Syntactic Analysis (Parsing)

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Summary

- Methods of Linear Parsing
  - Top-down Parsers (LL(1))
  - Bottom-up Parsers (LR(1))
- Types of Top-down Parsers
  - Table Driven Parsers (iterative)
  - Recursive Predictive Parsers
- Example of Recursive Parser (ANTLR style)
- Recursive Predictive Parser Generation
- Bottom-up Parsers
  - Introduction
  - Example of Bottom-up Parsing
  - SLR(1) Table Construction

Methods of Linear Parsing

The list of tokens will be traversed *left-to-right*. Decisions to proceed take into account one token of lookahead.

- Top-down parsers (LL(1))
  - Build the AST from the root to the leaves (top-down)
  - Follow a left-most derivation in forward direction
  - More intuitive: can be *manually* written
  - **Cannot use left-recursion, and need left-factoring**
- Bottom-up parsers (LR1))
  - Build the AST from the leaves to the root (bottom-up)
  - Follow a right-most derivation in *backward* direction
  - Less intuitive than top-down parsers
  - Slightly more powerful

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Types of Top-down Parsers

- Table Driven Parsers (iterative)
  - Parsing algorithm is fixed, driven by a decision table
  - Table M is built from the grammar G.
    Empty boxes correspond to syntax errors

M	a_1	$\ldots$	a	$\ldots$	a_n	$\$$
A_1						
$\vdots$						
A			$A \rightarrow \alpha_k$			
$\vdots$						
A_m						

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Types of Top-down Parsers

- Table Driven Parsers (iterative)
  - Parsing algorithm is fixed, driven by a decision table
  - Table M is built from the grammar G.
    Empty boxes correspond to syntax errors
- Recursive Predictive Parsers
  - Parsing algorithm is formed by a set of mutually recursive functions
  - Each rule $A \to \alpha$ generates the code of its function

```
void A(void) {
    // Code generated from α
}
```

  - `Gencode` describes how to translate a rule to the associated function

Example of Recursive Parser (ANTLR)

Simple grammar in ANTLR:

```
instruction_list :  ( instruction )*
                 ;
instruction :  IDENT  ASSIG  expr
            |  IF  expr  THEN  instruction_list
            ;
expr :  (IDENT | NUM)   ( PLUS  (IDENT | NUM) )*
     ;
```

Example of Recursive Parser (ANTLR)

Simple grammar in ANTLR:

```
instruction_list :  ( instruction )*
                 ;
instruction :  IDENT   ASSIG   expr
            |  IF   expr   THEN   instruction_list
            ;
expr :   expr_simple   ( PLUS   expr_simple )*
     ;
expr_simple :  IDENT
            |  NUM
            ;
```

Example of Recursive Parser (ANTLR)

- Production rule

  expr : (IDENT | NUM) (PLUS (IDENT | NUM))* ;

- Parser *by hand*

```
void expr ( ) {
    if ( token == IDENT || token == NUM ) {
        token = nextToken( );
        while ( token == PLUS ) {
            token = nextToken( );
            if ( token == IDENT || token == NUM ) {
                token = nextToken( );
            } else syntaxError()
        }
    } else syntaxError()
}
```

Example of Recursive Parser (ANTLR)

- Production rule

```
instruction_list :   ( instruction ) *
                 ;
```

- Parser

```
void  instruction_list ( )  {
        while   ( token == IDENT || token == IF )  {
                instruction( );
        }
}
```

Example of Recursive Parser (ANTLR)

- Production rule

```
instruction :  IDENT  ASSIG  expr
            |  IF  expr  THEN  instruction_list
            ;
```

- Parser

```
void instruction ( ) {
    if ( token == IDENT ) {
        MATCH(IDENT);  MATCH(ASSIG);  expr( );
    } else if ( token == IF ) {
        MATCH(IF);  expr( );  MATCH(THEN);  instruction_list( );
    } else syntaxError()
}
```

Example of Recursive Parser (ANTLR)

- Production rule

```
expr :    expr_simple   ( PLUS   expr_simple )*   ;
```

- Parser

```
void  expr ( )  {
        expr_simple( );
        while   ( token == PLUS )  {
            MATCH(PLUS);
            expr_simple( );
        }
}
```

Recursive Predictive Parsers Generation

- Firstly check that the grammar is LL(1), building the table $M[A, a]$ without conflicts.
- $Genrule(\ A \rightarrow \alpha\ )$ generates the code of a function A associated to the production rule
- $Gencode(\ e\ )$ generates the code that recognizes in the input an expression e

$$Genrule(\ A \rightarrow \alpha\ ) \ \equiv$$

$\texttt{void}\ A\,(\,\texttt{void}\,)\ \{$

$\quad\quad /*\ Gencode(\,\alpha\,)\ */$

$\}$

Recursive Predictive Parsers Generation

$Gencode(\ e_1 \,|\, e_2 \,|\, \ldots \,|\, e_n\ ) \ \equiv$

$\texttt{if}\ (\,\texttt{token} \in first(e_1)\,)\ \{$

$\quad\quad /*\ Gencode(\ e_1\ )\ */$

$\}\ \texttt{else if}\ (\,\texttt{token} \in first(e_2)\,)\ \{$

$\quad\quad /*\ Gencode(\ e_2\ )\ */$

$\ldots$

$\}\ \texttt{else if}\ (\,\texttt{token} \in first(e_n)\,)\ \{$

$\quad\quad /*\ Gencode(\ e_n\ )\ */$

$\}\ \texttt{else}\ \ syntaxError();\qquad //\ \texttt{if}\ \nexists\, i : 1 \leq i \leq n : nullable?(e_i)$

