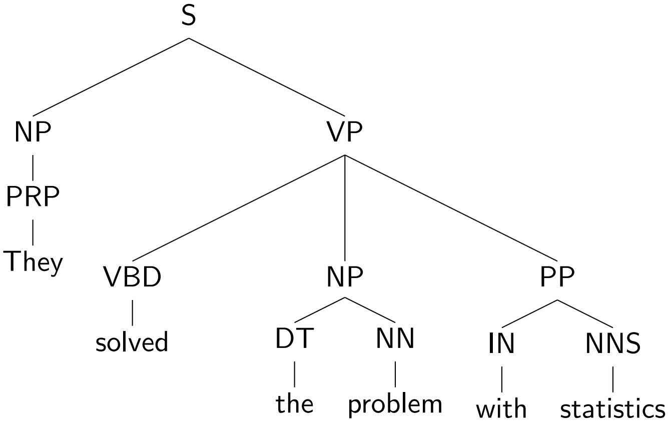
Advanced Human Language Technologies

Constituent Parsing

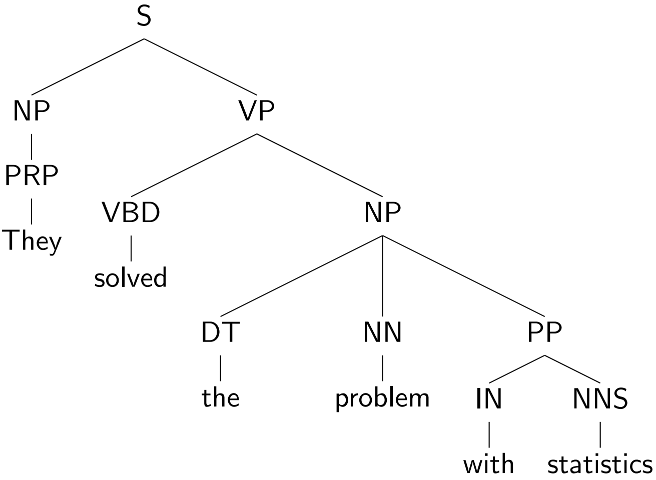
UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Facultat d'Informàtica de Barcelona

Outline

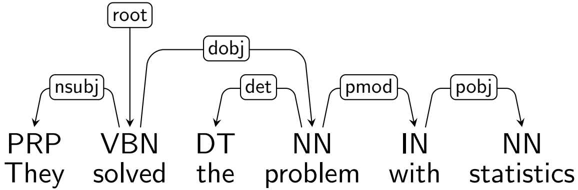
A Syntactic Tree



Another Syntactic Tree



Dependency Trees



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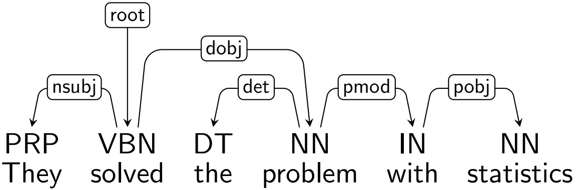
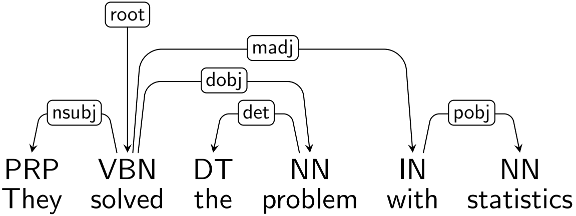
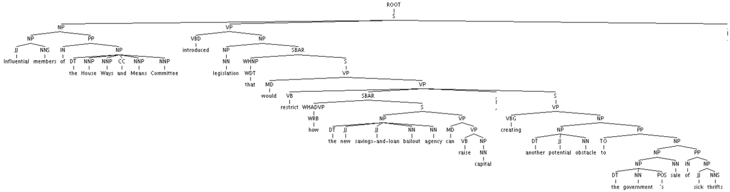




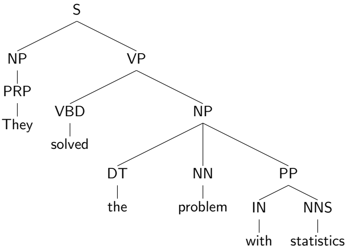
A “real” sentence



Influential members of the House Ways and Means Committee introduced legislation that would restrict how the new savings-and-loan bailout agency can raise capital, creating another potential obstacle to the government's sale of sick thrifts.

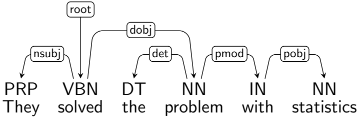
Theories of Syntactic Structure

Constituent Trees



- Main element: constituents (or phrases, or bracketings)
- Constituents = abstract linguistic units
- Results in nested trees

Dependency Trees



- Main element: dependency
- Focus on relations between words
- Handles *free word order* nicely.

Context Free Grammars (CFGs)

A context-free grammar is defined as a tuple $G = \langle N, \Sigma, R, S \rangle$ where:

- N is a set of non-terminal symbols
- $S \in N$ is a distinguished start symbol
- Σ is a set of terminal symbols
- R is a set of rules of the form $X \rightarrow Y_1 Y_2 \ldots Y_n$ where $n \geq 0,\ X \in N,\ Y_i \in N \cup \Sigma$

Context Free Grammars, Example

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\begin{aligned}
N &= \{\mathrm{S}, \mathrm{VP}, \mathrm{NP}, \mathrm{PP}, \mathrm{DT}, \mathrm{Vi}, \mathrm{Vt}, \mathrm{NN}, \mathrm{IN}\}^{1} \\
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R &= \left\{ \begin{array}{ll}
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\mathrm{VP} \rightarrow \mathrm{Vi} & \mathrm{NN} \rightarrow \mathrm{telescope} \\
\mathrm{VP} \rightarrow \mathrm{Vt\ NP} & \mathrm{DT} \rightarrow \mathrm{the} \\
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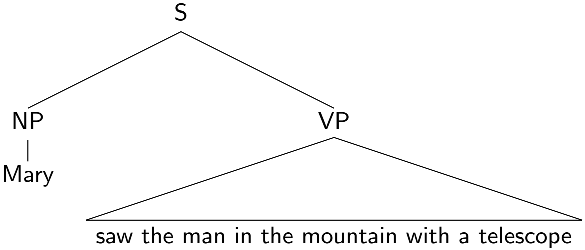
[1] S=sentence, VP=verb phrase, NP=noun phrase, PP=prepositional phrase, DT=determiner, Vi=intransitive verb, Vt=transitive verb, NN=noun, IN=preposition

Properties of CFGs

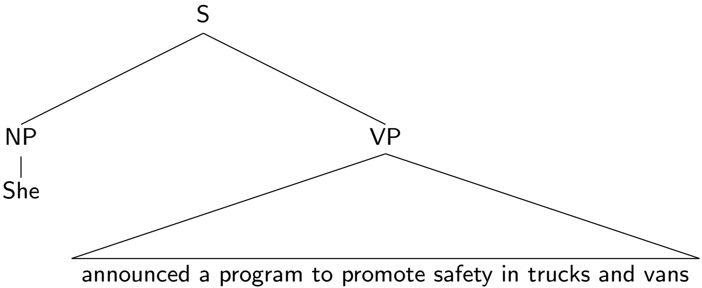
- A CFG defines a set of possible *derivations* (i.e. unique trees)
- A sequence of terminals $s \in \Sigma^*$ is *generated* by the CFG (or *recognized* by it, or *belongs* to the language defined by it) if there is at least a derivation that produces s.
- Some sequences of terminals generated by the CFG may have more than one derivation (*ambiguity*).

Ambiguity



- *Mary used a telescope to see a man who was in the mountain*
- *Mary saw a man who was in the mountain and carried a telescope*
- *Mary was in the mountain and used a telescope to see a man*
- *Mary was in the mountain that has a telescope and saw a man*
- *Mary saw a man who was in the mountain that has a telescope*
- *Mary was in the mountain and saw a man carrying a telescope*

Ambiguity



- *She announced a program aimed to make trucks and vans safer*
- *She used trucks and vans to announce a program aimed to promote safety*
- *She announced a program aimed to make trucks safer. She also announced vans*
- *She used trucks to announce a program aimed to promote safety. She also announced vans*
- *She announced a program. She did so in order to promote satefy in trucks and vans*

Ambiguity

Some trees are more likely than others...

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Can we model that?

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Probabilistic Context Free Grammar (**P**CFGs)

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- R is a set of rules of the form $X \rightarrow Y_1 Y_2 \ldots Y_n$ where $n \geq 0,\ X \in N,\ Y_i \in N \cup \Sigma$
- q is a set of non-negative parameters, one for each rule $X \rightarrow \alpha \in R$ such that, for any $X \in N$,

$$\sum_{(X \rightarrow \alpha) \in R} q(X \rightarrow \alpha) = 1$$

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\mathrm{NP} \rightarrow \mathrm{DT\ NN} & 0.4 & \mathrm{Vt} \rightarrow \mathrm{saw} & 1.0 \\
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Properties of PCFGs

- The probability of a parse tree $t \in \mathcal{T}_G$ is computed as:

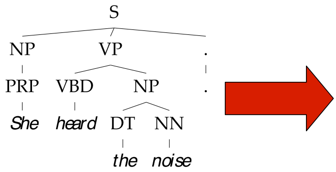
$$p(t) = \prod_{r \in t} q(r)$$

- If there is more than one tree for a sentence, we can rank them by probability.
- The most likely tree for a sentence s is:

$$\arg\max_{t \in \mathcal{T}(s)} p(t)$$

Learning Treebank Grammars

- Read the grammar rules from a treebank



S → NP VP .	1
NP → PRP	0.5
NP → DT NN	0.5
VP → VBD NP	1
PRP → She	1
…	

- Set rule weights by maximum likelihood

$$q(\alpha \to \beta) = \frac{\text{Count}(\alpha \to \beta)}{\text{Count}(\alpha)}$$

- Smoothing issues apply
- Having the appropriate CFG is critical to success

Outline

1. Trees and Grammars
2. Constituency Parsing
   - CKY Algorithm
   - Earley Algorithm

Parsing Natural Language Sentences

Goal of a parser:

- Find all possible trees

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- Find all possible trees
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- Many of the possible trees will share subtrees that we don't need to re-parse.
- Define a dynammic programming table (*aka* **chart**) to store intermediate results.

Outline

1 Trees and Grammars

2 Constituency Parsing
- CKY Algorithm
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CKY Algorithm

- Bottom-up
- Requires a grammar in Chomsky Normal Form (CNF).
- Dynammic programming: Store partial results that can be reused in different candidate solutions.
- Analogous to Viterbi in HMMs.
- Intermediate results stored in a *chart* structure.

