Syntactic Analysis (Parsing)

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Summary

- Linear Parsing Algorithms. (Counter)example
- Notations: BNF and Extended BNF
- Applying a Rule $A \rightarrow \alpha_i$
- Nullable, First and Follow
- Methods of Linear Parsing
  - Top-down Parsers
    - Grammar Restrictions in Top-down Parsing
      - Elimination of Left Recursion
      - Left Factoring
    - Types of Top-down Parsers
      - Table-driven Top-down Parser
      - Predictive Recursive Top-down Parser
  - Bottom-up Parsers

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Linear Parsing Algorithms

- The list of tokens w will be visited only once, usually in a *left-to-right* traversal. The current token receives the name of *lookahead*
- Compute the derivation $S \Rightarrow^* w$ without *backtracking*. Sources of indeterminism at this point:

$$S \Rightarrow^* w_0 A_1 w_1 \ldots A_n w_n \Rightarrow$$

  - Which non-terminal A_i choose to expand?
  - If we have a rule of the form $A \rightarrow \gamma_1 | \ldots | \gamma_k$, which γ_j —if any— will be used?
    $\Rightarrow$ bear in mind the *lookahead* token.

at most one γ_j may make sense

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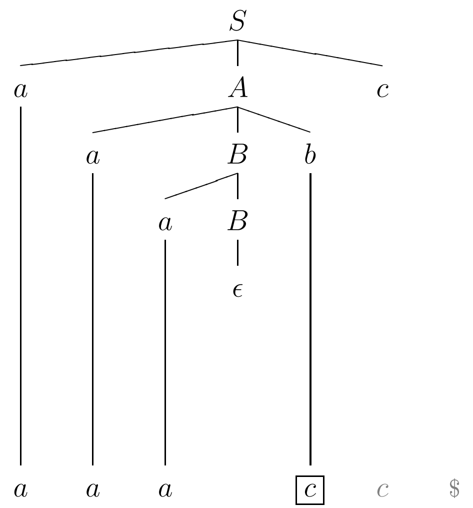
(Counter)example: backtracking

$$S \rightarrow aAc \mid c$$
$$A \rightarrow aBb \mid Bc$$
$$B \rightarrow aB \mid \epsilon$$

$$w = a\,a\,a\,c\,c$$



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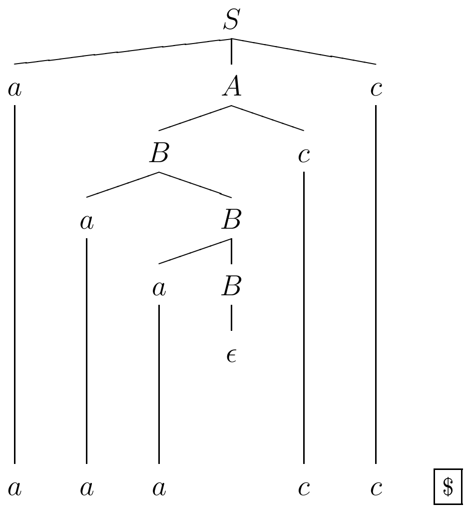
(Counter)example: backtracking

$$
\begin{array}{rcl}
S & \rightarrow & aAc \\
  & | & c \\
A & \rightarrow & aBb \\
  & | & Bc \\
B & \rightarrow & aB \\
  & | & \epsilon
\end{array}
$$

$w = a\,a\,a\,c\,c$



Exercises

- Find another simpler grammar that cannot be parsed with a linear algorithm.
  Give an input that demonstrate it.
- Characterize some conflicting grammars.

Notation. Backus-Naur Form (BNF)

- S, S' are the initial symbol of the grammar
- A, B, C, ..., are non-terminal symbols
- a, b, c, ..., and $\$$ are terminal symbols (tokens)
- X, Y, Z are terminal or non-terminal symbols
- u, v, w are words formed by terminal symbols, possibly empty (ϵ)
- α, β, γ are words formed by terminal and non-terminal symbols, possibly empty (ϵ)
- $A \rightarrow \alpha$ is a production rule.
- $A \rightarrow \alpha_1 \,|\, \ldots \,|\, \alpha_n$ is the group of rules for the non-terminal symbol A

Notation. Backus-Naur Form (BNF)