Recursive Predictive Parsers Generation

$Gencode(\ e_1 \,|\, e_2 \,|\, \ldots \,|\, e_n\ ) \ \equiv$

```
if ( token ∈ first(e1) ) {
        /* Gencode( e1 ) */
} else if ( token ∈ first(e2) ) {
        /* Gencode( e2 ) */
...
} else if ( token ∈ first(en) ) {
        /* Gencode( en ) */
}                              // if ∃i : 1 ≤ i ≤ n : nullable?(ei)
```

Recursive Predictive Parsers Generation

$Gencode(\ e_1\, e_2\ \ldots\ e_n\ )\ \equiv$

$/*\ Gencode(\ e_1\ )\ */$

$/*\ Gencode(\ e_2\ )\ */$

$\ldots$

$/*\ Gencode(\ e_n\ )\ */$

Recursive Predictive Parsers Generation

$Gencode(\ e_1^*\ )\ \equiv$

```
while ( token ∈ first(e1) ) {
        /* Gencode( e1 ) */
}
```

$Gencode(\ e_1^+\ )\ \equiv$

```
do {
        /* Gencode( e1 ) */
} while ( token ∈ first(e1) );
```

$Gencode(\ e_1?\ )\ \equiv$

```
if ( token ∈ first(e1) {
        /* Gencode( e1 ) */
}
```

$Gencode(\ \epsilon\ )\ \equiv$

```
;   // do nothing
```

Recursive Predictive Parsers Generation

$Gencode(\,A\,) \equiv$ `// for a non-terminal` A

$A(\,);$

$Gencode(\,a\,) \equiv$ `// for a terminal` a

$MATCH(\,a\,);$

Where $MATCH(\,a\,)$ is defined as follows:

$\texttt{if}\ (\,\texttt{token} == a\,)\ \{$

$\texttt{token} = nextToken(\,);$

$\}\ \texttt{else}\ \ syntaxError();$

Bottom-up LR(1) Parsers

- Characteristics
- Example of Bottom-up Parsing
- Shift-Reduce Parsing Algorithm
- Viable Prefixes. LR(0) DFA
- **action** and **goto** Tables Construction
- Shift/reduce and reduce/reduce conflicts
- Types of Bottom-up Parsing
  - SLR(1)
  - LR(1)
  - LALR(1)

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Example of Bottom-up Parsing

$$
\begin{aligned}
E \rightarrow\ & E + T \\
\mid\ & T \\
T \rightarrow\ & T * F \\
\mid\ & F \\
F \rightarrow\ & (\, E\,) \\
\mid\ & \texttt{id}
\end{aligned}
$$

$$w = \texttt{id}_1 + \texttt{id}_2 * \texttt{id}_3$$

Example of Bottom-up Parsing

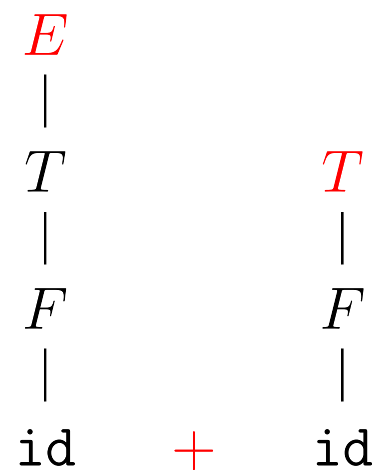
$$
\begin{aligned}
E \rightarrow\ & E + T \\
\mid\ & T \\
T \rightarrow\ & T * F \\
\mid\ & F \\
F \rightarrow\ & (\, E\,) \\
\mid\ & \texttt{id}
\end{aligned}
$$

$shift$ $\texttt{id}_1$ $\quad$ $\boxed{\texttt{id}_1}$ $+$ $\texttt{id}_2$ $*$ $\texttt{id}_3$ $\$$

$$
\begin{aligned}
E &\Rightarrow_{rd} E + T \Rightarrow_{rd} E + T * F \Rightarrow_{rd} E + T * id_3 \\
&\Rightarrow_{rd} E + F * id_3 \Rightarrow_{rd} E + id_2 * id_3 \Rightarrow_{rd} T + id_2 * id_3 \\
&\Rightarrow_{rd} F + id_2 * id_3 \Rightarrow_{rd} id_1 + id_2 * id_3
\end{aligned}
$$

Example of Bottom-up Parsing

$$
\begin{aligned}
E &\rightarrow E + T \\
&\mid T \\
T &\rightarrow T * F \\
&\mid F \\
F &\rightarrow (E) \\
&\mid \texttt{id}
\end{aligned}
$$



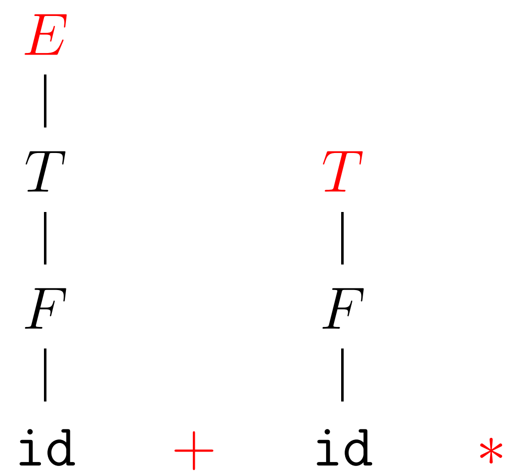
$shift\ *$

$\texttt{id}_1 \quad + \quad \texttt{id}_2 \quad \boxed{*} \quad \texttt{id}_3 \quad \$$

$$E \Rightarrow_{rd} E + T \Rightarrow_{rd} E + T * F \Rightarrow_{rd} \boxed{E + T} * id_3$$
$$\Rightarrow_{rd} E + F * id_3 \Rightarrow_{rd} E + id_2 * id_3 \Rightarrow_{rd} T + id_2 * id_3$$
$$\Rightarrow_{rd} F + id_2 * id_3 \Rightarrow_{rd} id_1 + id_2 * id_3$$