CKY Algorithm

Chart content:

- Maximum probability of a subtree with root X spanning words $i \dots j$:

$$\pi(i, j, X)$$

- Backpath to recover which rules produced the maximum probability tree:

$$\psi(i, j, X)$$

The goal is to compute:

- $\max_{t \in \mathcal{T}(s)} p(t) = \pi(1, n, S)$
- $\psi(1, n, S)$
- It is possible to use it without probabilities to get all parse trees (with higher complexity)

CKY Algorithm

Base case: Tree leaves

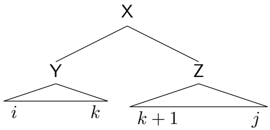
- $\forall i = 1 \dots n$, $\forall X \to w_i \in R$, $\pi(i, i, X) = q(X \to w_i)$

Recursive case: Non-terminal nodes

- $\forall i = 1 \dots n$, $\forall j = (i+1) \dots n$, $\forall X \in N$

$$\pi(i, j, X) = \max_{\substack{X \to Y Z \in R \\ k : i \leq k < j}} q(X \to Y Z) \times \pi(i, k, Y) \times \pi(k+1, j, Z)$$

$$\psi(i, j, X) = \arg \max_{\substack{X \to Y Z \in R \\ k : i \leq k < j}} q(X \to Y Z) \times \pi(i, k, Y) \times \pi(k+1, j, Z)$$



Output:

- Return $\pi(1, n, S)$ and recover backpath trough $\psi(1, n, S)$

CKY Algorithm - Example

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CKY Algorithm - Example

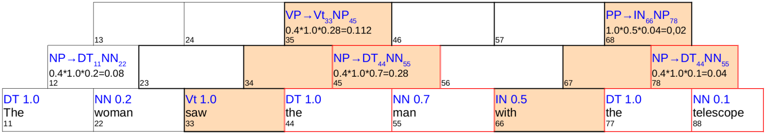
DT 1.0 The 11	NN 0.2 woman 22	Vt 1.0 saw 33	DT 1.0 the 44	NN 0.7 man 55	IN 0.5 with 66	DT 1.0 the 77	NN 0.1 telescope 88

CKY Algorithm - Example

12	23	34	45	56	67	78
$NP \rightarrow DT_{11} NN_{22}$ 0.4*1.0*0.2=0.08			$NP \rightarrow DT_{44} NN_{55}$ 0.4*1.0*0.7=0.28			$NP \rightarrow DT_{44} NN_{55}$ 0.4*1.0*0.1=0.04

11	22	33	44	55	66	77	88
DT 1.0 The	NN 0.2 woman	Vt 1.0 saw	DT 1.0 the	NN 0.7 man	IN 0.5 with	DT 1.0 the	NN 0.1 telescope

CKY Algorithm - Example



CKY Algorithm - Example

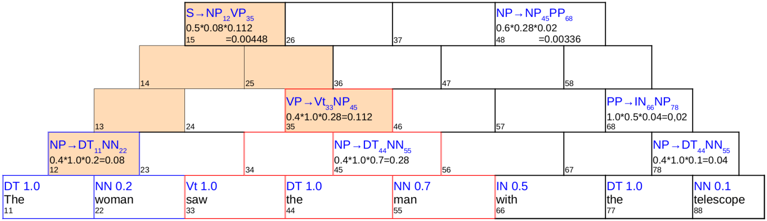
14	25	36	47	58

13	24	$VP \rightarrow Vt_{33} NP_{45}$ 0.4*1.0*0.28=0.112 35	46	57	$PP \rightarrow IN_{66} NP_{78}$ 1.0*0.5*0.04=0,02 68

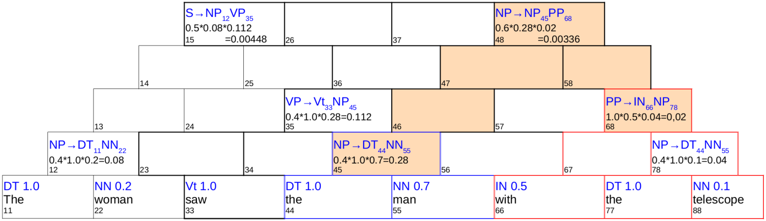
$NP \rightarrow DT_{11} NN_{22}$ 0.4*1.0*0.2=0.08 12	23	34	$NP \rightarrow DT_{44} NN_{55}$ 0.4*1.0*0.7=0.28 45	56	67	$NP \rightarrow DT_{44} NN_{55}$ 0.4*1.0*0.1=0.04 78

DT 1.0 The 11	NN 0.2 woman 22	Vt 1.0 saw 33	DT 1.0 the 44	NN 0.7 man 55	IN 0.5 with 66	DT 1.0 the 77	NN 0.1 telescope 88

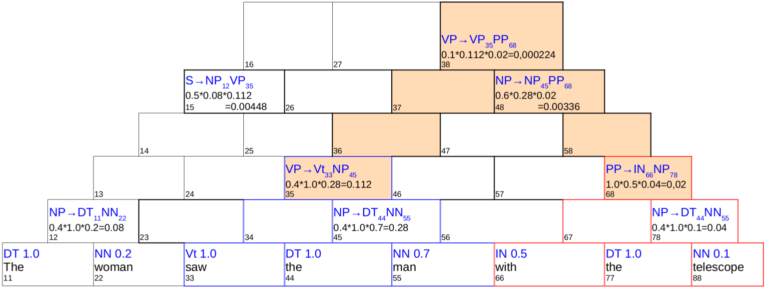
CKY Algorithm - Example



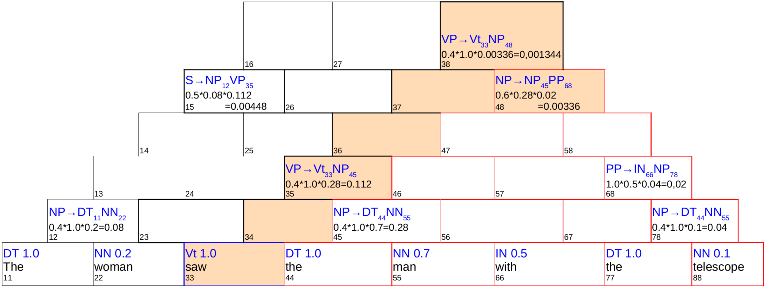
CKY Algorithm - Example



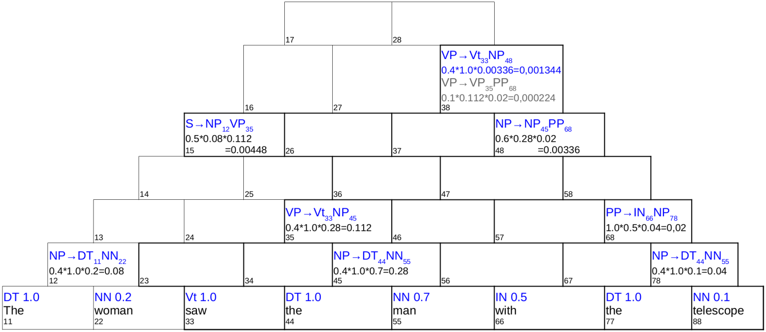
CKY Algorithm - Example



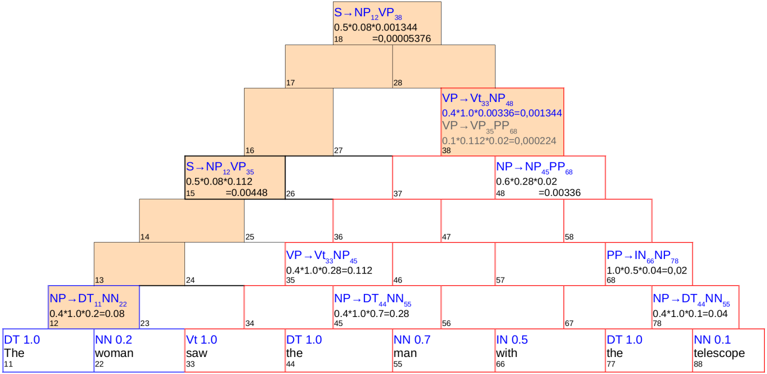
CKY Algorithm - Example



CKY Algorithm - Example



CKY Algorithm - Example



Outline

Earley Algorithm

- Top-down
- Can deal with any CFG (even left-recursive)
- Dynammic programming: Store partial results that can be reused in different candidate solutions.
- Intermediate results stored in a *chart* structure.

Earley Algorithm

Chart content:

- Set of items (aka *states*), each describing the applicability status of each rule after each word:

$$[i, j, X \to \alpha \bullet \beta]$$

- Backpath to recover which rules produced the complete tree:

$$\psi(i, j, X)$$

The goal is:

- Find if it is possible to reach $[1, n, S \to \alpha \bullet]$
- Recover $\psi(0, n, S)$ if it is
- Probabilistic versions exist, though not as straightforward as in CKY

Earley Algorithm

Parsing state examples:

$[0, 0, \mathrm{S} \rightarrow \bullet\ \mathrm{NP}\ \mathrm{VP}]$	A NP is expected at the beginning of the sentence
$[1, 2, \mathrm{NP} \rightarrow \mathrm{DT} \bullet \mathrm{NN}]$	A NP has been partially matched (DT was found between positions 1 and 2)
$[0, 3, \mathrm{VP} \rightarrow \mathrm{V}\ \mathrm{NP}\ \bullet]$	A VP has been completed between positions 0 and 3

Earley Algorithm

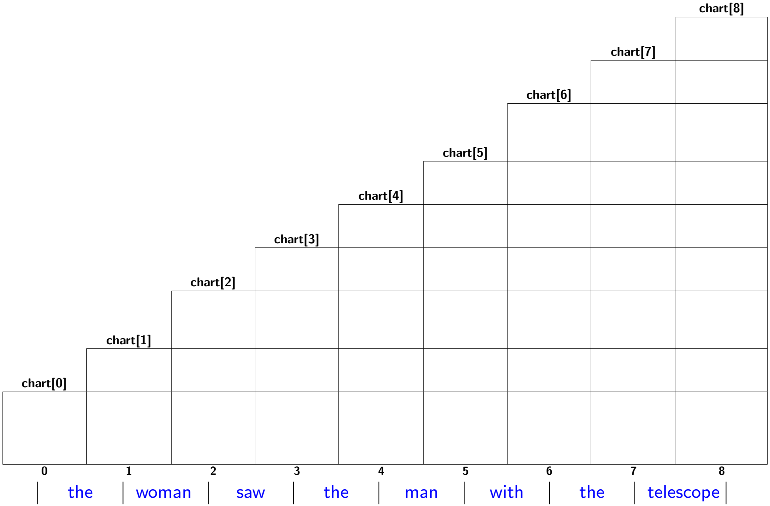
def Earley(words,grammar):
  chart = [[ ] for i in range(len(words)+1)]
  chart[0].append([0,0,$\gamma \rightarrow \bullet$S])
  for i in range(len(words)+1) :
    for state in chart[i] :
      if state.complete() : Complete(state)
      elif is_PoS(state.next()) : Scan(state)
      else : Predict(state)
  return chart

def Scan([i,j,$A \rightarrow \alpha \bullet B\beta$]):
  if B in words[j].PoS() : chart[j+1].append([j,j+1,$B \rightarrow$word[j]$\bullet$])

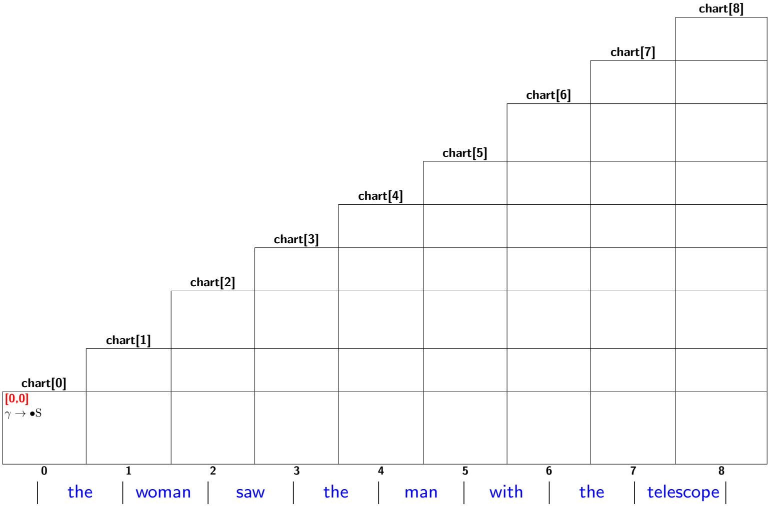
def Predict([i,j,$A \rightarrow \alpha \bullet B\beta$]):
  for $B \rightarrow \gamma$ in grammar : chart[j].append([j,j,$B \rightarrow \bullet\gamma$])

def Complete([k,j,$B \rightarrow \gamma\bullet$]):
  for [i,k,$A \rightarrow \alpha \bullet B\beta$] in chart[k] : chart[j].append([i,j,$A \rightarrow \alpha B \bullet \beta$])

