

# Master in Artificial Intelligence

Syntactic  
parsers  
CKY  
algorithm

## Introduction to Human Language Technologies

### 9. Syntactic parsing: parsers



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# Outline

Syntactic  
parsers

CKY  
algorithm

- 1 Syntactic parsers
  - Background
  - Chart-based methods

- 2 CKY algorithm

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# Factors in parsing

Parsing performance depends on many aspects:

- Grammar expressivity (combination of symbols)
- Coverage (words)
- Parsing strategy (bottom-up, top-down)
- Rule application order (largest rule, most likely rule)
- Ambiguity management (keep all, select one - probabilities, semantics, pragmatics)
- ...

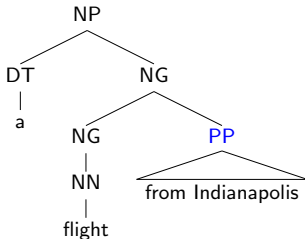
# The problem of repeating derivations

- Top-down and bottom-up strategies both lead to repeated derivations when using backtracking

Ex: "a flight from Indianapolis to Houston [on TWA...]"

NG  $\rightarrow$  NN

NG  $\rightarrow$  NG PP



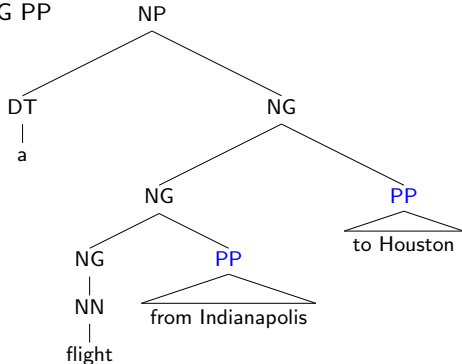
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# Properties

Syntactic  
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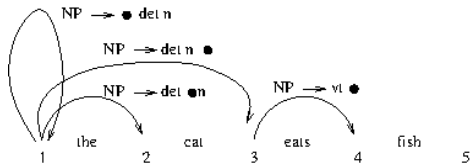
- They avoid re-doing derivations using dynamic programming.
- They represent derivations as a directed graph named **chart**.
- They use a dynamic programming table to build the chart.



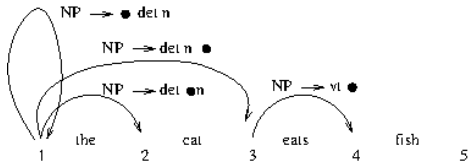
# Chart

- Nodes: positions between words of the input sentence
- Edges: **dotted rules** subsuming a sequence of words of the input sentence
- Dotted rules represent rules states:
  - Passive rules:  $A \rightarrow B_1 \dots B_k \bullet$
  - Active rules:  $A \rightarrow B_1 \dots B_i \bullet B_{i+1} \dots B_k$

Ex:



# Chart as a dynamic programming table



					[1,5]
				[1,4]	[2,5]
		[1,3]		[2,4]	[3,5]
	[1,2]	NP → det • n		[3,4]	[4,5]
[1,1]	NP → • det n	[2,3]		[4,4]	[5,5]
1	2	3	4	5	
	the	cat	eats	fish	

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# Popular chart-based algorithms

- **CKY algorithm** (Younger, 1967)
  - introduced dynamic programming
  - limited to CFGs in Chomsky Normal Form
  - passive bottom-up chart parser (only passive rules)
  - straightforward probabilistic version
- Earley algorithm (Earley, 1970)
  - any CFG
  - active top-down parser (active/passive rules)
  - non-straightforward probabilistic version
- Generalized chart parsing (Kay, 1980)

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# Chomsky Normal Form (CNF)

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A CFG  $G = (N, \Sigma, R, S)$  expressed in CNF is as follows:

- $N$  is a set of non-terminal symbols
- $\Sigma$  is a set of terminal symbols
- $R$  is a set of rules which take one of two forms:
  - $X \rightarrow Y_1 Y_2$  for  $X, Y_1, Y_2 \in N$
  - $X \rightarrow \alpha$  for  $X \in N$  and  $\alpha \in \Sigma$
- $S \in N$  is a start symbol

Any CFG can be converted into CNF

# CNF conversion

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- 1 Convert Hybrid rules: replace terminals with new non-terminals

Ex:  $INF\_VP \rightarrow to\ VP \implies$   
 $INF\_VP \rightarrow TO\ VP$   
 $TO \rightarrow to$

