Lógica en la Informática / Logic in Computer Science
June 22nd, 2018. Time: 2h30min. No books or lecture notes.

Note on evaluation: eval(propositional logic) = max{ eval(Problems 1,2,3), eval(partial exam) }.
eval(first-order logic) = eval(Problems 4,5,6).

1a) Let F be a formula. Is it true that F is satisfiable if, and only if, all logical consequences of F are satisfiable formulas? Prove it using only the definitions of propositional logic.

1b) Is it true that a formula F is a tautology if, and only if, its Tseitin transformation $Tseitin(F)$ is a tautology? Prove it using only the definitions of propositional logic. Important note: all your answers should be as short, clean and simple as possible.

2a) Notation: we consider clauses C and sets S of clauses over a set of propositional symbols $\mathcal{P}$. We define $negateAll(C) = \{negate(lit) \mid lit \in C\}$, that is, the clause obtained by flipping (changing the sign) of *all* literals. For example, $negateAll(p \vee \neg q \vee \neg r)$ is $\neg p \vee q \vee r$. Similarly, we define $negateAll(S) = \{negateAll(C) \mid C \in S\}$, i.e, all literals in S are flipped. Explain in two lines: Is it true that S is satisfiable iff $negateAll(S)$ is satisfiable?

2b) Now, for $N \subseteq \mathcal{P}$, $negate(N, C)$ negates the literals whose symbol is in N. For example, $negate(\ \{p, q\},\ p \vee \neg q \vee \neg r\ )$ is $\neg p \vee q \vee \neg r$. We extend this to $negate(N, S)$ as before. Explain in two lines: Is it true that S is satisfiable iff $negate(N, S)$ is satisfiable?

2c) S is called *renamable Horn* if there is some $N \subseteq \mathcal{P}$ such that $negate(N, S)$ is Horn. Explain in two lines: Given S and N such that $negate(N, S)$ is Horn, what would you do to efficiently decide whether S is satisfiable?

2d) Assume you are given a renamable Horn S but you do not know the set N. Explain in two lines: Can you still decide the satisfiability of S with the same cost as in 2c)? We mean the same asymptotical cost, in $O(...)$-notation.

3) Write the clauses obtained by encoding $AtMostOne(x_0, x_1, x_2, x_3)$ using the logarithmic encoding (only write the clauses, give no explanations).

4a) Assume we have a binary predicate symbol P and two interpretations I_1 and I_2, where D_{I_1} is the natural numbers, D_{I_2} is the integers, and $P_{I_1}(n, m) = P_{I_2}(n, m) = \ n > m$. Write a formula F, using no other predicate symbols than P, such that exactly one of the two interpretations is a model of F and say which one. Give no explanations.

4b) Same question if D_{I_1} is the integers, D_{I_2} is the rational numbers.

4c) Same question if D_{I_1} is the real numbers, D_{I_2} the complex numbers, with two binary symbols: a predicate symbol Eq interpreted as equality, and a function symbol p interpreted as the product.

5) Assume we have a yes/no question Q, based on some input data. Explain **in a few words** each one of the following cases:

5a) What does it mean that Q is decidable?

5b) What does it mean that Q is semi-decidable?

5c) What does it mean that Q is co-semi-decidable?

5d) Is SAT in first-order logic decidable? semi-decidable? co-semi-decidable?

5e) Same question for logical equivalence.

5f) Give an (as simple as you can!) example of non-termination of resolution in first-order logic.

6) Formalize and prove by resolution that sentence E is a logical consequence of the other four.

A: Cristiano is a real madrid player

B: Messi and Cristiano are world-class football players

C: To be a world-class football player, one has to be modest

D: Real madrid has no modest players

E: This year Germany will win the world cup