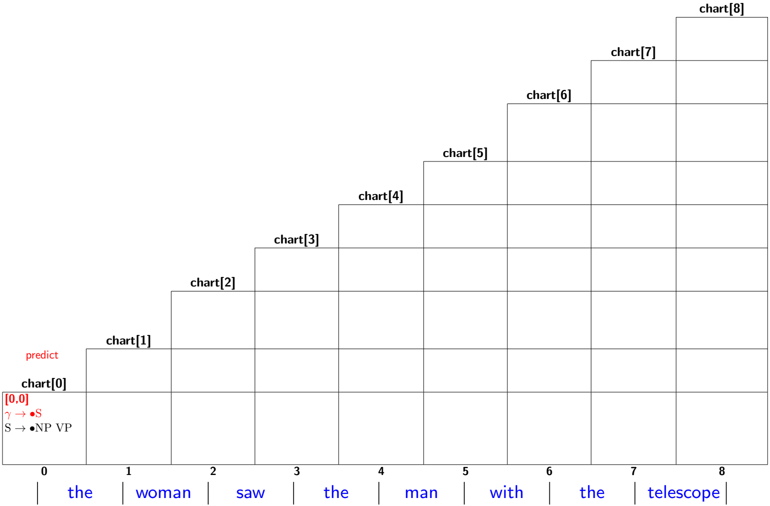
Earley Algorithm



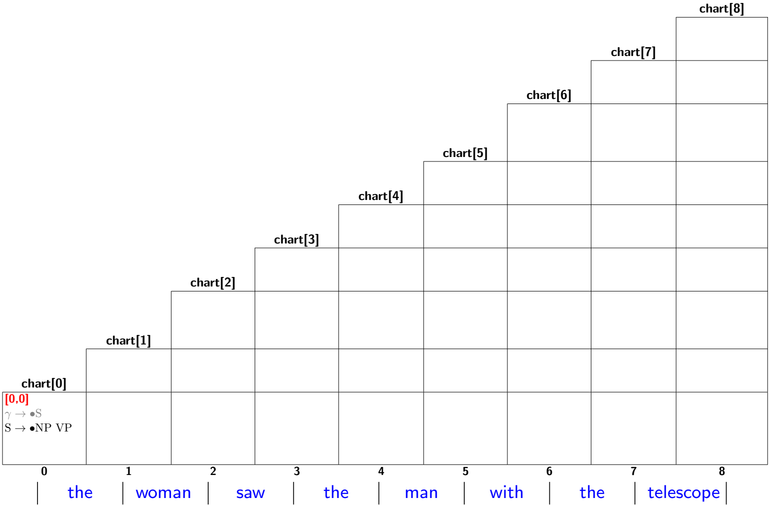
Earley Algorithm



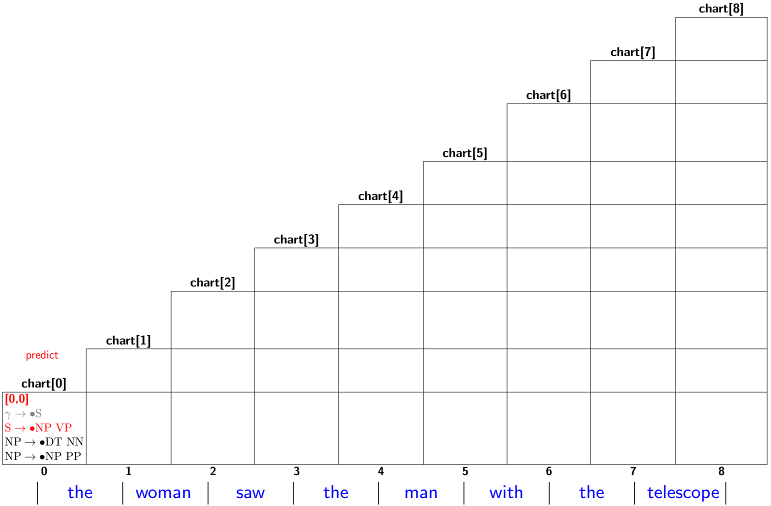
Earley Algorithm



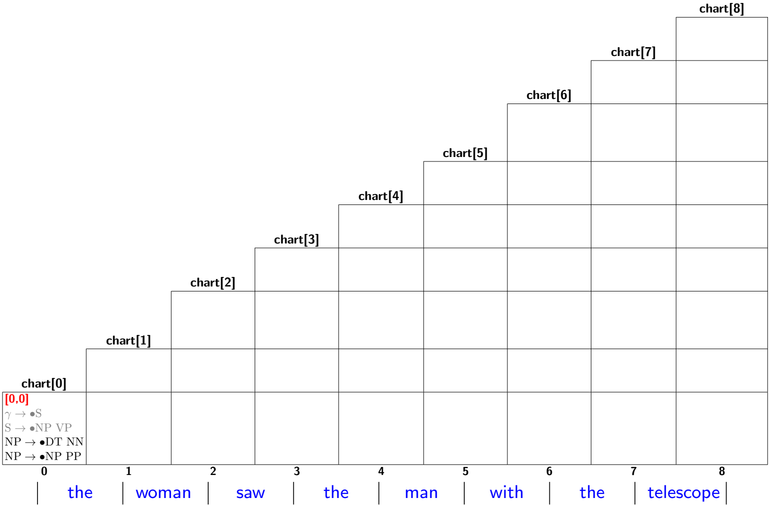
Earley Algorithm



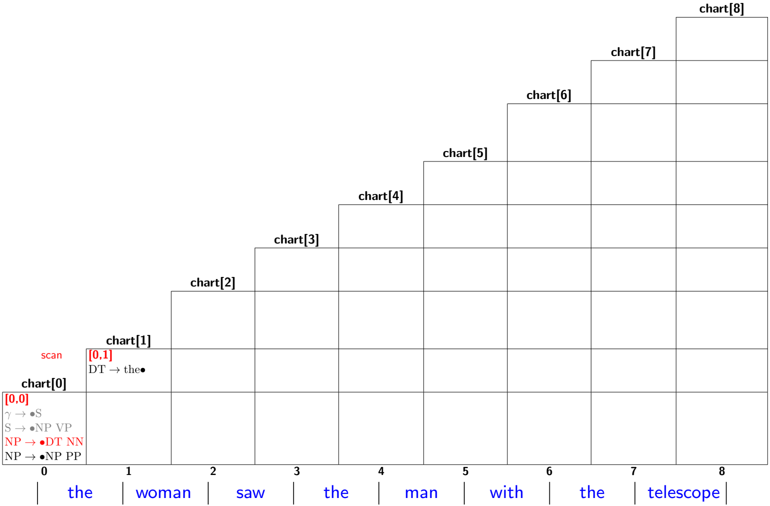
Earley Algorithm



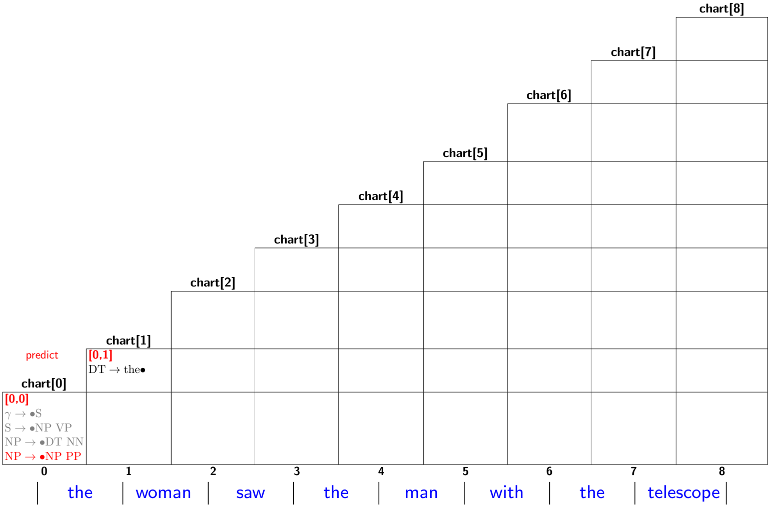
Earley Algorithm



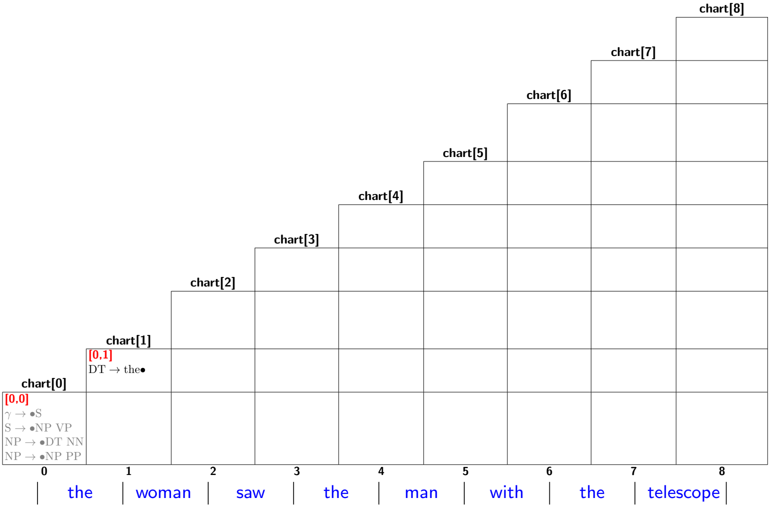
Earley Algorithm



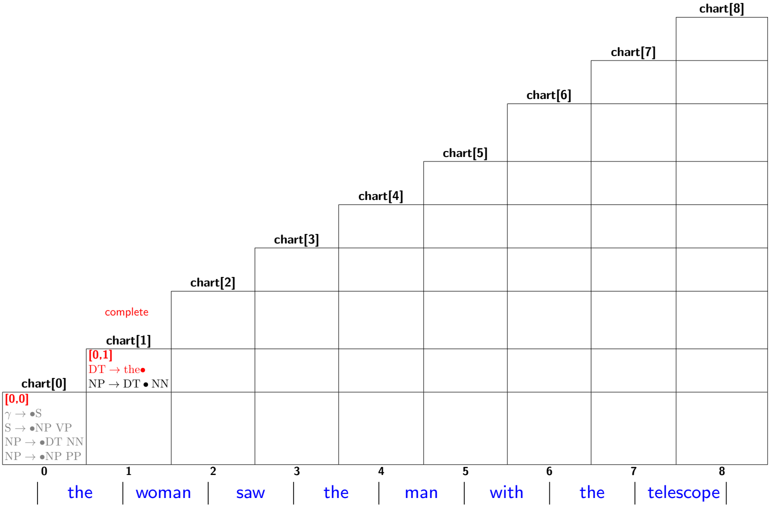
Earley Algorithm



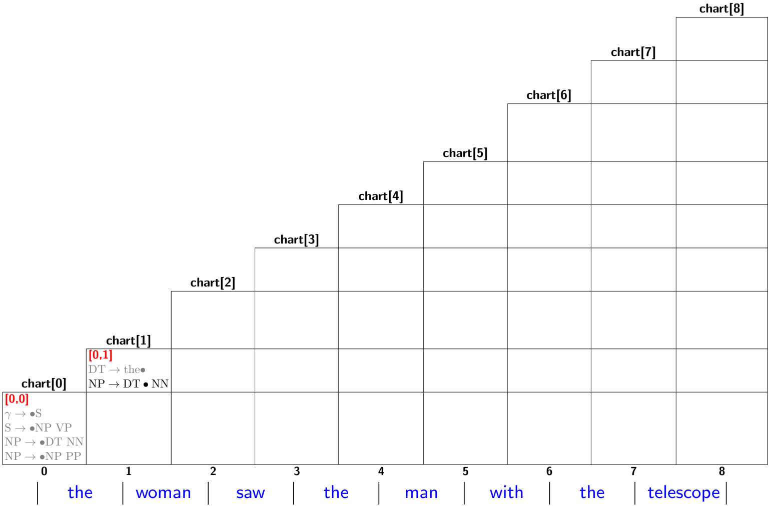
Earley Algorithm



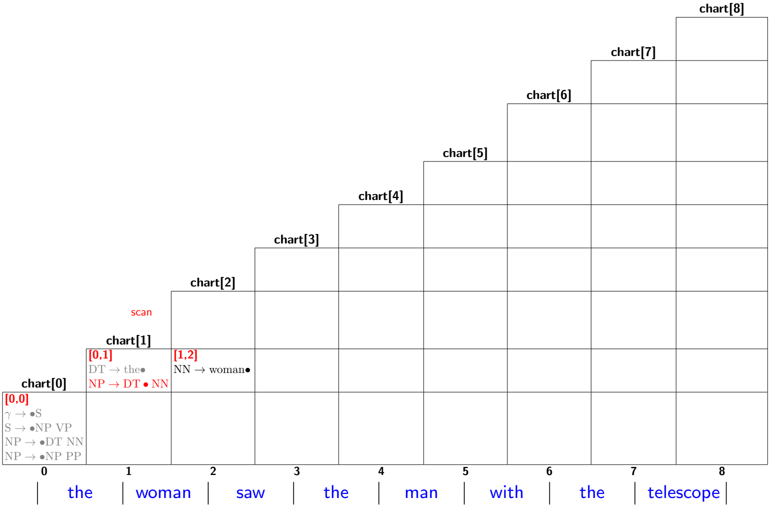
Earley Algorithm



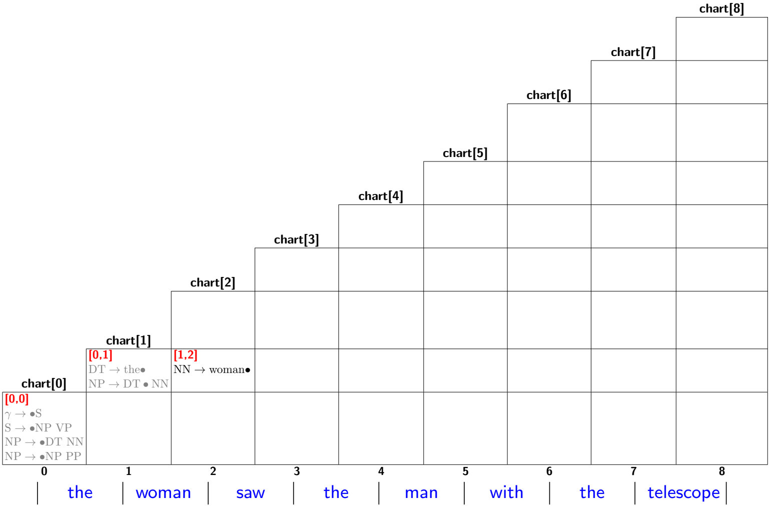
Earley Algorithm



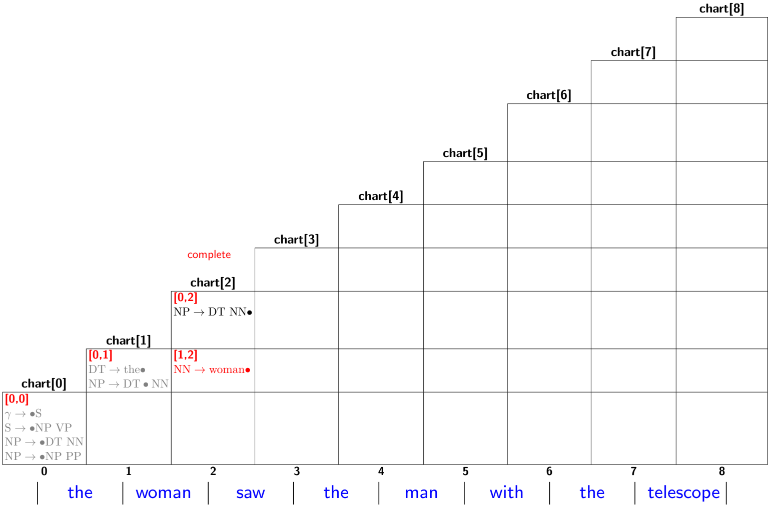