Example of Bottom-up Parsing

$$
\begin{aligned}
E \rightarrow\ & E + T \\
\mid\ & T \\
T \rightarrow\ & T * F \\
\mid\ & F \\
F \rightarrow\ & (\ E\ ) \\
\mid\ & \texttt{id}
\end{aligned}
$$



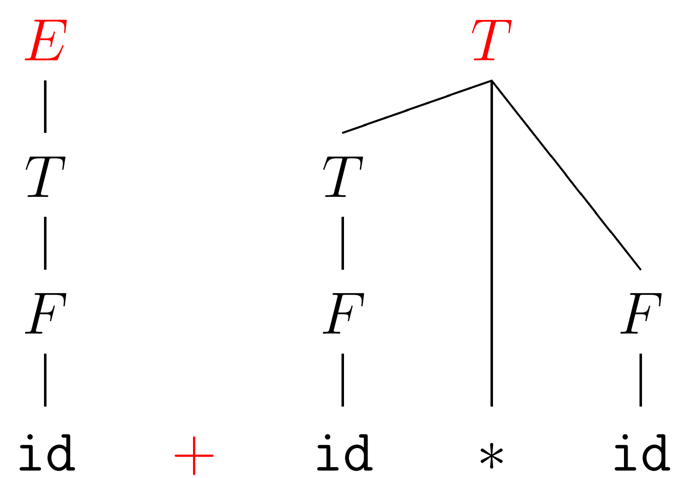
$shift$ $\texttt{id}_3$

$\texttt{id}_1 \quad + \quad \texttt{id}_2 \quad * \quad \boxed{\texttt{id}_3} \quad \$$

$$E \Rightarrow_{rd} E + T \Rightarrow_{rd} E + T * F \Rightarrow_{rd} \boxed{E + T *}\, id_3$$
$$\Rightarrow_{rd} E + F * id_3 \Rightarrow_{rd} E + id_2 * id_3 \Rightarrow_{rd} T + id_2 * id_3$$
$$\Rightarrow_{rd} F + id_2 * id_3 \Rightarrow_{rd} id_1 + id_2 * id_3$$

Example of Bottom-up Parsing

$$
\begin{aligned}
E &\rightarrow E + T \\
&\mid T \\
T &\rightarrow T * F \\
&\mid F \\
F &\rightarrow (E) \\
&\mid \texttt{id}
\end{aligned}
$$



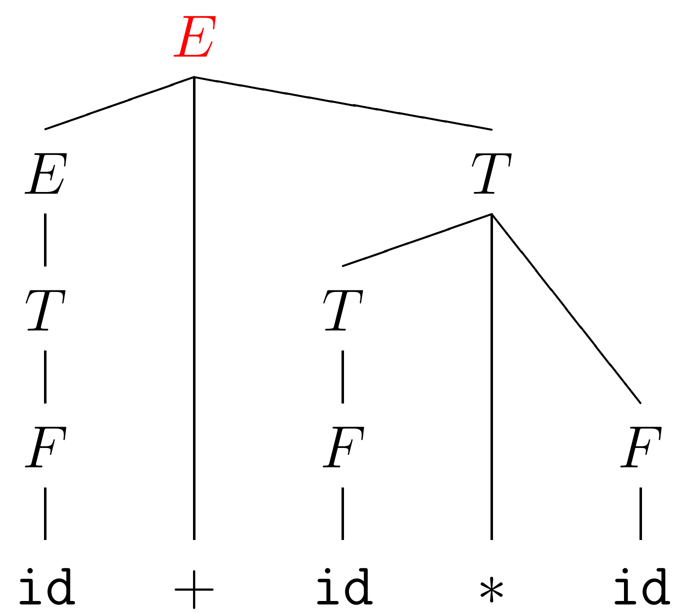
reduce with $E \rightarrow E + T$

$\texttt{id}_1 \; + \; \texttt{id}_2 \; * \; \texttt{id}_3 \; \boxed{\$}$

$$
E \Rightarrow_{rd} \boxed{E + T} \Rightarrow_{rd} E + T * F \Rightarrow_{rd} E + T * id_3
$$
$$
\Rightarrow_{rd} E + F * id_3 \Rightarrow_{rd} E + id_2 * id_3 \Rightarrow_{rd} T + id_2 * id_3
$$
$$
\Rightarrow_{rd} F + id_2 * id_3 \Rightarrow_{rd} id_1 + id_2 * id_3
$$

Example of Bottom-up Parsing

$$
\begin{aligned}
E &\rightarrow E + T \\
  &\mid T \\
T &\rightarrow T * F \\
  &\mid F \\
F &\rightarrow (E) \\
  &\mid \texttt{id}
\end{aligned}
$$



accept

$\texttt{id}_1 \quad + \quad \texttt{id}_2 \quad * \quad \texttt{id}_3 \quad \boxed{\$}$

$$
\boxed{E} \Rightarrow_{rd} E + T \Rightarrow_{rd} E + T * F \Rightarrow_{rd} E + T * id_3
$$
$$
\Rightarrow_{rd} E + F * id_3 \Rightarrow_{rd} E + id_2 * id_3 \Rightarrow_{rd} T + id_2 * id_3
$$
$$
\Rightarrow_{rd} F + id_2 * id_3 \Rightarrow_{rd} id_1 + id_2 * id_3
$$