- $\gamma_1 A \gamma_2 \Rightarrow \gamma_1 \alpha \gamma_2$ is a derivation step with the rule $A \rightarrow \alpha$
- $\Rightarrow^*$ is the reflexive transitive closure of $\Rightarrow$
- Leftmost-derivation ($\Rightarrow^*_{ld}$) if every step:

  $$w A \beta \Rightarrow_{A\rightarrow\alpha} w \alpha \beta$$
- Rightmost-derivation ($\Rightarrow^*_{rd}$) if every step:

  $$\beta A w \Rightarrow_{A\rightarrow\alpha} \beta \alpha w$$

Extended Backus-Naur Form (EBNF)

Rules $A \rightarrow \alpha$ where α can take the form of a regular expression:

- $\alpha_1 \cdots \alpha_n$: concatenation, or ϵ
- $\alpha_1 | \ldots | \alpha_n$: alternatives
- α_1^* : 0 or more times α_1
- α_1^+ : 1 or more times α_1 $(\alpha_1 \alpha_1^*)$
- $\alpha_1?$: 0 or 1 times α_1 $(\alpha_1 | \epsilon)$
- (α_1) : parenthesis for breaking the standard precedence between operators
  $$\{\,|\,\} \prec_p \{\cdot\} \prec_p \{^*, ^+, ?\}$$
- a non-terminal symbol A, or a terminal symbol a

Extended Backus-Naur Form (EBNF)

In *ANTLR*, identifiers in capital letters denote terminals symbols, and identifiers beginning in lower case denote non-terminals.

Rules take the form $\quad id : reg\_exp \; ;$

Example:

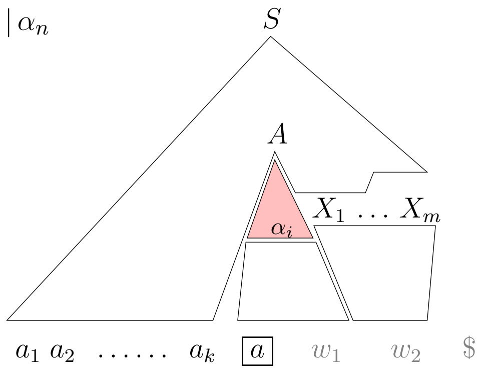
```
expr : term ( (PLUS | MINUS) term)* ;

term : factor ( (TIMES | QUOTIENT) factor)* ;

factor : IDENT ( LEFT_BRK expr RIGHT_BRK )?
       | NUM
       | LEFT_PAR expr RIGHT_PAR
       ;
```


Applying a Rule $A \rightarrow \alpha_i$

$A \rightarrow \alpha_1 | \ldots | \alpha_n$

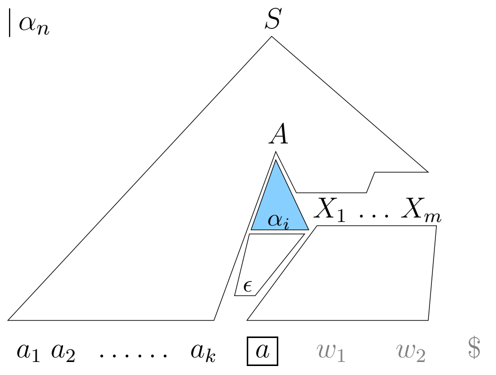


$A \Rightarrow \alpha_i \Rightarrow^* a\,w_1 \qquad X_1 \ldots X_m \Rightarrow^* w_2$

$a \in first(\alpha_i)$

Applying a Rule $A \rightarrow \alpha_i$

$A \rightarrow \alpha_1 | \ldots | \alpha_n$

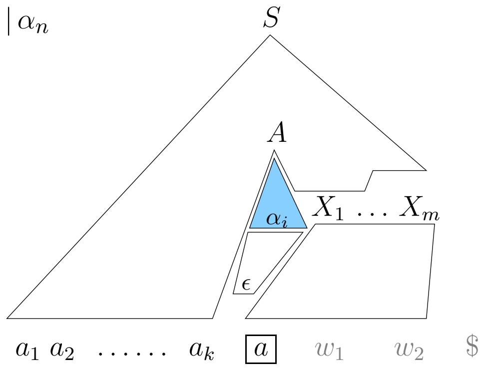


$A \Rightarrow \alpha_i \Rightarrow^* \epsilon \qquad X_1 \ldots X_m \Rightarrow^* a\,w_1\,w_2$

$nullable?(\alpha_i) \qquad S\,\$ \Rightarrow^* a_1 \ldots a_k\, A\, a\, w_1\, w_2\, \$$

Applying a Rule $A \to \alpha_i$

$A \;\to\; \alpha_1 \,|\, \ldots \,|\, \alpha_n$



$A \Rightarrow \alpha_i \Rightarrow^* \epsilon$
$nullable?(\alpha_i)$

$S\,\$ \Rightarrow^* \gamma_1\, A\, a\, \gamma_2$
$a \in follow(A)$

Nullable, First, Follow

- Can α derive ϵ?

