Lógica en la Informática / Logic in Computer Science
January 16th, 2019. Time: 2h30min. No books or lecture notes.

Note on evaluation: eval(propositional logic) = max{ eval(Problems 1,2,3,4), eval(partial exam) }.
eval(first-order logic) = eval(Problems 5,6,7).

1a) Let F and G be propositional tautologies. Is it true that, for every propositional formula H, we have $H \models F \land G$? Prove it using only the definitions of propositional logic.

1b) Is it true that the formula p is a logical consequence of the set S of three clauses $\{\ p \lor q \lor r,\quad \neg q \lor r,\quad \neg r\ \}$? Prove it in the simplest and shortest way you know. You may use any well-known property of propositional logic, even without proving that property.

2) Let $Res(S)$ denote the closure under resolution of a set S of propositional two-literal clauses. Which three properties of $Res(S)$ do you find essential to prove that 2-SAT is polynomial? Answer in three lines like this:
1. ...
2. ...
3. ...

3) Given a propositional CNF, that is, a set of propositional clauses S, explain in two lines your best method to decide wether S is a tautology.

4) Write the clauses needed for expressing $x_1 + \ldots + x_4 \leq 1$ using the ladder encoding. (Please write them in a clean and ordered way; give no explanations.)

5) Let F be the folowing formula of first-order logic with equality:
$\forall x\, \forall y\, \forall z\; f(x, f(y,z)) = f(f(x,y),z) \ \land\ \forall x\; f(e,x) = x \ \land\ \forall x\; f(i(x),x) = e \ \land\ \forall x\, \forall y\; f(x,y) = f(y,x)$.
Any model of F is called a *conmutative group* (where e is the *neutral element* for f and i its *inverse*).

5a) Give a well-known example of a conmutative group with *infinite* domain. Please write it as clean and simple as possible; give no explanations.

5b) Give an *as simple as possible* example of a conmutative group with a *finite* domain. Please write it as clean and simple as possible; give no explanations.

6) Formalize and prove by resolution that sentence D is a logical consequence of the other three:
A: Everybody loves his father and his mother.
B: John is stupìd.
C: When someone is stupid, at least one of his parents is stupid too.
D: There are stupid people that are loved by someone.
Mandatory: use function symbols $f(x)$ and $m(x)$ meaning "father of x" and "mother of x".

7) Consider a 1-ary function symbol f and a 3-ary predicate symbol P and a first-order interpretation I with a finite domain $D_I = \{a, b\}$ and the (finite) definition of the functions f_I and P_I. Answer in a few words: Is it decidable whether I satisfies a given formula F (over f and P)? If so, what do you think is the complexity of this? (hint: any relationship with 3-SAT?).