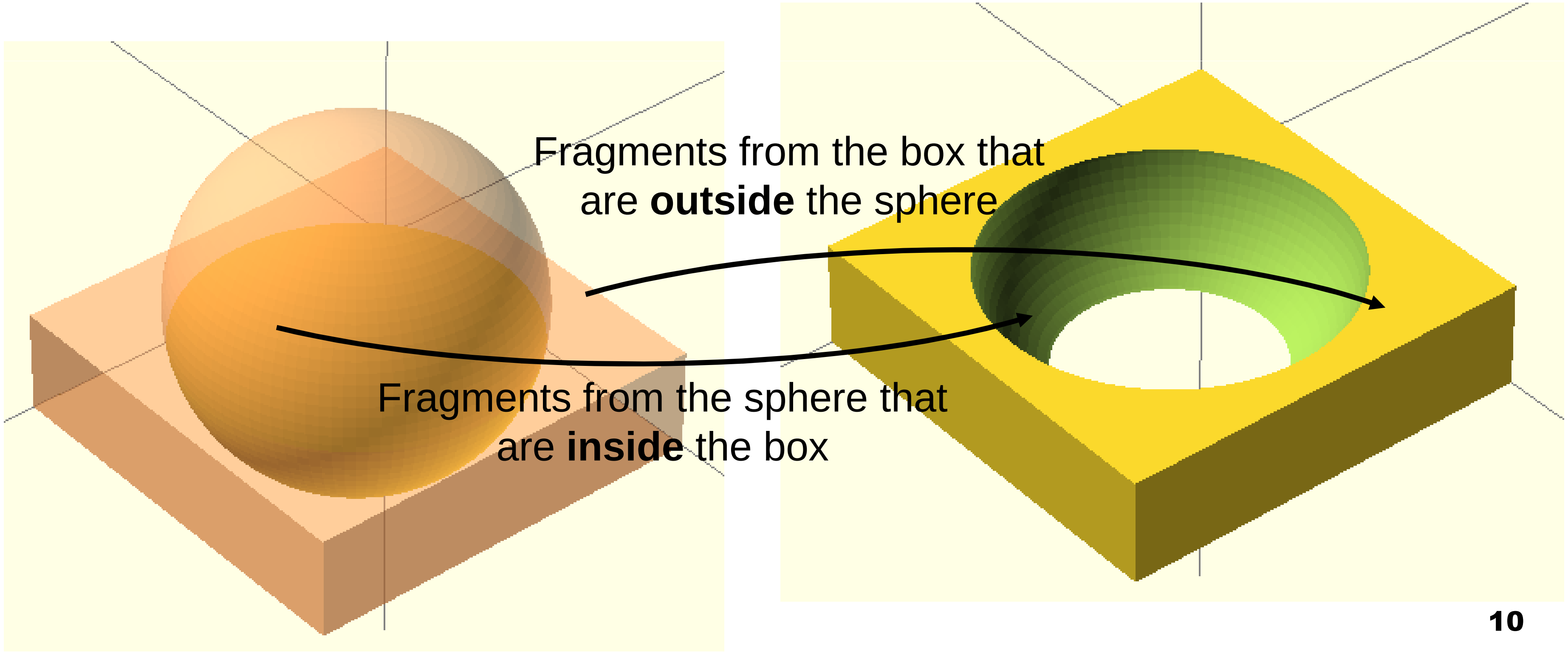


Fragments from the box that are **inside** the sphere

Fragments from the sphere that are **inside** the box

CSG rendering - difference

- Visible fragments of A-B = **subset** of the fragments produced by **front faces** of A and **back faces** of B
- A - B can be rendered using e.g. parity count.