  $nullable?(\alpha)$ iff $\alpha \Rightarrow^* \epsilon$

- Set of terminals a that can be the first symbol of words derived from α

  $$first(\alpha) \;=\; \{\, a \mid \alpha \Rightarrow^* a\, w \,\}$$

- Set of terminals a that can follow A in a derivation from $S\,\$$

  $$follow(A) \;=\; \{\, a \mid S\,\$ \Rightarrow^* \gamma_1 A\, a\, \gamma_2 \,\}$$

To Sum Up . . .

$$\begin{array}{rcl} S & \to & aAc \\ & | & c \\ A & \to & aBb \\ & | & Bc \\ B & \to & aB \\ & | & \epsilon \end{array}$$

$$\begin{array}{lcl} first(aB) & = & \{\, a \,\} \\ first(\epsilon) & = & \emptyset \\ first(B) & = & first(aB) \cup first(\epsilon) \\ & = & \{\, a \,\} \\ first(aBb) & = & \{\, a \,\} \\ first(Bc) & = & \{\, a,\ c \,\} \\ first(A) & = & first(aBb) \cup first(Bc) \\ & = & \{\, a,\ c \,\} \\ first(aAc) & = & \{\, a \,\} \\ first(c) & = & \{\, c \,\} \\ first(S) & = & first(aAc) \cup first(c) \\ & = & \{\, a,\ c \,\} \end{array}$$

To Sum Up . . .

$$\begin{array}{rcl} S & \to & aAc \\ & | & c \\ A & \to & aBb \\ & | & Bc \\ B & \to & aB \\ & | & \epsilon \end{array}$$

$$\begin{array}{lcl} first(aB) & = & \{\, a \,\} \\ first(\epsilon) & = & \emptyset \\ nullable?(\epsilon) & = & true \\ follow(B) & = & \{\, b,\ c \,\} \end{array}$$

$nullable?(\alpha)$ (BNF)

$$nullable?(\alpha) \text{ iff } \alpha \Rightarrow^* \epsilon$$

- $nullable?(a) = false$
- $nullable?(A) = true$ if $\exists\, A \to \alpha \in G \wedge nullable(\alpha)$
  $= false$ otherwise
- $nullable?(X_1 \ldots X_n) = true$ if $\forall i : 1 \leq i \leq n : nullable(X_i)$
  $= false$ otherwise

$nullable?(\alpha)$ (Extended BNF)

$$nullable?(\alpha) \text{ iff } \alpha \Rightarrow^* \epsilon$$

- $nullable?(\alpha_1\, \alpha_2) = true$ if $nullable?(\alpha_1) \wedge nullable?(\alpha_2)$
  $= false$ otherwise
- $nullable?(\alpha_1 \,|\, \alpha_2) = true$ if $nullable?(\alpha_1) \vee nullable?(\alpha_2)$
  $= false$ otherwise
- $nullable?(\alpha_1^*) = true$
- $nullable?(\alpha_1^+) = true$ if $nullable?(\alpha_1)$
  $= false$ otherwise
- $nullable?(\alpha_1?) = true$

$first(\alpha)$ (BNF)

$$first(\alpha) = \{\, a \mid \alpha \Rightarrow^* a\, w \,\}$$

- $first(a) = \{a\}$
- $first(A) \supseteq first(\alpha_j)$ if $\exists\, A \to \alpha_j \in G$
- $first(X_1 \ldots X_n) \supseteq first(X_i)$ if $nullable?(X_1 \ldots X_{i-1})$