- 2 Convert non-binary rules:

Ex:  $S \rightarrow VP\ NP\ PP \implies$   
 $S \rightarrow VP\ X$   
 $X \rightarrow NP\ PP$

- 3 Convert unit productions:  $A \rightarrow^* B$  and  $B \rightarrow \alpha \implies A \rightarrow \alpha$

Ex:  $NP \rightarrow N$  and  $N \rightarrow dog \implies NP \rightarrow dog$

# Exercise

Convert the following CFG to CNF

- 1  $S \rightarrow NP VP$
- 2  $NP \rightarrow det\ n$
- 3  $NP \rightarrow n$
- 4  $VP \rightarrow vt\ NP\ PP$
- 5  $VP \rightarrow vi$
- 6  $PP \rightarrow with\ NP$
- 7  $det \rightarrow the|a$
- 8  $n \rightarrow cat|fish|knife$
- 9  $vt \rightarrow eats$
- 10  $vi \rightarrow eats$

# CKY Algorithm

Chart content:

- Maximum probability of a subtree with root  $X$  spanning words  $i \dots j$ :

$$\pi(i, j, X)$$

- Backpath to recover which rules produced the maximum probability tree:

$$\psi(i, j, X)$$

The goal is to compute:

- $\max_{t \in \mathcal{T}(s)} p(t) = \pi(1, n, S)$
- $\psi(1, n, S)$
- It is possible to use it without probabilities to get all parse trees (with higher complexity)



# CKY Algorithm

Base case: Tree leaves

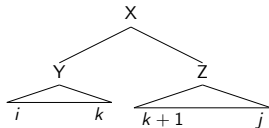
$$\blacksquare \forall i = 1 \dots n, \forall X \rightarrow w_i \in R, \pi(i, i, X) = q(X \rightarrow w_i)$$

Recursive case: Non-terminal nodes

$$\blacksquare \forall i = 1 \dots n, \forall j = (i + 1) \dots n, \forall X \in N$$

$$\pi(i, j, X) = \max_{\substack{X \rightarrow YZ \in R \\ k: i \leq k < j}} q(X \rightarrow YZ) \times \pi(i, k, Y) \times \pi(k + 1, j, Z)$$

$$\psi(i, j, X) = \arg \max_{\substack{X \rightarrow YZ \in R \\ k: i \leq k < j}} q(X \rightarrow YZ) \times \pi(i, k, Y) \times \pi(k + 1, j, Z)$$

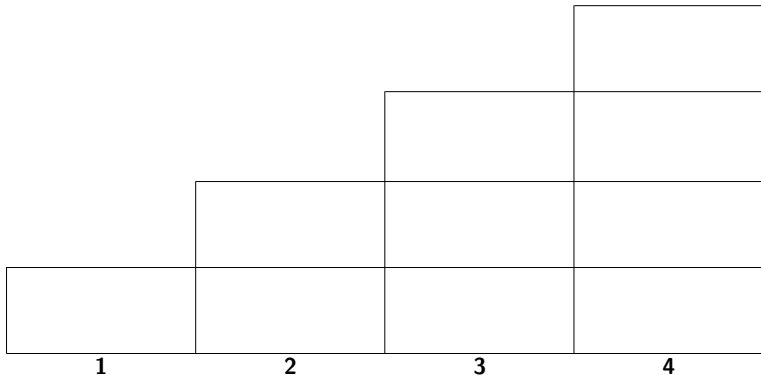


Output:

$$\blacksquare \text{Return } \pi(1, n, S) \text{ and recover backpath through } \psi(1, n, S)$$

## Syntactic parsers

## CKY algorithm



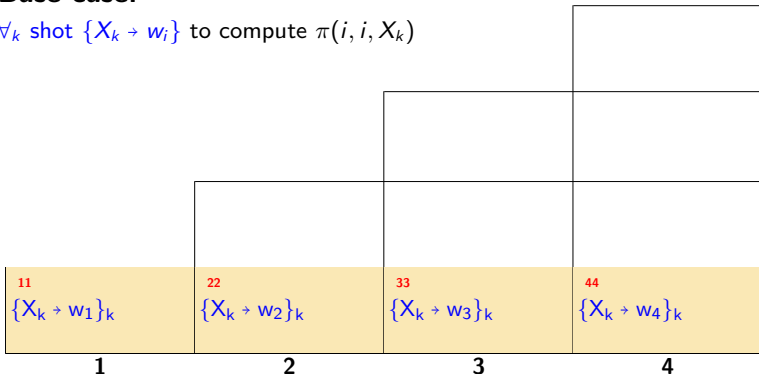
# CKY Algorithm

Suppose  $s = w_1 w_2 w_3 w_4$  and  $G = \langle N, \Sigma, S, R, q \rangle$  a PCFG