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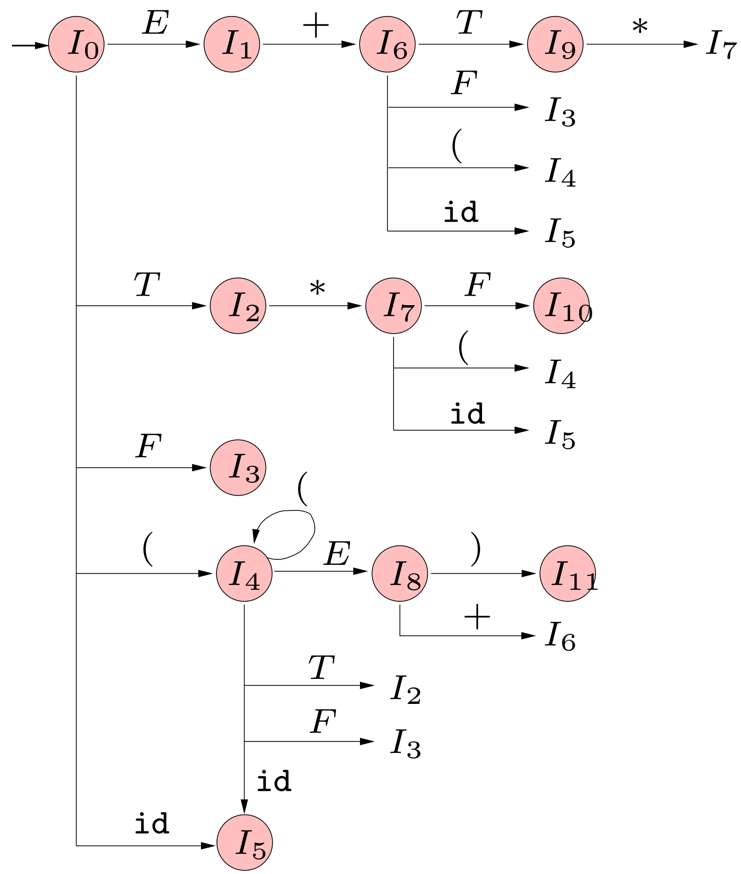
Example of Bottom-up Parsing (cont.)

1. $E \rightarrow E + T$
2. $E \rightarrow T$
3. $T \rightarrow T * F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow \texttt{id}$

$\texttt{id}_1 + \texttt{id}_2 * \texttt{id}_3$

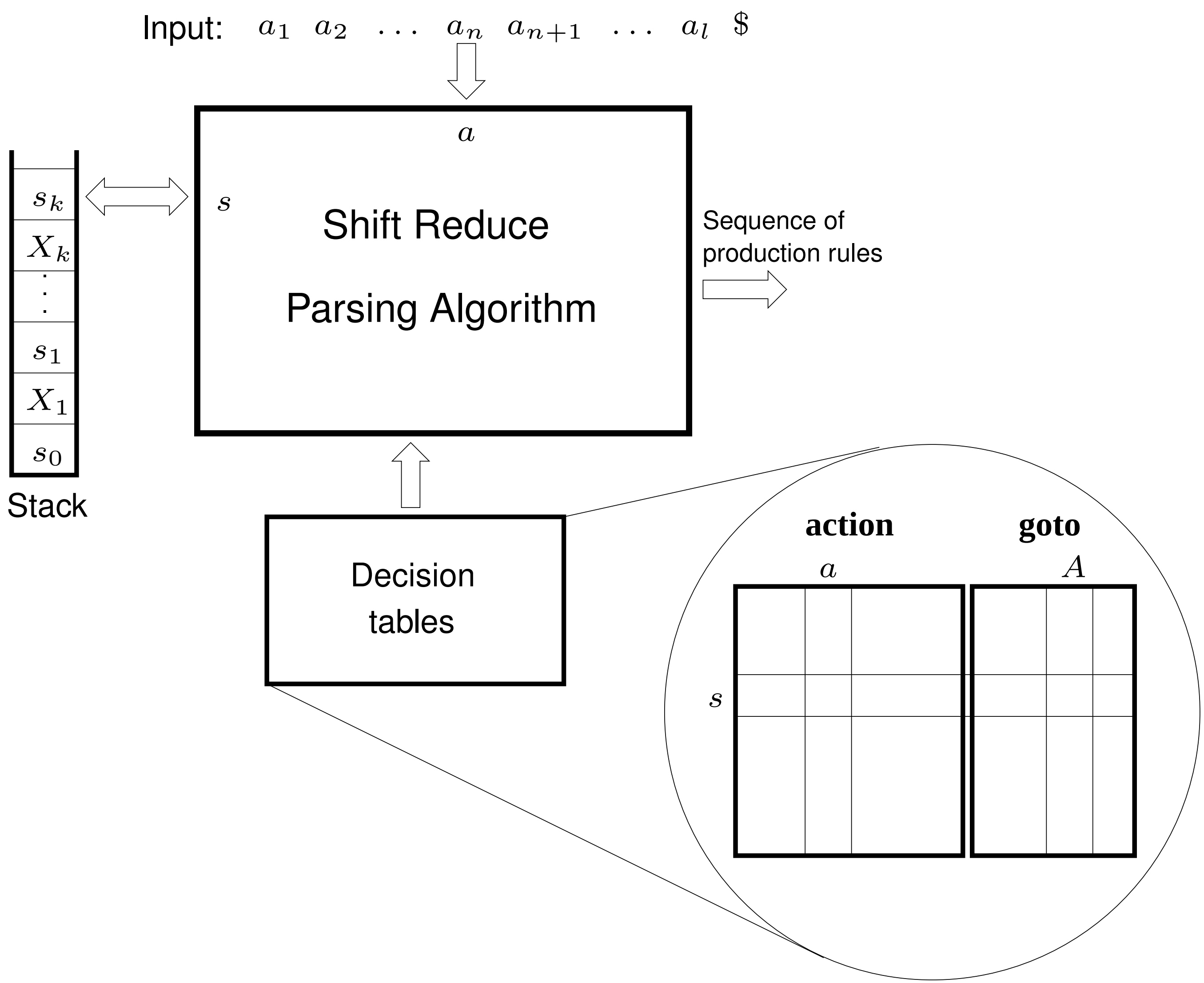
Step	Stack	Input	Action
(1)		$id_1 + id_2 * id_3\ \$$	shift $\texttt{id}_1$
(2)	id_1	$+ id_2 * id_3\ \$$	reduce with $6.\ F \rightarrow id$
(3)	F	$+ id_2 * id_3\ \$$	reduce with $4.\ T \rightarrow F$
(4)	T	$+ id_2 * id_3\ \$$	reduce with $2.\ E \rightarrow T$
(5)	E	$+ id_2 * id_3\ \$$	shift $+$
(6)	$E +$	$id_2 * id_3\ \$$	shift $\texttt{id}_2$
(7)	$E + id_2$	$* id_3\ \$$	reduce with $6.\ F \rightarrow id$
(8)	$E + F$	$* id_3\ \$$	reduce with $4.\ T \rightarrow F$
(9)	$E + T$	$* id_3\ \$$	shift $*$
(10)	$E + T *$	$id_3\ \$$	shift $\texttt{id}_3$
(11)	$E + T * id_3$	$\$$	reduce with $6.\ F \rightarrow id$
(12)	$E + T * F$	$\$$	reduce with $3.\ T \rightarrow T * F$
(13)	$E + T$	$\$$	reduce with $1.\ E \rightarrow E + T$
(14)	E	$\$$	accept