Earley Algorithm



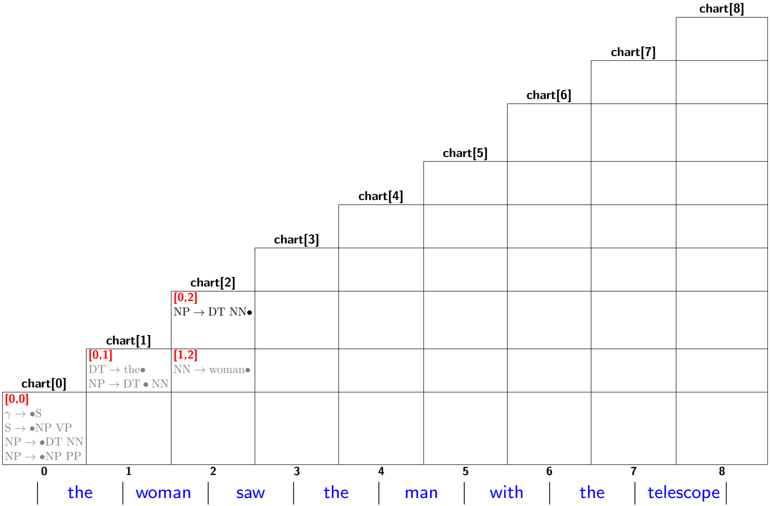
Earley Algorithm



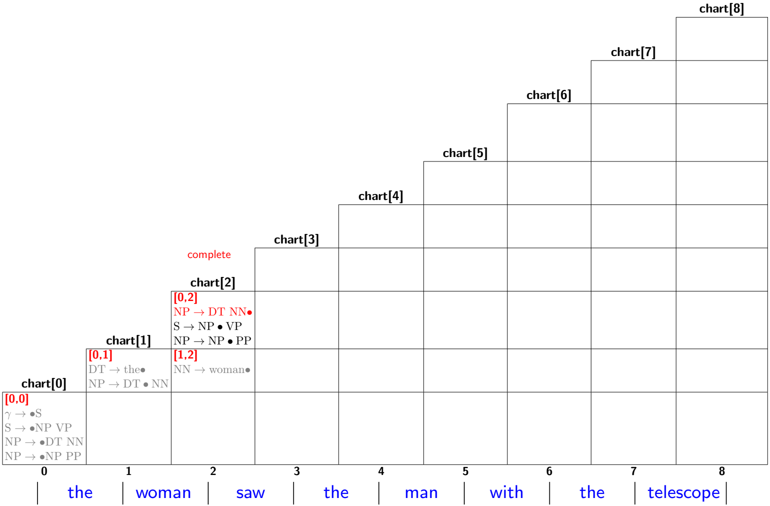
Earley Algorithm



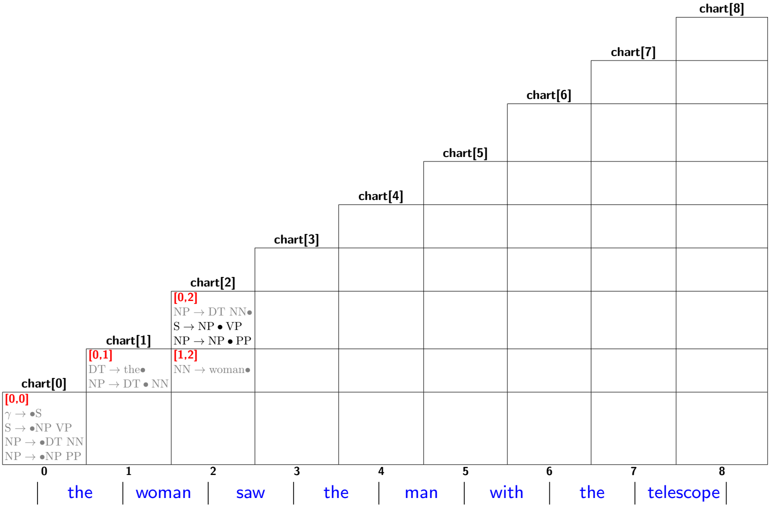
Earley Algorithm



Earley Algorithm



Earley Algorithm



Earley Algorithm

predict

chart[0]

[0,0]
$\gamma \to \bullet S$
$S \to \bullet NP\ VP$
$NP \to \bullet DT\ NN$
$NP \to \bullet NP\ PP$

chart[1]

[0,1]
$DT \to the \bullet$
$NP \to DT \bullet NN$

chart[2]

[0,2]
$NP \to DT\ NN \bullet$
$S \to NP \bullet VP$
$NP \to NP \bullet PP$

[1,2]
$NN \to woman \bullet$

[2,2]
$VP \to \bullet Vi$
$VP \to \bullet Vt\ NP$
$VP \to \bullet VP\ PP$

chart[3]

chart[4]

chart[5]

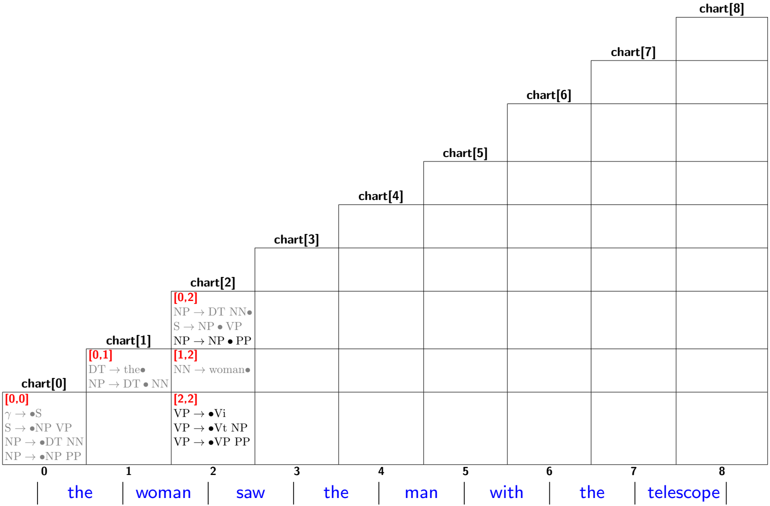
chart[6]

chart[7]