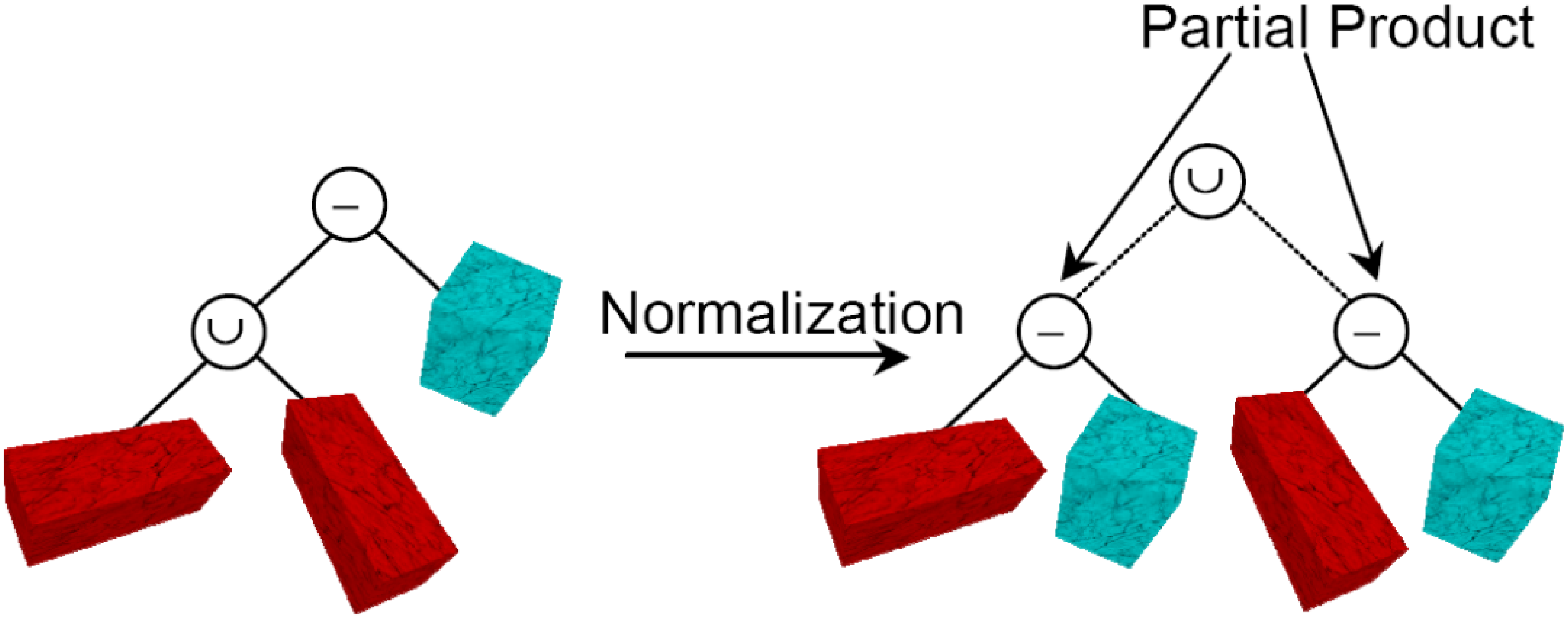
Fragments from the box that are **outside** the sphere

Fragments from the sphere that are **inside** the box

OpenSCAD demos

- cexample000.scad
- cexample001.scad
- cexample002.scad
- cHinge.scad
- worm_Gear.scad

CSG Normalization



[Image credit: Florian Kirsch]