Viable Prefixes. LR(0) DFA



Step	Stack	Input	Action
(10)	${}_0\ E\ {}_1 + {}_6\ T\ {}_9 * {}_7$	$\boxed{id_3}$ \$	shift $\texttt{id}_3$ (**s5**)

Shift / Reduce Parsing Algorithm



Shift / Reduce Parsing Algorithm

```
Stk := EmptyStack(); PushStack(Stk, 0);    // Initial state 0
a := FirstToken();
loop
    s := TopStack(Stk);                     // Current state
    if action[s, a] = si then               // Shift and go to state i
        PushStack(Stk, a); PushStack(Stk, i);
        a := NextToken();
    else if action[s, a] = rj then          // Reduce with rule j) A → α
        for i := 1 to |α| do
            PopStack(Stk); PopStack(Stk);           // Pop states and symbols
        s′ := TopStack(Stk); s′ := goto[s′, A];     // New state s′
        PushStack(Stk, A); PushStack(Stk, s′);      // Push symbol A and s′
        emit production rule A → α
    else if action[s, a] = acc then        // Accept
        accept
    else throw syntax error
endloop
```

LR(0) Items

- An LR(0) item has the form $A \to \alpha \cdot \beta$
  - at this moment α is on [top of] the stack
  - it is expected [at the beginning of the rest of the input] something derivable from β
- For example, at state I_7 of the previous automata:
  - we have $T *$ on top of the stack, and we are expecting something that can be a factor F in order to get a term T of the form $T * F$.
    So item $T \to T * \cdot F \in I_7$
  - we are also directly expecting an identifier $\mathtt{id}$ to get that factor F, or a left parenthesis $($ to get a factor of the form (E).
    So also items $F \to \cdot\, \mathtt{id}$ and $F \to \cdot\, (E) \in I_7$

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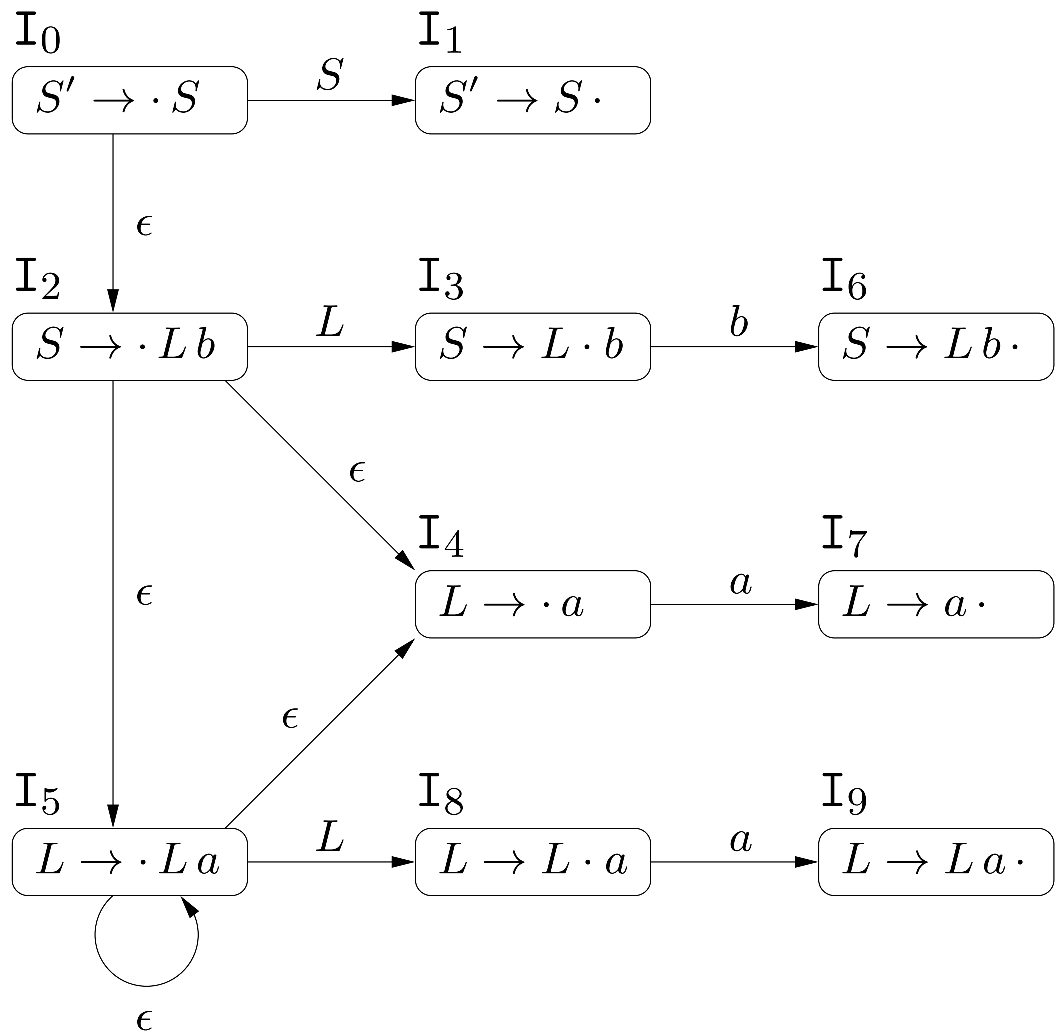
LR(0) NFA