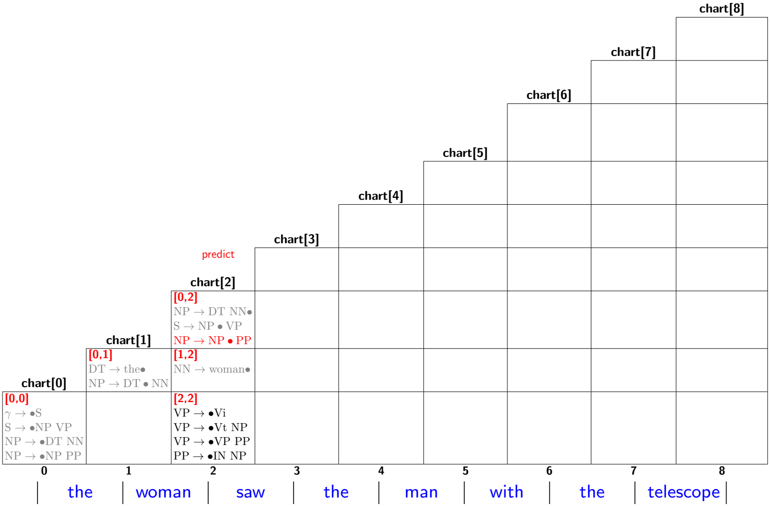
chart[8]

0		1		2		3		4		5		6		7		8
	the		woman		saw		the		man		with		the		telescope	

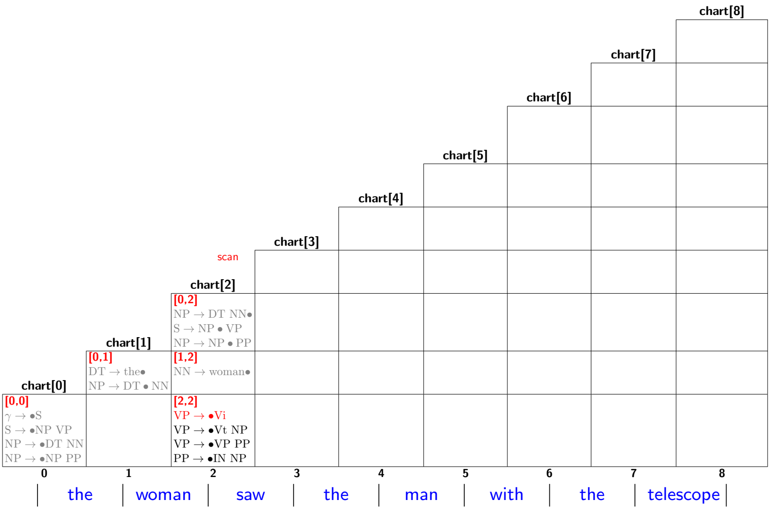
Earley Algorithm



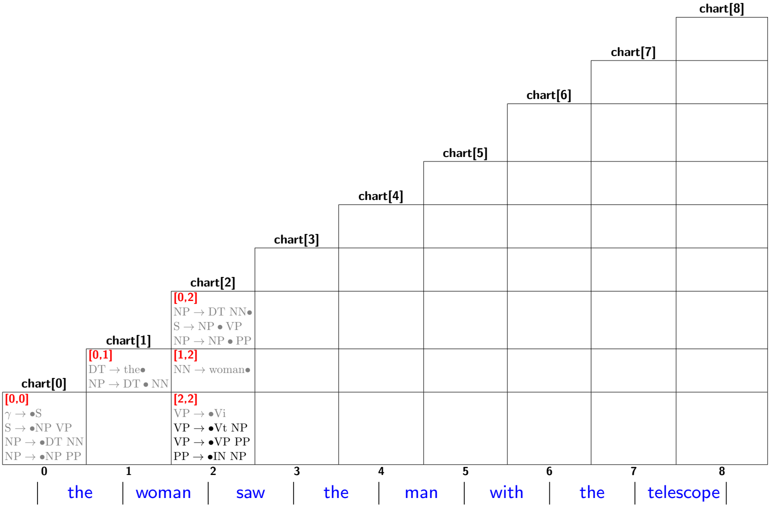
Earley Algorithm



Earley Algorithm



Earley Algorithm



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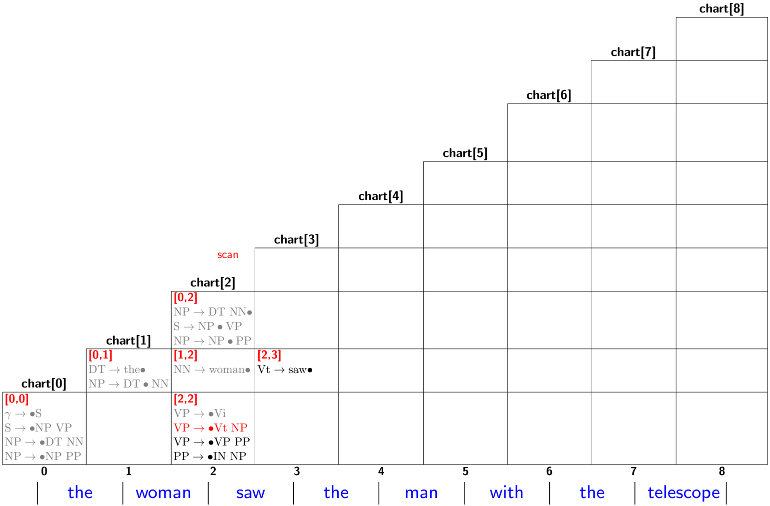

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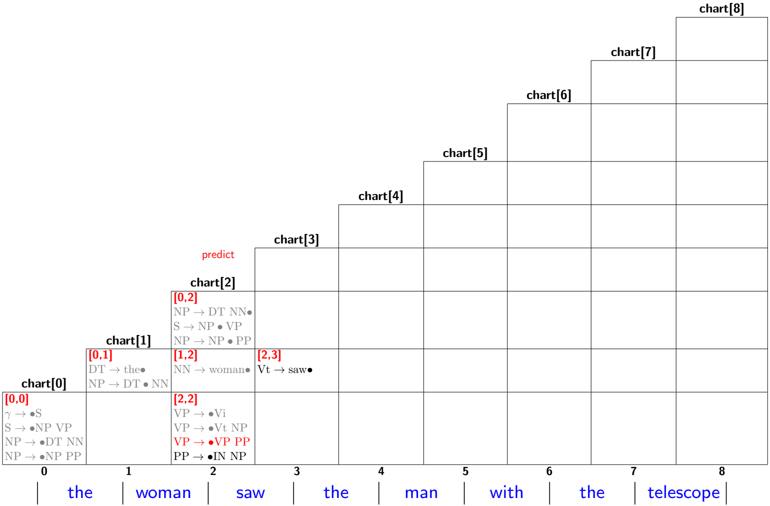


Earley Algorithm

chart[8]

chart[7]

chart[6]

chart[5]

chart[4]

chart[3]

scan

chart[2]

[0,2]
NP → DT NN•
S → NP • VP
NP → NP • PP

chart[1]

[0,1]
DT → the•
NP → DT • NN

[1,2]
NN → woman•

[2,3]
Vt → saw•

chart[0]

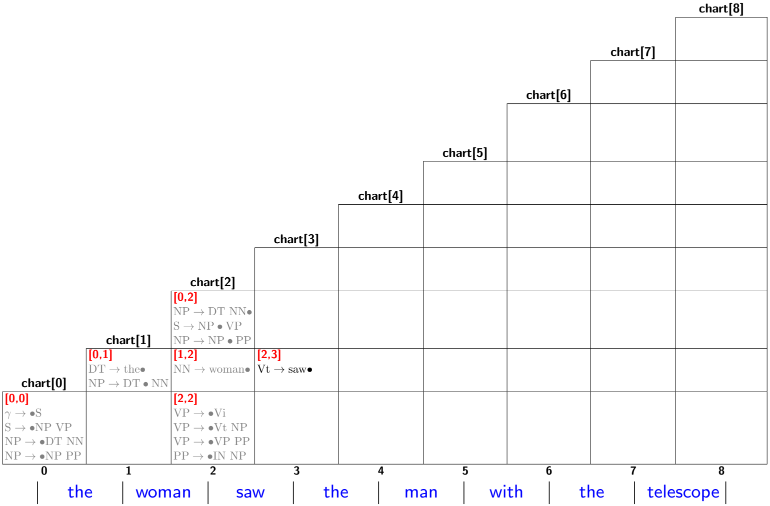
[0,0]
γ → •S
S → •NP VP
NP → •DT NN
NP → •NP PP