$first(\alpha)$ (Extended BNF)

$$first(\alpha) = \{\, a \mid \alpha \Rightarrow^* a\, w \,\}$$

- $first(\alpha_1\, \alpha_2) = first(\alpha_1)$ if $\neg\, nullable?(\alpha_1)$
  $first(\alpha_1\, \alpha_2) = first(\alpha_1) \cup first(\alpha_2)$ if $nullable?(\alpha_1)$
- $first(\alpha_1 \,|\, \alpha_2) = first(\alpha_1) \cup first(\alpha_2)$
- $first(\alpha_1^*) = first(\alpha_1)$
- $first(\alpha_1^+) = first(\alpha_1)$
- $first(\alpha_1?) = first(\alpha_1)$

$follow(A)$ (BNF)

$$follow(A) = \{ a \mid S\$ \Rightarrow^* \gamma_1 A\, a\, \gamma_2 \}$$

- $follow(S) \supseteq \{\$\}$
- $follow(B) \supseteq first(\beta)$ if $\exists\, A \rightarrow \alpha B \beta \in G$
- $follow(B) \supseteq follow(A)$ if $\exists\, A \rightarrow \alpha B \in G$
  $follow(B) \supseteq follow(A)$ if $\exists\, A \rightarrow \alpha B \beta \in G \,\wedge$ $nullable?(\beta)$

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$follow(\alpha)$ (Extended BNF)

If α occurs in G: $follow(\alpha) = \{ a \mid S\$ \Rightarrow^* \gamma_1 \alpha\, a\, \gamma_2 \}$

- $follow(\alpha) \supseteq follow(A)$ if $\exists\, A \rightarrow \alpha \in G$
- $follow(\alpha_1) \supseteq first(\alpha_2)$ if $\alpha_1\, \alpha_2$ occurs in G
- $follow(\alpha_2) \supseteq follow(\alpha_1\, \alpha_2)$ if $\alpha_1\, \alpha_2$ occurs in G
- $follow(\alpha_1) \supseteq follow(\alpha_1\, \alpha_2)$ if $\alpha_1\, \alpha_2$ occurs in G $\wedge\, nullable?(\alpha_2)$
- $follow(\alpha_1) \supseteq follow(\alpha_1 \mid \alpha_2)$ if $\alpha_1 \mid \alpha_2$ occurs in G
- $follow(\alpha_2) \supseteq follow(\alpha_1 \mid \alpha_2)$ if $\alpha_1 \mid \alpha_2$ occurs in G
- $follow(\alpha_1) \supseteq first(\alpha_1) \cup follow(\alpha_1^*)$ if α_1^* occurs in G
- $follow(\alpha_1) \supseteq first(\alpha_1) \cup follow(\alpha_1^+)$ if α_1^+ occurs in G
- $follow(\alpha_1) \supseteq follow(\alpha_1?)$ if $\alpha_1?$ occurs in G

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Methods of Linear Parsing

The list of tokens will be traversed *left-to-right*. Decisions to proceed take into account one token of lookahead.

- Top-down parsers (LL(1))
  - Build the AST from the root to the leaves (top-down)
  - Follow a left-most derivation in forward direction
  - More intuitive: can be *manually* written
  - **Grammars may need preprocessing**
- Bottom-up parsers (LR(1))
  - Build the AST from the leaves to the root (bottom-up)
  - Follow a right-most derivation in *backward* direction
  - Less intuitive than top-down parsers
  - Slightly more powerful

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Elimination of Left Recursion

Grammar G:

$E \rightarrow E + \mathtt{id}$
$\quad\mid \mathtt{id}$

$first(E + \mathtt{id}) = \{\mathtt{id}\}$
$first(\mathtt{id}) = \{\mathtt{id}\}$
$first(E + \mathtt{id}) \cap first(\mathtt{id}) \neq \emptyset$

$w = \mathtt{id} + \mathtt{id} + \mathtt{id}$

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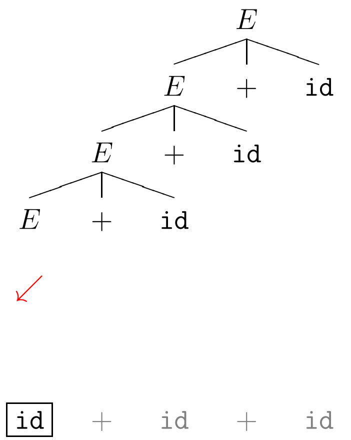
Elimination of Left Recursion

Grammar G:

$$\begin{array}{lcl} E & \rightarrow & E + \texttt{id} \\ & | & \texttt{id} \end{array}$$