CSG Normalization

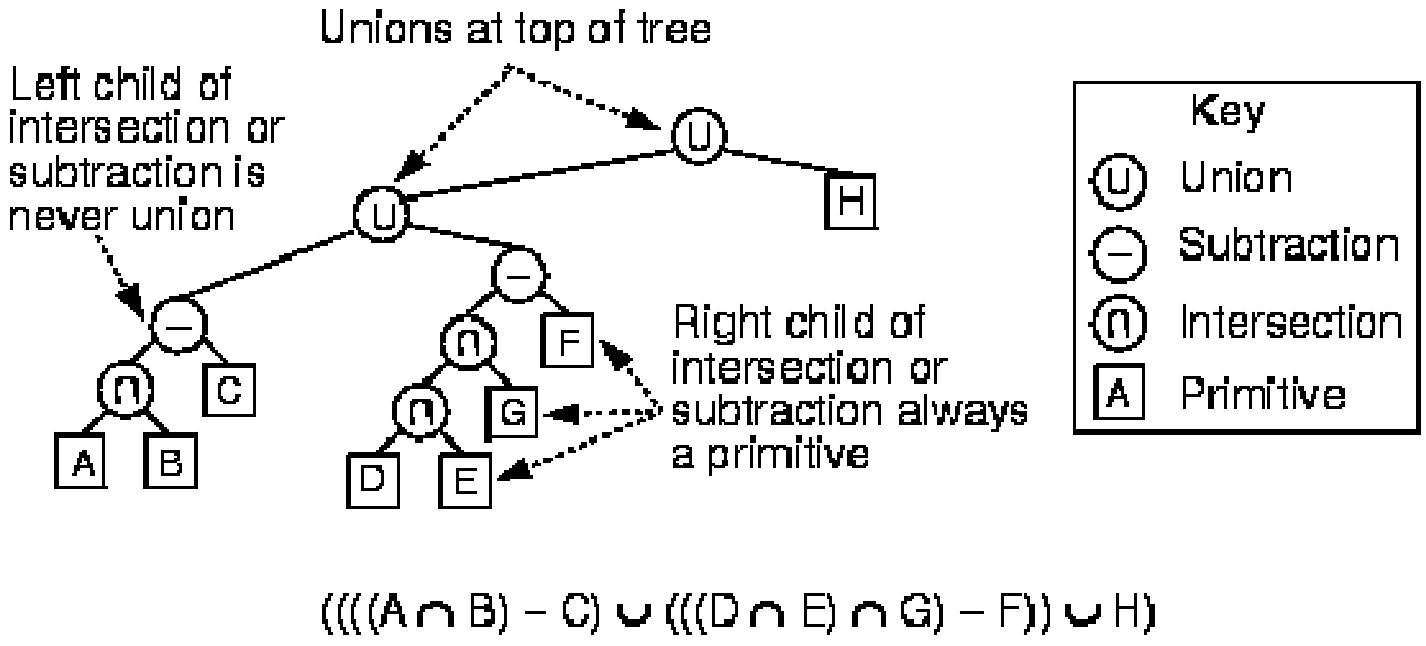

Unions at top of tree
Left child of intersection or subtraction is never union
U
U
H
–
–
∩
C
F
∩
∩
G
A
B
D
E
Right child of intersection or subtraction always a primitive
Key
U Union
– Subtraction
∩ Intersection
A Primitive
((((A ∩ B) – C) ∪ (((D ∩ E) ∩ G) – F)) ∪ H)

Figure 5. A CSG tree in normal form

CSG Normalization

1. $X - (Y \cup Z) \rightarrow (X - Y) - Z$
2. $X \cap (Y \cup Z) \rightarrow (X \cap Y) \cup (X \cap Z)$ ←
3. $X - (Y \cap Z) \rightarrow (X - Y) \cup (X - Z)$ ←
4. $X \cap (Y \cap Z) \rightarrow (X \cap Y) \cap Z$
5. $X - (Y - Z) \rightarrow (X - Y) \cup (X \cap Z)$ ←
6. $X \cap (Y - Z) \rightarrow (X \cap Y) - Z$
7. $(X - Y) \cap Z \rightarrow (X \cap Z) - Y$
8. $(X \cup Y) - Z \rightarrow (X - Z) \cup (Y - Z)$ ←
9. $(X \cup Y) \cap Z \rightarrow (X \cap Z) \cup (Y \cap Z)$ ←

Push unions towards
the root

CSG Normalization

1. $X - (Y \cup Z) \rightarrow (X - Y) - Z$
2. $X \cap (Y \cup Z) \rightarrow (X \cap Y) \cup (X \cap Z)$
3. $X - (Y \cap Z) \rightarrow (X - Y) \cup (X - Z)$
4. $X \cap (Y \cap Z) \rightarrow (X \cap Y) \cap Z$
5. $X - (Y - Z) \rightarrow (X - Y) \cup (X \cap Z)$
6. $X \cap (Y - Z) \rightarrow (X \cap Y) - Z$
7. $(X - Y) \cap Z \rightarrow (X \cap Z) - Y$
8. $(X \cup Y) - Z \rightarrow (X - Z) \cup (Y - Z)$
9. $(X \cup Y) \cap Z \rightarrow (X \cap Z) \cup (Y \cap Z)$

Eliminate redundant unions

CSG Normalization

1. $X - (Y \cup Z) \rightarrow (X - Y) - Z$
2. $X \cap (Y \cup Z) \rightarrow (X \cap Y) \cup (X \cap Z)$
3. $X - (Y \cap Z) \rightarrow (X - Y) \cup (X - Z)$
4. $X \cap (Y \cap Z) \rightarrow (X \cap Y) \cap Z$
5. $X - (Y - Z) \rightarrow (X - Y) \cup (X \cap Z)$
6. $X \cap (Y - Z) \rightarrow (X \cap Y) - Z$
7. $(X - Y) \cap Z \rightarrow (X \cap Z) - Y$
8. $(X \cup Y) - Z \rightarrow (X - Z) \cup (Y - Z)$
9. $(X \cup Y) \cap Z \rightarrow (X \cap Z) \cup (Y \cap Z)$

Rearrange “products”