[2,2]
VP → •Vi
VP → •Vt NP
VP → •VP PP
PP → •IN NP

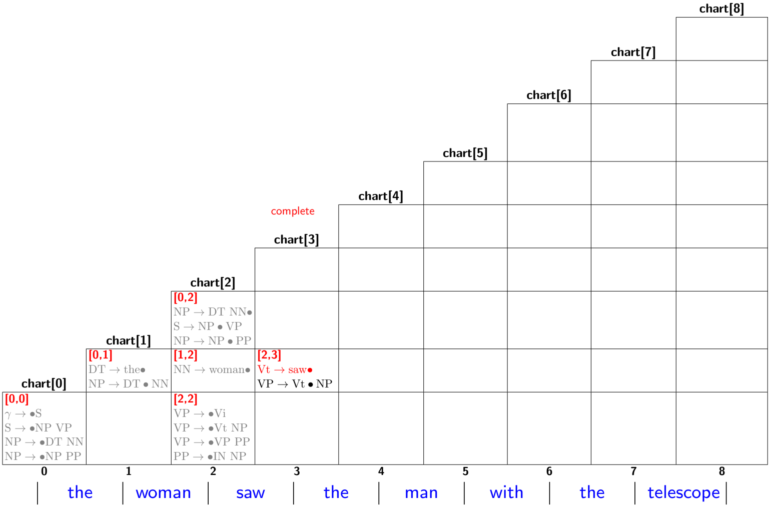
0 1 2 3 4 5 6 7 8

| the | woman | saw | the | man | with | the | telescope |

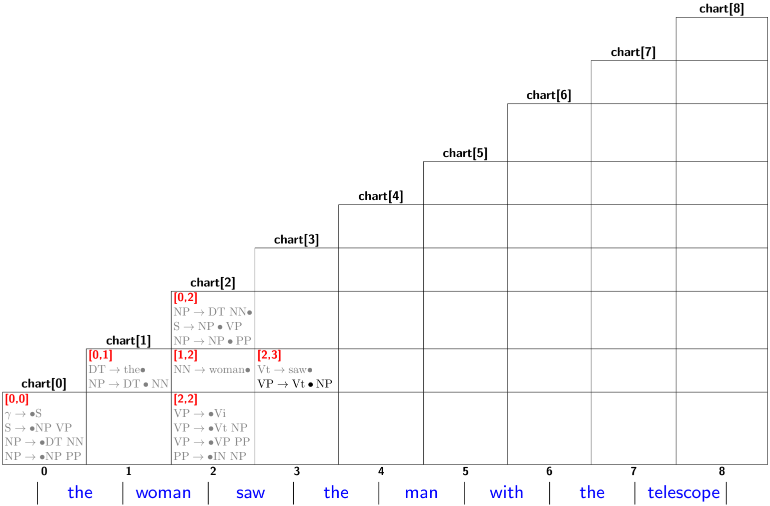
Earley Algorithm



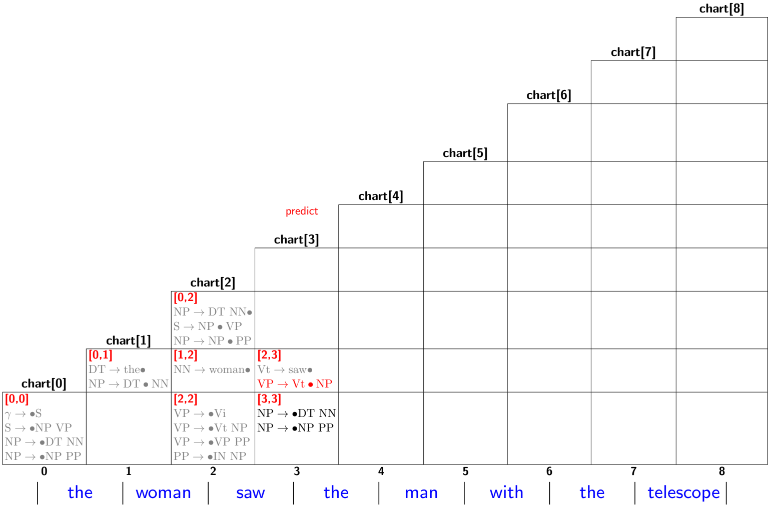
Earley Algorithm



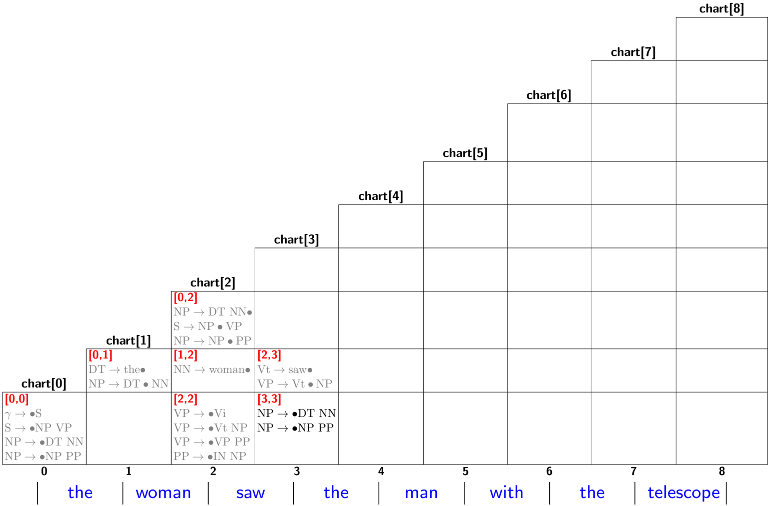
Earley Algorithm



Earley Algorithm



Earley Algorithm



Earley Algorithm

scan

chart[0] | chart[1] | chart[2] | chart[3] | chart[4] | chart[5] | chart[6] | chart[7] | chart[8]

[0,0]
$\gamma \to \bullet$S
S $\to \bullet$NP VP
NP $\to \bullet$DT NN
NP $\to \bullet$NP PP

[0,1]
DT $\to$ the$\bullet$
NP $\to$ DT $\bullet$ NN

[0,2]
NP $\to$ DT NN$\bullet$
S $\to$ NP $\bullet$ VP
NP $\to$ NP $\bullet$ PP

[1,2]
NN $\to$ woman$\bullet$

[2,2]
VP $\to \bullet$Vi
VP $\to \bullet$Vt NP
VP $\to \bullet$VP PP
PP $\to \bullet$IN NP

[2,3]
Vt $\to$ saw$\bullet$
VP $\to$ Vt $\bullet$ NP

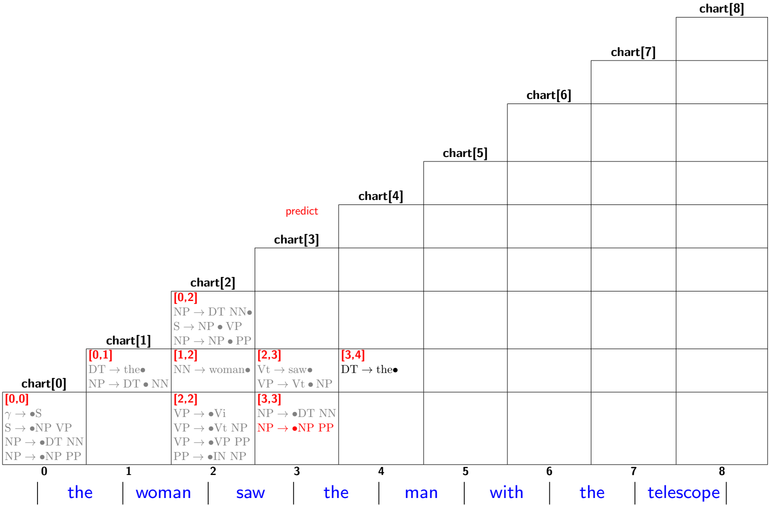
[3,3]
NP $\to \bullet$DT NN
NP $\to \bullet$NP PP

[3,4]
DT $\to$ the$\bullet$

0 1 2 3 4 5 6 7 8

| the | woman | saw | the | man | with | the | telescope |

Earley Algorithm



Earley Algorithm

chart[0] chart[1] chart[2] chart[3] chart[4] chart[5] chart[6] chart[7] chart[8]

[0,0]
$\gamma \rightarrow \bullet$S
S $\rightarrow \bullet$NP VP
NP $\rightarrow \bullet$DT NN
NP $\rightarrow \bullet$NP PP

[0,1]
DT $\rightarrow$ the$\bullet$
NP $\rightarrow$ DT $\bullet$ NN

[0,2]
NP $\rightarrow$ DT NN$\bullet$
S $\rightarrow$ NP $\bullet$ VP
NP $\rightarrow$ NP $\bullet$ PP

[1,2]
NN $\rightarrow$ woman$\bullet$

[2,2]
VP $\rightarrow \bullet$Vi
VP $\rightarrow \bullet$Vt NP
VP $\rightarrow \bullet$VP PP
PP $\rightarrow \bullet$IN NP

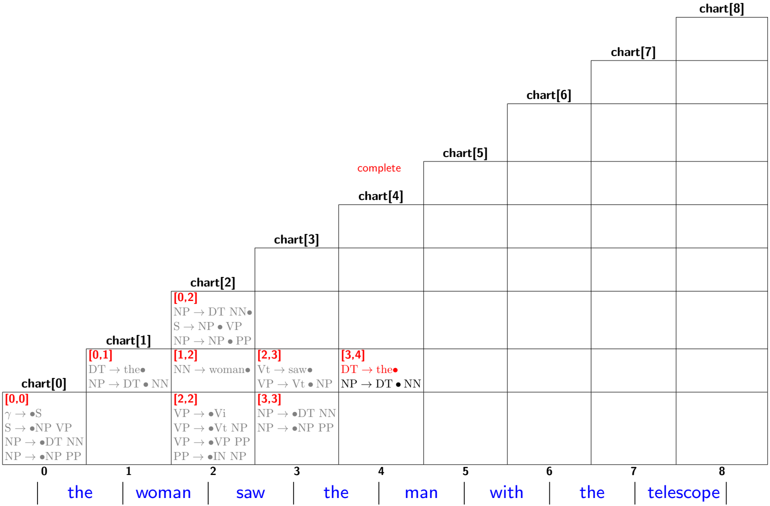
[2,3]
Vt $\rightarrow$ saw$\bullet$
VP $\rightarrow$ Vt $\bullet$ NP

