Lógica en la Informática / Logic in Computer Science

Tuesday November 22nd, 2016

Time: 1h45min. No books, lecture notes or formula sheets allowed.

1) Let S be a set of propositional clauses over a set of n predicate symbols, and let $Res(S)$ be its closure under resolution. For each one of the following cases, indicate whether $Res(S)$ is infinite or finite, and, if finite, of which size. Very briefly explain why.

1a) If clauses in S have at most two literals.

1b) S is a set of Horn clauses.

1c) Every clause in S has either two literals or is a Horn clause.

1d) S is an arbitrary set of propositional clauses.

2) Let C_1 and C_2 be propositional clauses, and let D be the conclusion by resolution of C_1 and C_2.

2a) Is D a logical consequence of $C_1 \vee C_2$? Prove it formally, using only the definitions of propositional logic.

2b) Is D a logical consequence of $C_1 \wedge C_2$? Prove it formally, using only the definitions of propositional logic.

2c) Let S be a set of propositional clauses and let $Res(S)$ be its closure under resolution. Is it true that S is satisfiable if, and only if, the empty clause is not in $Res(S)$? Very briefly explain why.

3) Consider the well-known NP-complete *vertex cover* problem: Given a natural number k and a graph with n vertices and m edges $\{(u_1, v_1), \ldots, (u_m, v_m)\}$ with $u_i, v_i \in \{1 \ldots n\}$, it asks whether the graph has a k-cover, that is, a subset of size k of the vertices such that for each edge (u_i, v_i) at least one of u_i and v_i is in the cover.

3a) How would you use a SAT solver to decide it?

3b) How would you use a SAT solver for the optimization version of vertex cover, that is, given only the graph, to find its smallest possible k-cover?

3c) In one exam last year, we considered the following decision problem, called *minOnes*: given a natural number k and a set S of clauses over variables $\{x_1, \ldots, x_n\}$, it asks whether S has any model I with at most k ones, that is, with $I(x_1) + \ldots + I(x_n) \leq k$. Its optimization version is, given only S, to find its model with the minimal number of ones. If the set of clauses S only has clauses with at most two literals, does the optimization version of minOnes become polynomial? Explain briefly why.