Lógica en la Informática / Logic in Computer Science

Friday April 16th, 2021

Time: 1h20min. No books, lecture notes or formula sheets allowed.

1) (3 points) Prove your answers using only the formal definitions of propositional logic.
1a) Is it true that if F, G, H are formulas such that $F \land G \not\models H$ then $F \land G \land H$ is unsatisfiable?

1b) Let F be a tautology, and let G an unsatisfiable formula. Is it true true that $F \land \neg G$ is a tautology?

2) (2 points) The problem called "minOnes" takes as input a natural number k and a propositional formula F over propositional variables $\{x_1 \ldots, x_n\}$. Its aim is to decide if there is any model I of F with at most k ones, that is, any model I such that $I(x_1) + \ldots + I(x_n) \leq k$.
Answer in a few words: Is minOnes NP-hard? Why?

3) (2 points) Every propositional formula F over n variables can also expressed by a Boolean circuit with n inputs and one output. In fact, sometimes the circuit can be much smaller than F because each subformula only needs to be represented once. For example, if F is

$$x_1 \land (x_3 \land x_4 \ \lor\ x_3 \land x_4) \quad \lor \quad x_2 \land (x_3 \land x_4 \ \lor\ x_3 \land x_4),$$

a circuit C for F with only five gates exists. Giving names a_i to the output wires of each logical gate, and using a_0 as the output of C, we can write C as:

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a0 = or(a1,a2)     a1 = and(x1,a3)        a3 = or(a4,a4)
                   a2 = and(x2,a3)        a4 = and(x3,x4)
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Explain **very briefly** what do you think is the best way to use a standard SAT solver for CNFs to determine whether two circuits C_1 and C_2, represented like this, are logically equivalent.
Note: assume different names $b_0, b_1, b_2 \ldots$ are used for the internal wires of C_2.

4) (3 points) Consider the cardinality constraint $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \leq 4$ (expressing that at most 4 of the propositional symbols $\{x_1, x_2, x_3, x_4, x_5, x_6\}$ are true).
4a) Write the clauses needed to encode this constraint using no auxiliary variables.

4b) In general, in terms of n and k, how many clauses are needed to encode a cardinality constraint $x_1 + \ldots + x_n \leq k$ using no auxiliary variables? (give no explanations here).

4c) Write the names of any other encoding you know for cardinality constraints $x_1 + \ldots + x_n \leq k$, an encoding that do use auxiliary variables. In terms of n and k, how many clauses are needed? (give no explanations here).