[3,3]
NP $\rightarrow \bullet$DT NN
NP $\rightarrow \bullet$NP PP

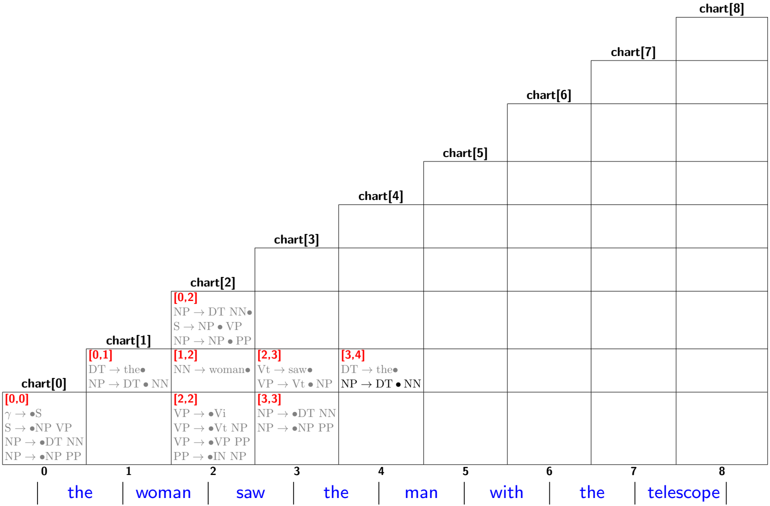
[3,4]
DT $\rightarrow$ the$\bullet$

0		1		2		3		4		5		6		7		8
	the		woman		saw		the		man		with		the		telescope	

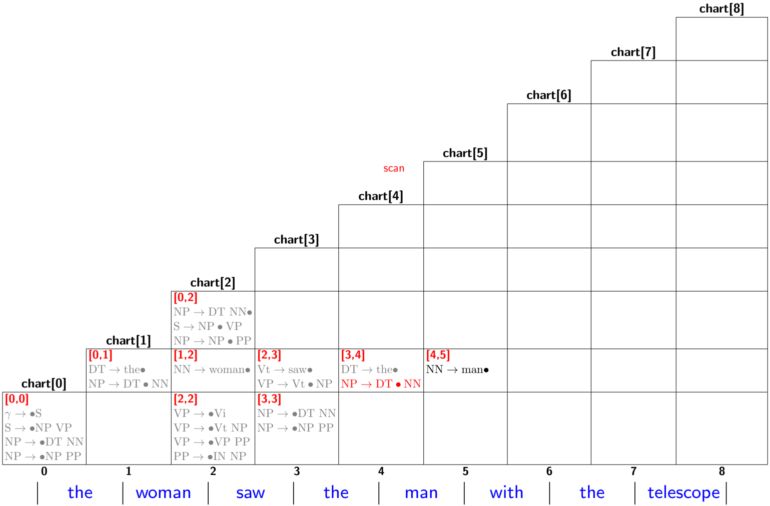
Earley Algorithm



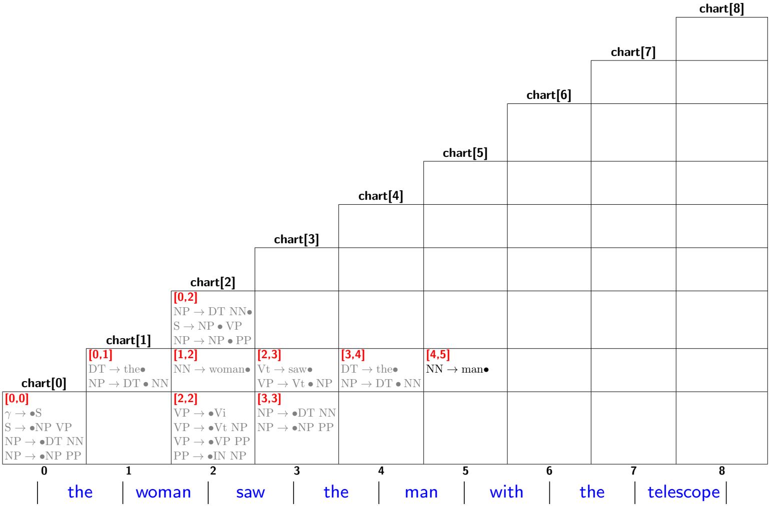
Earley Algorithm



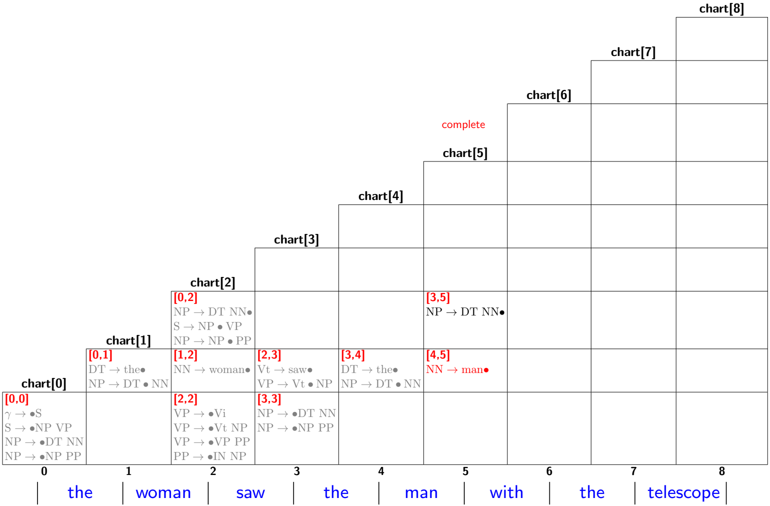
Earley Algorithm



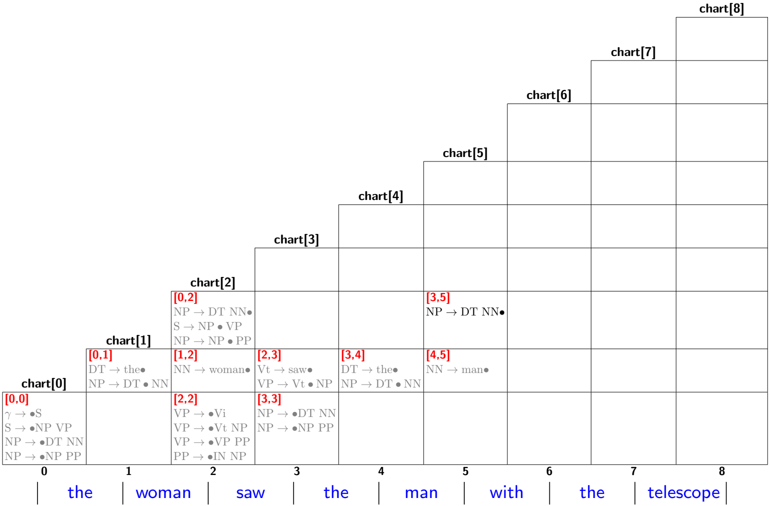
Earley Algorithm



Earley Algorithm



Earley Algorithm



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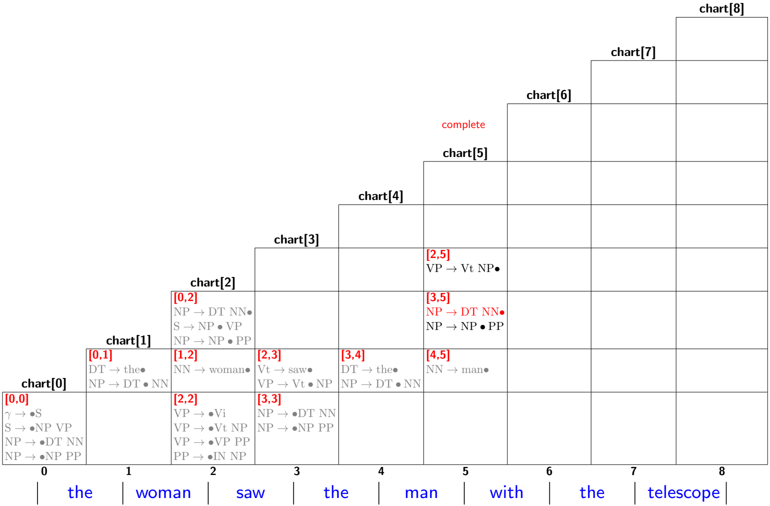

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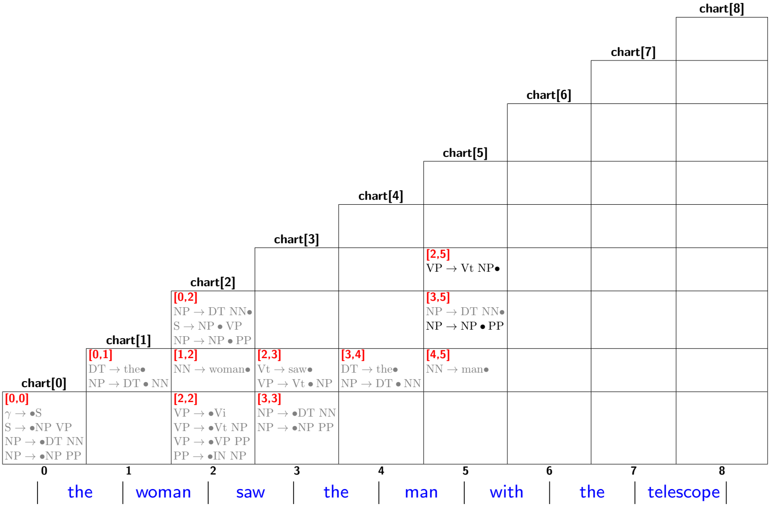
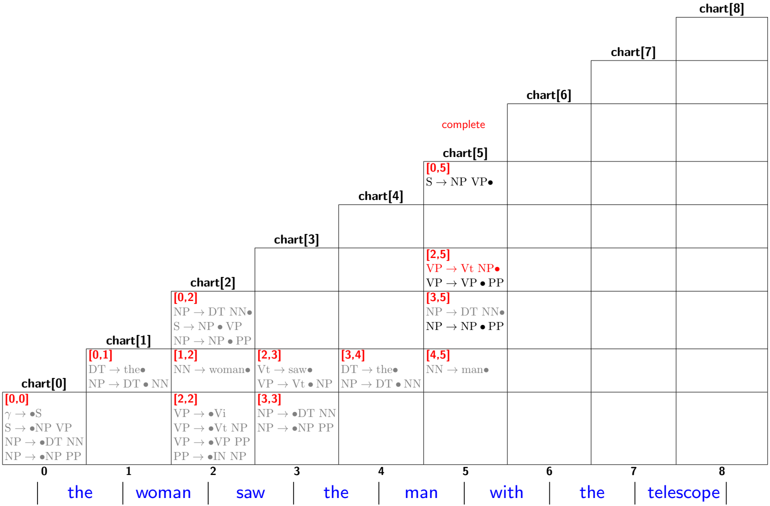
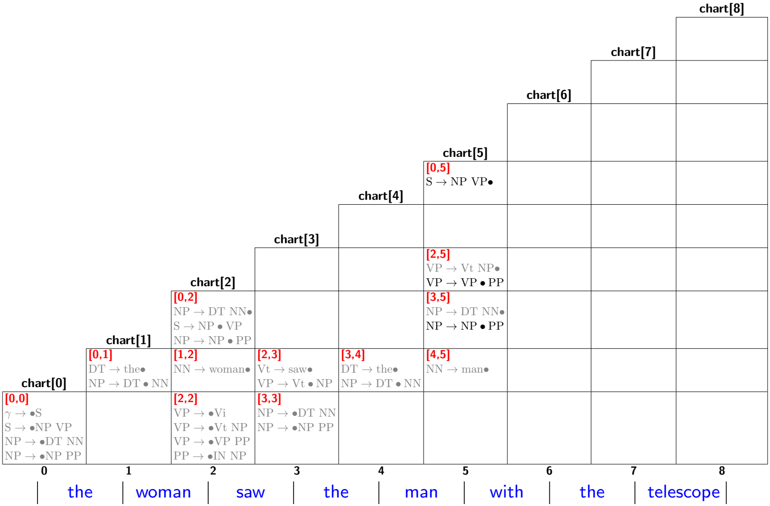


Earley Algorithm



Earley Algorithm



Earley Algorithm

complete

chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]	chart[7]	chart[8]
					[0,5] S → NP VP• γ → S•			
					[2,5] VP → Vt NP• VP → VP • PP			
		[0,2] NP → DT NN• S → NP • VP NP → NP • PP			**[3,5]** NP → DT NN• NP → NP • PP			
	[0,1] DT → the• NP → DT • NN	**[1,2]** NN → woman•	**[2,3]** Vt → saw• VP → Vt • NP	**[3,4]** DT → the• NP → DT • NN	**[4,5]** NN → man•			
[0,0] γ → •S S → •NP VP NP → •DT NN NP → •NP PP		**[2,2]** VP → •Vi VP → •Vt NP VP → •VP PP PP → •IN NP	**[3,3]** NP → •DT NN NP → •NP PP					

0 1 2 3 4 5 6 7 8

| the | woman | saw | the | man | with | the | telescope |

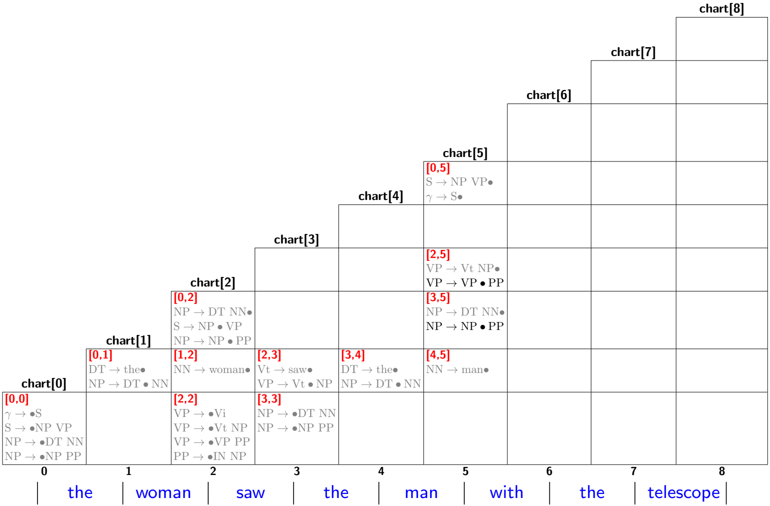
Earley Algorithm

complete

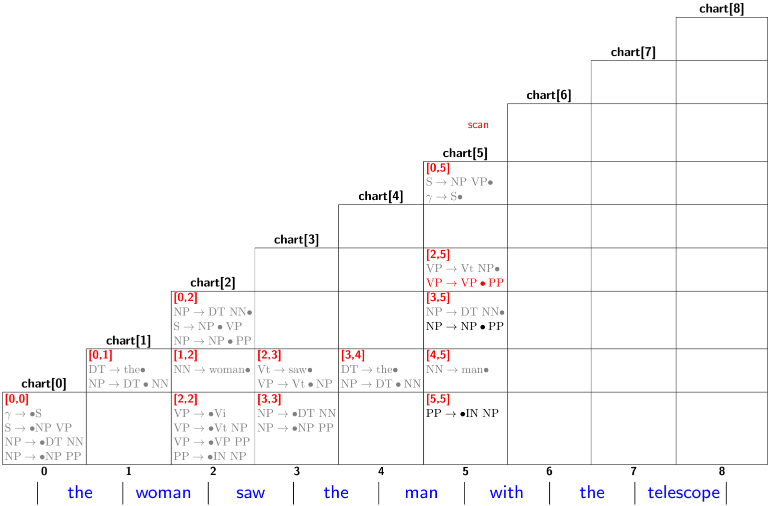
start	chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]	chart[7]	chart[8]
0	**[0,0]** $\gamma \to \bullet S$ $S \to \bullet NP\ VP$ $NP \to \bullet DT\ NN$ $NP \to \bullet NP\ PP$	**[0,1]** $DT \to the \bullet$ $NP \to DT \bullet NN$	**[0,2]** $NP \to DT\ NN \bullet$ $S \to NP \bullet VP$ $NP \to NP \bullet PP$			**[0,5]** $S \to NP\ VP \bullet$ $\gamma \to S \bullet$			
1			**[1,2]** $NN \to woman \bullet$						
2			**[2,2]** $VP \to \bullet Vi$ $VP \to \bullet Vt\ NP$ $VP \to \bullet VP\ PP$ $PP \to \bullet IN\ NP$	**[2,3]** $Vt \to saw \bullet$ $VP \to Vt \bullet NP$		**[2,5]** $VP \to Vt\ NP \bullet$ $VP \to VP \bullet PP$			
3				**[3,3]** $NP \to \bullet DT\ NN$ $NP \to \bullet NP\ PP$	**[3,4]** $DT \to the \bullet$ $NP \to DT \bullet NN$	**[3,5]** $NP \to DT\ NN \bullet$ $NP \to NP \bullet PP$			
4						**[4,5]** $NN \to man \bullet$			
5									
6									
7									
8									