$$R = \{X_k \rightarrow Y_s Z_t\} \cup \{X_k \rightarrow \alpha\}$$

**Base case:**

$\forall_k$  shot  $\{X_k \rightarrow w_i\}$  to compute  $\pi(i, i, X_k)$



# CKY Algorithm

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**Recursive case:**

$\forall_k$  shot  $\{X_k \rightarrow Y_s Z_t\}$  to get  $\varphi(i, j, X_k)$  and  
compute  $\pi(i, j, X_k)$

	12	23	34	
11 $\{X_k \rightarrow w_1\}_k$	22 $\{X_k \rightarrow w_2\}_k$	33 $\{X_k \rightarrow w_3\}_k$	44 $\{X_k \rightarrow w_4\}_k$	
1	2	3	4	

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			13	24
		12	23	34
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			14
		13	24
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Example for (1,3)

		13 $\{X_k \rightarrow Y_{s,11} Z_{t,23}\}_k$ 23	24
	12		34
11 $\{X_k \rightarrow w_1\}_k$	22 $\{X_k \rightarrow w_2\}_k$	33 $\{X_k \rightarrow w_3\}_k$	44 $\{X_k \rightarrow w_4\}_k$
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**Recursive case:**

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compute  $\pi(i, j, X_k)$

Example for (1,3)

		13 $\{X_k \rightarrow Y_{s,12} Z_{t,33}\}_k$	24
	12	23	34
11 $\{X_k \rightarrow w_1\}_k$	22 $\{X_k \rightarrow w_2\}_k$	33 $\{X_k \rightarrow w_3\}_k$	44 $\{X_k \rightarrow w_4\}_k$
1	2	3	4



# CKY Algorithm

**Input:** a sentence  $s = x_1 \dots x_n$ , a PCFG  $G = (N, \Sigma, S, R, q)$ .

**Initialization:**

For all  $i \in \{1 \dots n\}$ , for all  $X \in N$ ,

$$\pi(i, i, X) = \begin{cases} q(X \rightarrow x_i) & \text{if } X \rightarrow x_i \in R \\ 0 & \text{otherwise} \end{cases}$$

**Algorithm:**

- For  $l = 1 \dots (n - 1)$ 
  - For  $i = 1 \dots (n - l)$ 
    - \* Set  $j = i + l$
    - \* For all  $X \in N$ , calculate

$$\pi(i, j, X) = \max_{\substack{X \rightarrow YZ \in R, \\ s \in \{i \dots (j-1)\}}} (q(X \rightarrow YZ) \times \pi(i, s, Y) \times \pi(s + 1, j, Z))$$

and

$$bp(i, j, X) = \arg \max_{\substack{X \rightarrow YZ \in R, \\ s \in \{i \dots (j-1)\}}} (q(X \rightarrow YZ) \times \pi(i, s, Y) \times \pi(s + 1, j, Z))$$

**Output:** Return  $\pi(1, n, S) = \max_{t \in \mathcal{T}(s)} p(t)$ , and backpointers  $bp$  which allow recovery of  $\arg \max_{t \in \mathcal{T}(s)} p(t)$ .

## Exercise

Compute the best parse tree and its probability for the following input sentence using the PCFG:

“the woman saw the man with the telescope”

Syntactic  
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$S \rightarrow NP VP$	0.5	$Vi \rightarrow \text{sleeps}$	1.0
$S \rightarrow NP Vi$	0.5	$Vt \rightarrow \text{saw}$	1.0
$NP \rightarrow DT NN$	0.4	$NN \rightarrow \text{man}$	0.7
$NP \rightarrow NP PP$	0.6	$NN \rightarrow \text{woman}$	0.2
$PP \rightarrow IN NP$	1.0	$NN \rightarrow \text{telescope}$	0.1
$VP \rightarrow Vt NP$	0.4	$DT \rightarrow \text{the}$	1.0
$VP \rightarrow VP PP$	0.1	$IN \rightarrow \text{with}$	0.5
$VP \rightarrow Vi PP$	0.5	$IN \rightarrow \text{in}$	0.5