$w = \texttt{id} + \texttt{id} + \texttt{id}$



Elimination of Left Recursion (BNF)

$$\begin{array}{lcl} A & \rightarrow & A\,\alpha_1 \\ & | & \dots \\ & | & A\,\alpha_n \\ & | & \beta_1 \\ & | & \dots \\ & | & \beta_m \end{array}$$

Transform into right recursion:

$$\begin{array}{lcl} A & \rightarrow & \beta_1\,A' \\ & | & \dots \\ & | & \beta_m\,A' \end{array}$$

$$\begin{array}{lcl} A' & \rightarrow & \alpha_1\,A' \\ & | & \dots \\ & | & \alpha_n\,A' \\ & | & \epsilon \end{array}$$

Elimination of Left Recursion (EBNF)

$$\begin{array}{lcl} A & \rightarrow & A\,\alpha_1 \\ & | & \dots \\ & | & A\,\alpha_n \\ & | & \beta_1 \\ & | & \dots \\ & | & \beta_m \end{array}$$

Extended BNF: use regular expressions

$$A \rightarrow (\,\beta_1 \,|\, \dots \,|\, \beta_m\,)\,(\,\alpha_1 \,|\, \dots \,|\, \alpha_n\,)^*$$

$$\begin{array}{lcl} A & \rightarrow & B\,(A')^* \\ B & \rightarrow & \beta_1 \,|\, \dots \,|\, \beta_m \\ A' & \rightarrow & \alpha_1 \,|\, \dots \,|\, \alpha_n \end{array}$$

Exercises

$$\begin{array}{lcl} LI & \rightarrow & LI\;\; I \\ & | & I \end{array}$$

$$\begin{array}{lcl} E & \rightarrow & E + T \\ & | & T \\ T & \rightarrow & T * F \\ & | & F \\ F & \rightarrow & (\,E\,) \\ & | & \texttt{id} \end{array}$$

Indirect left recursion:

$$\begin{array}{lcl} A & \rightarrow & B\,d \\ B & \rightarrow & C\,e \\ C & \rightarrow & A\,f \\ & | & g \end{array}$$

Left Factoring

$$
\begin{aligned}
E &\rightarrow T + E \\
&\mid T \\
T &\rightarrow \texttt{id} \\
&\mid (E)
\end{aligned}
$$

$first(T + E) = first(T) = \{ \texttt{id}, (\}$

$first(T) = \{ \texttt{id}, (\}$

$first(T + E) \cap first(T) \neq \emptyset$

$first(\texttt{id}) = \{ \texttt{id} \}$

$first((E)) = \{ (\}$

$first(\texttt{id}) \cap first((E)) = \emptyset$

Left Factoring (BNF)

$$
\begin{aligned}
A &\rightarrow \beta\, \alpha_1 \\
&\mid \ldots \\
&\mid \beta\, \alpha_n \\
&\mid \gamma_1 \\
&\mid \ldots \\
&\mid \gamma_m
\end{aligned}
\qquad
\begin{aligned}
A &\rightarrow \beta\, A' \\
&\mid \gamma_1 \\
&\mid \ldots \\
&\mid \gamma_m \\
A' &\rightarrow \alpha_1 \\
&\mid \ldots \\
&\mid \alpha_n
\end{aligned}
$$

Left Factoring (EBNF)

$$
\begin{aligned}
A &\rightarrow \beta\, \alpha_1 \\
&\mid \ldots \\
&\mid \beta\, \alpha_n \\
&\mid \gamma_1 \\
&\mid \ldots \\
&\mid \gamma_m
\end{aligned}
$$

Extended BNF:

$$A \rightarrow \beta\, (\alpha_1 \mid \ldots \mid \alpha_n) \mid \gamma_1 \mid \ldots \mid \gamma_m$$

$$
\begin{aligned}
A &\rightarrow \beta\, A' \mid \gamma_1 \mid \ldots \mid \gamma_m \\
A' &\rightarrow \alpha_1 \mid \ldots \mid \alpha_n
\end{aligned}
$$

