Lógica en la Informática / Logic in Computer Science
January 14th, 2021. Time: 2h30min. No books or lecture notes.

Note on evaluation: eval(propositional logic) = max{ eval(Problems 1,2,3), eval(partial exam) }.
eval(first-order logic) = eval(Problems 4,5,6).

1) (4 points) Prove your answers to the following questions, using only the formal definitions of propositional logic.

1a) Given two propositional formulas F and G, is it true that $F \to G$ is a tautology iff $F \models G$?

1b) Let F and G be propositional formulas. Is it true that if $F \to G$ is satisfiable and F is satisfiable, then G is satisfiable?

2) (3 points) 2-SAT is the satisfiability problem for sets of clauses where each clause has at most 2 literals. Similarly 3-SAT is defined for at most 3 literals.

2a) Explain very briefly what the precise computational complexity of 2-SAT is, and why.

2b) Same question for 3-SAT. In particular, explain why 3-SAT is at least as hard as SAT for arbitrary formulas.

3) (3 points) Let S be a satisfiable set of propositional Horn clauses. Answer the following two questions, explaining very, very, briefly why.

3a) What is the complexity of finding the *minimal* model of S, that is, the model I with the minimal number of symbols p such that $I(p) = 1$?

3b) What is the complexity of deciding whether S has only one model or more than one?

4) (3 points) For 4a and 4b, just write the simplest and cleanest possible formula F. Use no more predicate or function symbols than just p. Give no explanations.

4a) Write a satisfiable first-order formula F, using only a *binary* predicate p, such that all models I of F have an infinite domain D_I.

4b) Write a satisfiable formula F of first-order logic with equality, using only a *unary* predicate p, such that F expresses that there is a single element satisfying p, that is, all models I of F have a single (unique) element e in its domain D_I such that $p_I(e) = 1$.

5) (3 points) Let F be the first-order formula $\exists x \forall y \exists z \; (\; p(z, y) \wedge \neg p(x, y) \;)$.

5a) Give a model I of F with $D_I = \{a, b, c\}$.

5b) Is it true that $F \models \forall x \; p(x, x)$?

5c) Is there any model of F with a single-element domain?

6) (4 points) Formalize and prove by resolution that sentence F is a logical consequence of the first five:

A: All people that have electric cars are ecologists.
B: If someone has a grandmother, then that someone has a mother whose mother is that grandmother.
C: A person is an ecologist if his/her mother is an ecologist.
D: Mary is John's grandmother.
E: Mary has an electric car.
F: John is an ecologist.