0) $S' \to S$

1) $S \to L\ b$

2) $L \to L\ a$

3) $L \to a$



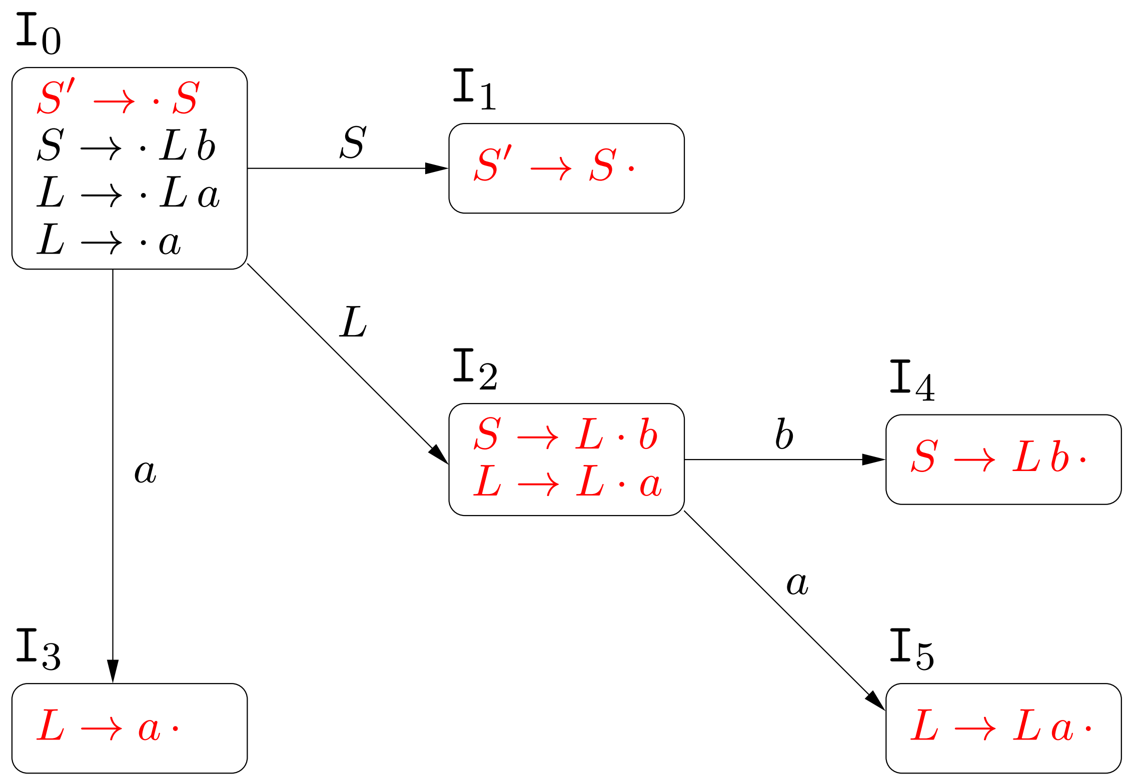
LR(0) DFA

0) $S' \rightarrow S$

1) $S \rightarrow L\ b$

2) $L \rightarrow L\ a$

3) $L \rightarrow a$



LR(0) Tables Construction

- if $A \rightarrow \alpha \cdot a \beta \in I_i$ and $\texttt{DTran}[I_i,\ a] = I_j$ then

  $action[i,\ a] \supseteq \{ \texttt{shift}\ a\ \texttt{and go to}\ j\ (\texttt{s}_j) \}$

- if $A \rightarrow \alpha \cdot \in I_i$ and $n)\ A \rightarrow \alpha \in G$ then

  $\forall a \in \Sigma \cup \{\$\}$:

  $action[i,\ a] \supseteq \{ \texttt{reduce with rule}\ n\ (\texttt{r}_n) \}$

- if $A \rightarrow \alpha \cdot A \beta \in I_i$ and $\texttt{DTran}[I_i,\ A] = I_j$ then

  $goto[i,\ A] = j$

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LR(0) Tables Construction

0) $S' \to S$

1) $S \to L\ b$

2) $L \to L\ a$

3) $L \to a$

state	action			goto	
	a	b	$\$$	S	L
0	$s3$			1	2
1			acc		
2	$s5$	$s4$			
3	$r3$	$r3$	$r3$		
4	$r1$	$r1$	$r1$		
5	$r2$	$r2$	$r2$		

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LR(0) Tables Construction. Example