Exercises

$$
\begin{aligned}
P &\rightarrow \texttt{if}\ C\ \texttt{then}\ P\ \texttt{endif} \\
&\mid \texttt{if}\ C\ \texttt{then}\ P\ \texttt{else}\ P\ \texttt{endif} \\
&\mid \texttt{p} \\
C &\rightarrow \texttt{c}
\end{aligned}
$$

$$
\begin{aligned}
I &\rightarrow LE\ \texttt{':='}\ E \\
&\mid \texttt{write}\ \texttt{'('}\ E\ \texttt{')'} \\
&\mid \texttt{id}\ \texttt{'('}\ E\ \texttt{')'} \\
E &\rightarrow \texttt{id} \\
&\mid \texttt{num} \\
LE &\rightarrow \texttt{id}
\end{aligned}
$$

Types of Top-down Parsers

- Table Driven parsers (iterative)
  - Parsing algorithm is fixed, driven by a decision table
  - Table M is built from the grammar G.
    Empty boxes correspond to syntax errors

M	a_1	$\dots$	a	$\dots$	a_n	\$
A_1						
$\vdots$						
A			$A \to \alpha_k$			
$\vdots$						
A_m						

Types of Top-down Parsers

- Table Driven parsers (iterative)
  - Parsing algorithm is fixed, driven by a decision table
  - Table M is built from the grammar G.
    Empty boxes correspond to syntax errors
- Recursive predictive parsers
  - Parsing algorithm is formed by a set of mutually recursive functions
  - Each rule $A \to \alpha$ generates the code of its function

```
void A(void) {
    // Code generated from α
}
```

  - `Gencode` describes how to translate a rule to the associated function

Table-driven Top-down Parser

M	a_1	$\dots$	a	$\dots$	a_n	\$
A_1						
$\vdots$						
A			$A \to \alpha_k$			
$\vdots$						
A_m						

$$A \to \alpha_1 \mid \dots \mid \alpha_k \mid \dots \mid \alpha_o$$

$A \to \alpha_k \in M[A, a]$ if :

- $a \in first(\alpha_k)$, or
- $nullable?(\alpha_k)$ and $a \in follow(A)$

Table-driven Top-down Parser

Algorithm to build the parser table $M[A, a]$

for all rule $A \to \alpha \in G$ do
  add $A \to \alpha$ to $M[A, a]$ if :

- $a \in first(\alpha)$ or,
- $nullable?(\alpha)$ and $a \in follow(A)$

Exercises

Complete the table M for the following grammars, indicating the possible problems.

Grammar G_1 :

$$
\begin{aligned}
E &\rightarrow E + T \\
&\mid T \\
T &\rightarrow T * F \\
&\mid F \\
F &\rightarrow \mathtt{id} \\
&\mid (E)
\end{aligned}
$$

Grammar G_2 :

$$
\begin{aligned}
E &\rightarrow T\, E' \\
E' &\rightarrow +\, T\, E' \\
&\mid \epsilon \\
T &\rightarrow F\, T' \\
T' &\rightarrow *\, F\, T' \\
&\mid \epsilon \\
F &\rightarrow \mathtt{id} \\
&\mid (E)
\end{aligned}
$$

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Exercises

Complete the table M for the following grammars, indicating the possible problems.

Grammar G_3 :

$$
\begin{aligned}
P &\rightarrow \mathtt{if}\ C\ \mathtt{then}\ P \\
&\mid \mathtt{if}\ C\ \mathtt{then}\ P\ \mathtt{else}\ P \\
&\mid \mathtt{p} \\
C &\rightarrow \mathtt{c}
\end{aligned}
$$

Grammar G_4 :

$$
\begin{aligned}
P &\rightarrow \mathtt{if}\ C\ \mathtt{then}\ P\ P' \\
&\mid \mathtt{p} \\
P' &\rightarrow \epsilon \\
&\mid \mathtt{else}\ P \\
C &\rightarrow \mathtt{c}
\end{aligned}
$$

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Exercises

Complete the table M for the following grammars, indicating the possible problems.

Grammar G_5 :

$$
\begin{aligned}
P &\rightarrow \mathtt{if}\ C\ \mathtt{then}\ P\ P' \\
&\mid \mathtt{p} \\
P' &\rightarrow \mathtt{endif} \\
&\mid \mathtt{else}\ P\ \mathtt{endif} \\
C &\rightarrow \mathtt{c}
\end{aligned}
$$

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Table-driven Top-down Parser Algorithm

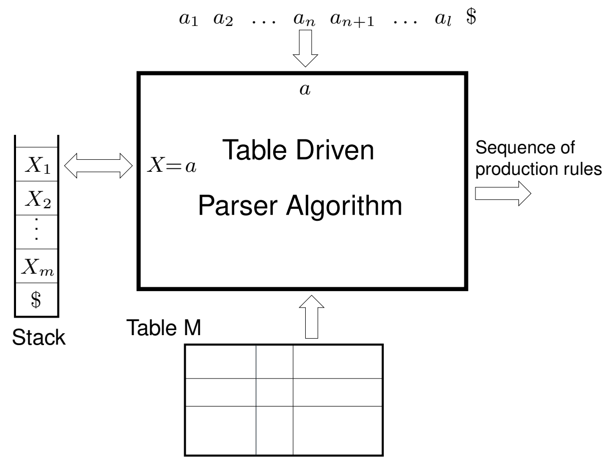

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Table-driven Top-down Parser Algorithm

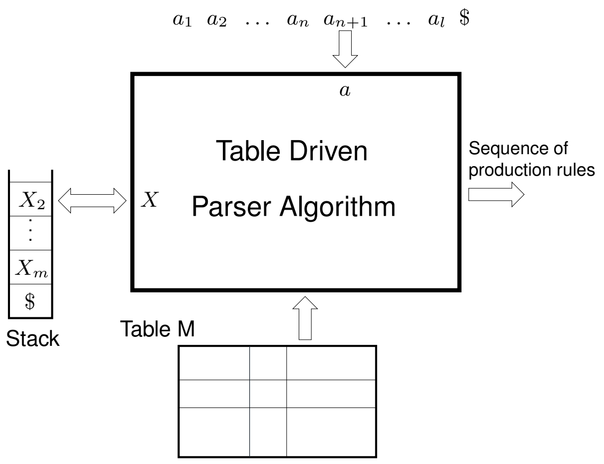

Table-driven Top-down Parser Algorithm

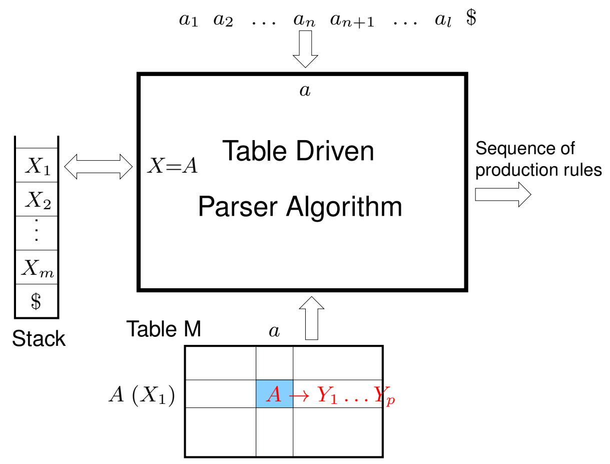

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Table-driven Top-down Parser Algorithm

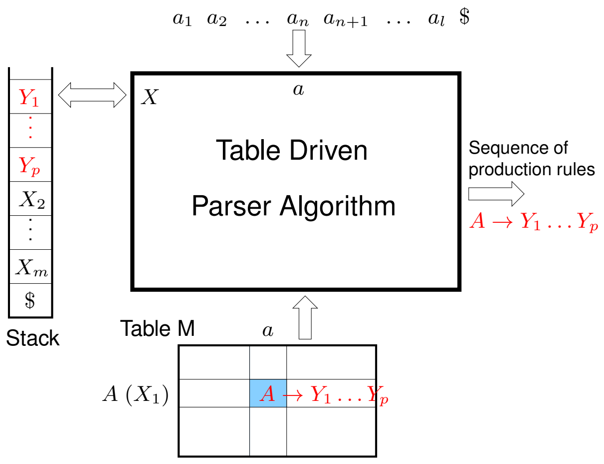

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Table-driven Top-down Parser Algorithm

```
Stk := EmptyStack(); PushStack(Stk, $); PushStack(Stk, S);
X := TopStack(Stk);  a := FirstToken();
while X ≠ $ do
    if X is terminal then
        if X = a then
            PopStack(Stk);  a := NextToken();
        else
            throw syntax error
    else  // X is non−terminal
        if M[X, a] is empty (is error) then
            throw syntax error
        else  // M[X, a] = X → Y1 ... Yp
            emit production rule X → Y1 ... Yp
            PopStack(Stk);  for i := p downto 1 do  PushStack(Stk, Yi);
    X := TopStack(Stk);
endwhile
```

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