0) $E' \rightarrow E$

1) $E \rightarrow E + T$

2) $E \rightarrow T$

3) $T \rightarrow T * F$

4) $T \rightarrow F$

5) $F \rightarrow (E)$

6) $F \rightarrow$ id

state	action						goto		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2	r2	r2	**r2** **s7**	r2	r2	r2			
3	r4	r4	r4	r4	r4	r4			
4	s5			s4			8	2	3
5	r6	r6	r6	r6	r6	r6			
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9	r1	r1	**r1** **s7**	r1	r1	r1			
10	r3	r3	r3	r3	r3	r3			
11	r5	r5	r5	r5	r5	r5			

SLR(1) Tables Construction

- if $A \to \alpha \cdot a \beta \in I_i$ and $\texttt{DTran}[I_i, a] = I_j$ then

  $action[i, a] \supseteq \{ \texttt{shift } a \texttt{ and go to } j \ (\texttt{s}_j) \}$

- if $A \to \alpha \cdot \in I_i$ and $n)\ A \to \alpha \in G$ then

  $\forall a \in follow(A)$:

  $action[i, a] \supseteq \{ \texttt{reduce with rule } n \ (\texttt{r}_n) \}$

- if $A \to \alpha \cdot A \beta \in I_i$ and $\texttt{DTran}[I_i, A] = I_j$ then

  $goto[i, A] = j$

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SLR(1) Tables Construction

0) $S' \to S$
1) $S \to L\ b$
2) $L \to L\ a$
3) $L \to a$

state	action			goto	
	a	b	$\$$	S	L
0	$s3$			1	2
1			acc		
2	$s5$	$s4$			
3	$r3$	$r3$			
4			$r1$		
5	$r2$	$r2$			

$$follow(S) = \{\,\$\,\}$$
$$follow(L) = \{\,a,\ b\,\}$$

SLR(1) Tables Construction. Example

0) $E' \to E$

1) $E \to E + T$

2) $E \to T$

3) $T \to T * F$

4) $T \to F$

5) $F \to (E)$

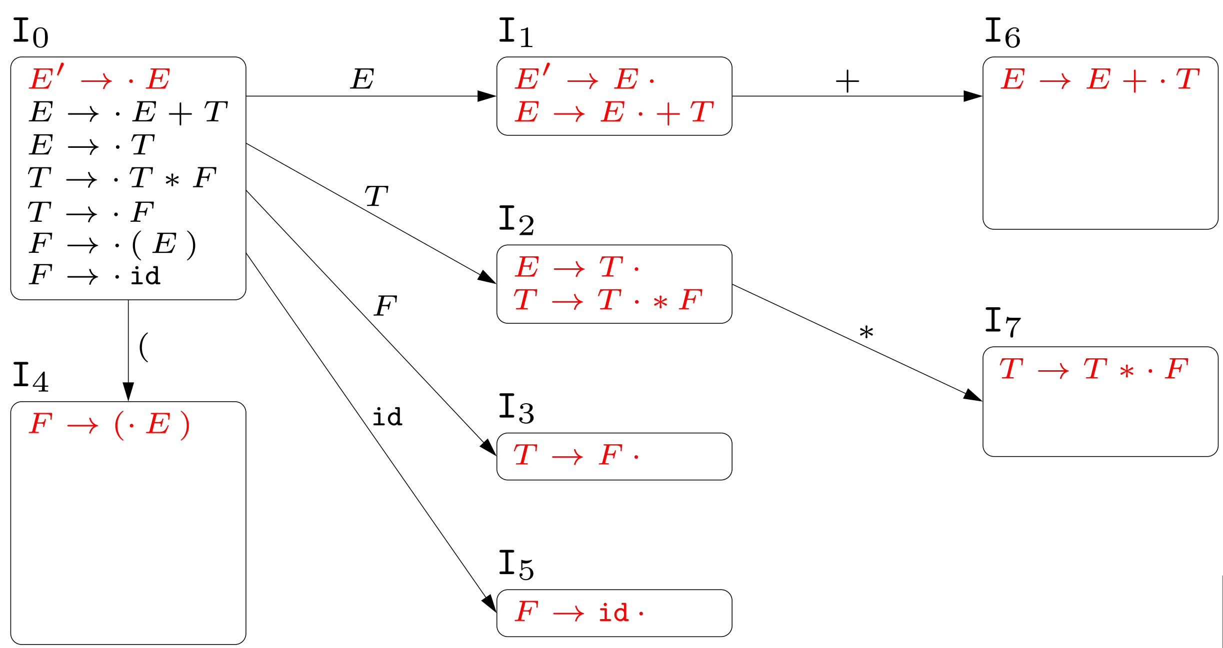
6) $F \to \texttt{id}$

state	action						goto		
	id	$+$	$*$	(	)	\$	E	T	F
0	$s5$			$s4$			1	2	3
1		$s6$				acc			
2		$r2$	**s7**		$r2$	$r2$			
3		$r4$	$r4$		$r4$	$r4$			

$follow(E) = \{ +,), \$ \}$

$follow(T) = \{ +, *,), \$ \}$

$follow(F) = \{ +, *,), \$ \}$



SLR(1) Tables Construction. Example

0) $E' \to E$

1) $E \to E + T$

2) $E \to T$

3) $T \to T * F$

4) $T \to F$

5) $F \to (E)$

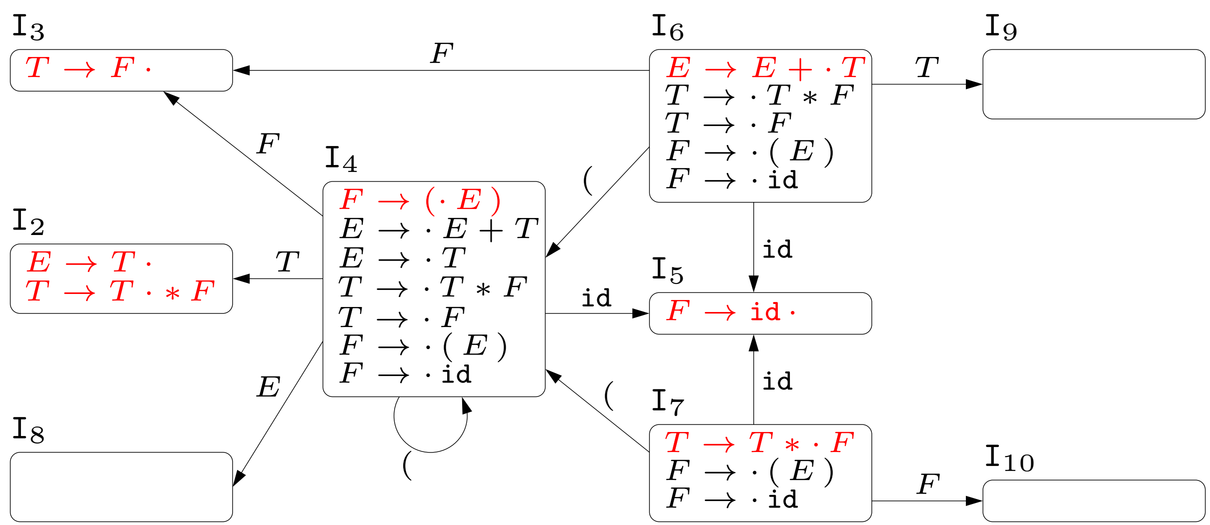
6) $F \to \texttt{id}$

state	*action*						*goto*		
	id	$+$	$*$	(	)	\$	E	T	F
4	$s5$			$s4$			8	2	3
5		$r6$	$r6$		$r6$	$r6$			
6	$s5$			$s4$				9	3
7	$s5$			$s4$					10

$follow(E) = \{ +,), \$ \}$

$follow(T) = \{ +, *,), \$ \}$

$follow(F) = \{ +, *,), \$ \}$



SLR(1) Tables Construction. Example

0) $E' \rightarrow E$

1) $E \rightarrow E + T$

2) $E \rightarrow T$

3) $T \rightarrow T * F$

4) $T \rightarrow F$

5) $F \rightarrow (E)$

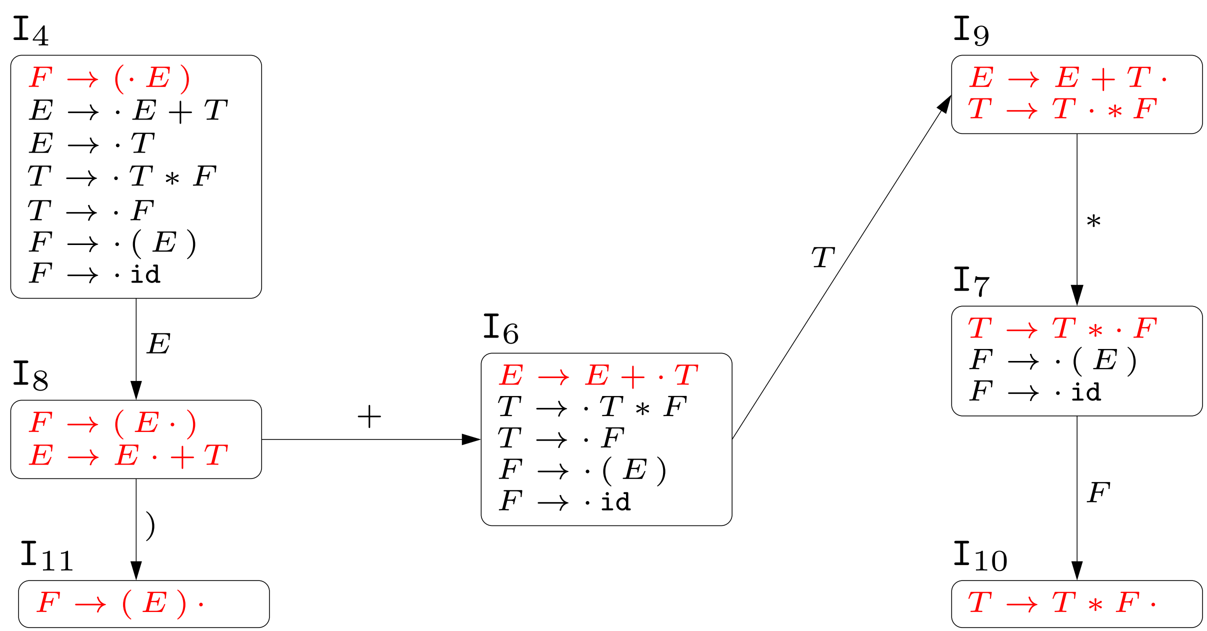
6) $F \rightarrow \texttt{id}$

state	*action*						*goto*		
	id	$+$	$*$	(	)	\$	E	T	F
8		$s6$			$s11$				
9		$r1$	**s7**		$r1$	$r1$			
10		$r3$	$r3$		$r3$	$r3$			
11		$r5$	$r5$		$r5$	$r5$			

$follow(E) = \{ +,), \$ \}$

$follow(T) = \{ +, *,), \$ \}$

$follow(F) = \{ +, *,), \$ \}$



SLR(1) Tables Construction. Example

0) $E' \to E$
1) $E \to E + T$
2) $E \to T$
3) $T \to T * F$
4) $T \to F$
5) $F \to (E)$
6) $F \to \texttt{id}$

$follow(E) = \{ +,), \$ \}$
$follow(T) = \{ +, *,), \$ \}$
$follow(F) = \{ +, *,), \$ \}$

state	action						goto		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	**s7**		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9		r1	**s7**		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

FIB