0 | the | 1 | woman | 2 | saw | 3 | the | 4 | man | 5 | with | 6 | the | 7 | telescope | 8

Earley Algorithm



Earley Algorithm



Earley Algorithm

scan

chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]	chart[7]	chart[8]
					[0,5] S → NP VP• γ → S•			
					[2,5] VP → Vt NP• VP → VP • PP			
		[0,2] NP → DT NN• S → NP • VP NP → NP • PP			[3,5] NP → DT NN• NP → NP • PP			
	[0,1] DT → the• NP → DT • NN	[1,2] NN → woman•	[2,3] Vt → saw• VP → Vt • NP	[3,4] DT → the• NP → DT • NN	[4,5] NN → man•			
[0,0] γ → •S S → •NP VP NP → •DT NN NP → •NP PP		[2,2] VP → •Vi VP → •Vt NP VP → •VP PP PP → •IN NP	[3,3] NP → •DT NN NP → •NP PP		[5,5] PP → •IN NP			

0 1 2 3 4 5 6 7 8

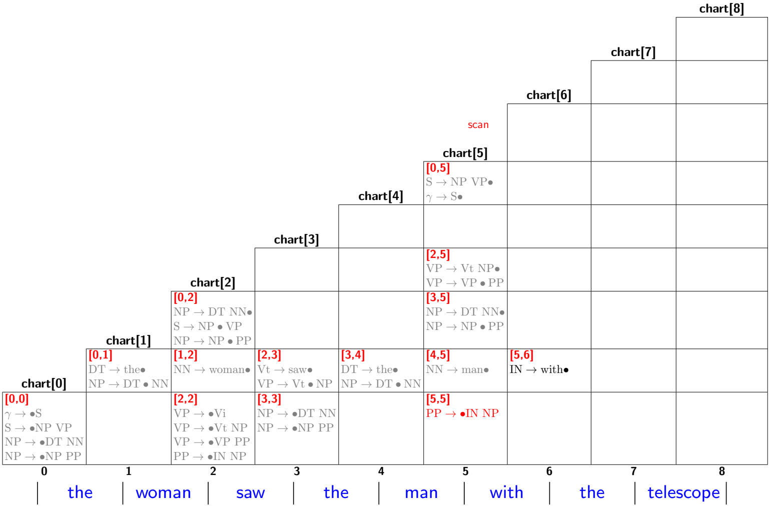
| the | woman | saw | the | man | with | the | telescope |

Earley Algorithm

chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]	chart[7]	chart[8]
					[0,5] S → NP VP• γ → S•			
					[2,5] VP → Vt NP• VP → VP • PP			
		[0,2] NP → DT NN• S → NP • VP NP → NP • PP			**[3,5]** NP → DT NN• NP → NP • PP			
	[0,1] DT → the• NP → DT • NN	**[1,2]** NN → woman•	**[2,3]** Vt → saw• VP → Vt • NP	**[3,4]** DT → the• NP → DT • NN	**[4,5]** NN → man•			
[0,0] γ → •S S → •NP VP NP → •DT NN NP → •NP PP		**[2,2]** VP → •Vi VP → •Vt NP VP → •VP PP PP → •IN NP	**[3,3]** NP → •DT NN NP → •NP PP		**[5,5]** PP → •IN NP			

0 | the | 1 | woman | 2 | saw | 3 | the | 4 | man | 5 | with | 6 | the | 7 | telescope | 8

Earley Algorithm



Earley Algorithm

	chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]	chart[7]	chart[8]
						[0,5] S $\rightarrow$ NP VP$\bullet$ $\gamma \rightarrow$ S$\bullet$			
						[2,5] VP $\rightarrow$ Vt NP$\bullet$ VP $\rightarrow$ VP $\bullet$ PP			
			[0,2] NP $\rightarrow$ DT NN$\bullet$ S $\rightarrow$ NP $\bullet$ VP NP $\rightarrow$ NP $\bullet$ PP			**[3,5]** NP $\rightarrow$ DT NN$\bullet$ NP $\rightarrow$ NP $\bullet$ PP			
		[0,1] DT $\rightarrow$ the$\bullet$ NP $\rightarrow$ DT $\bullet$ NN	**[1,2]** NN $\rightarrow$ woman$\bullet$	**[2,3]** Vt $\rightarrow$ saw$\bullet$ VP $\rightarrow$ Vt $\bullet$ NP	**[3,4]** DT $\rightarrow$ the$\bullet$ NP $\rightarrow$ DT $\bullet$ NN	**[4,5]** NN $\rightarrow$ man$\bullet$	**[5,6]** IN $\rightarrow$ with$\bullet$		
	[0,0] $\gamma \rightarrow \bullet$S S $\rightarrow \bullet$NP VP NP $\rightarrow \bullet$DT NN NP $\rightarrow \bullet$NP PP		**[2,2]** VP $\rightarrow \bullet$Vi VP $\rightarrow \bullet$Vt NP VP $\rightarrow \bullet$VP PP PP $\rightarrow \bullet$IN NP	**[3,3]** NP $\rightarrow \bullet$DT NN NP $\rightarrow \bullet$NP PP		**[5,5]** PP $\rightarrow \bullet$IN NP			
	0	1	2	3	4	5	6	7	8

| the | woman | saw | the | man | with | the | telescope |

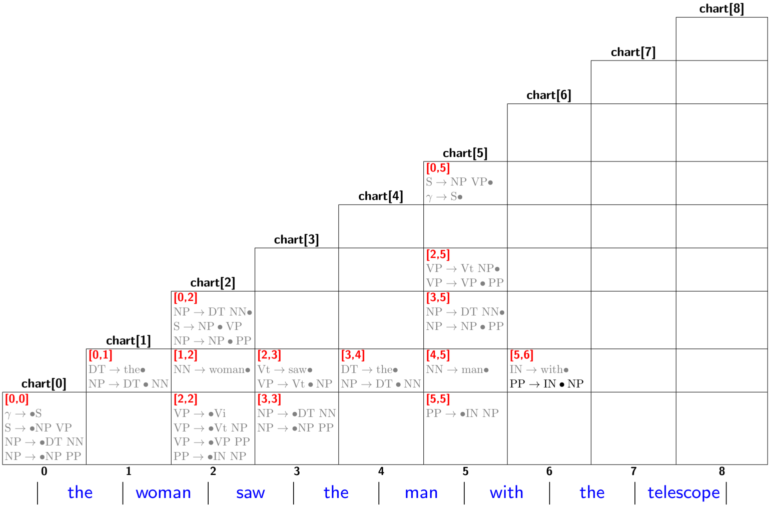
Earley Algorithm

complete

chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]	chart[7]	chart[8]
					[0,5] S → NP VP• γ → S•			
					[2,5] VP → Vt NP• VP → VP • PP			
		[0,2] NP → DT NN• S → NP • VP NP → NP • PP			**[3,5]** NP → DT NN• NP → NP • PP			
	[0,1] DT → the• NP → DT • NN	**[1,2]** NN → woman•	**[2,3]** Vt → saw• VP → Vt • NP	**[3,4]** DT → the• NP → DT • NN	**[4,5]** NN → man•	**[5,6]** IN → with• PP → IN • NP		
[0,0] γ → •S S → •NP VP NP → •DT NN NP → •NP PP		**[2,2]** VP → •Vi VP → •Vt NP VP → •VP PP PP → •IN NP	**[3,3]** NP → •DT NN NP → •NP PP		**[5,5]** PP → •IN NP			
0	1	2	3	4	5	6	7	8

| the | woman | saw | the | man | with | the | telescope |

Earley Algorithm



Earley Algorithm

predict

chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]	chart[7]	chart[8]
[0,0] γ → •S S → •NP VP NP → •DT NN NP → •NP PP	**[0,1]** DT → the• NP → DT • NN	**[0,2]** NP → DT NN• S → NP • VP NP → NP • PP			**[0,5]** S → NP VP• γ → S•			
		[1,2] NN → woman•						
		[2,2] VP → •Vi VP → •Vt NP VP → •VP PP PP → •IN NP	**[2,3]** Vt → saw• VP → Vt • NP		**[2,5]** VP → Vt NP• VP → VP • PP			
			[3,3] NP → •DT NN NP → •NP PP	**[3,4]** DT → the• NP → DT • NN	**[3,5]** NP → DT NN• NP → NP • PP			
					[4,5] NN → man•			
					[5,5] PP → •IN NP	**[5,6]** IN → with• PP → IN • NP		
						[6,6] NP → •DT NN NP → •NP PP		

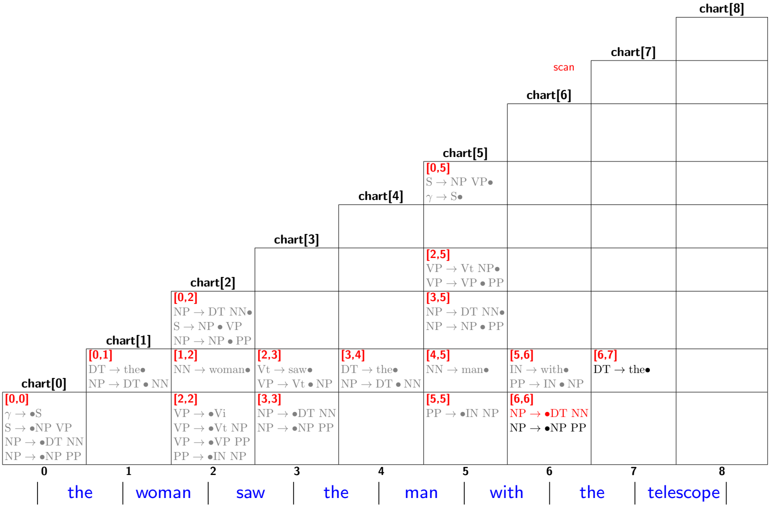
0 | the | 1 | woman | 2 | saw | 3 | the | 4 | man | 5 | with | 6 | the | 7 | telescope | 8

Earley Algorithm

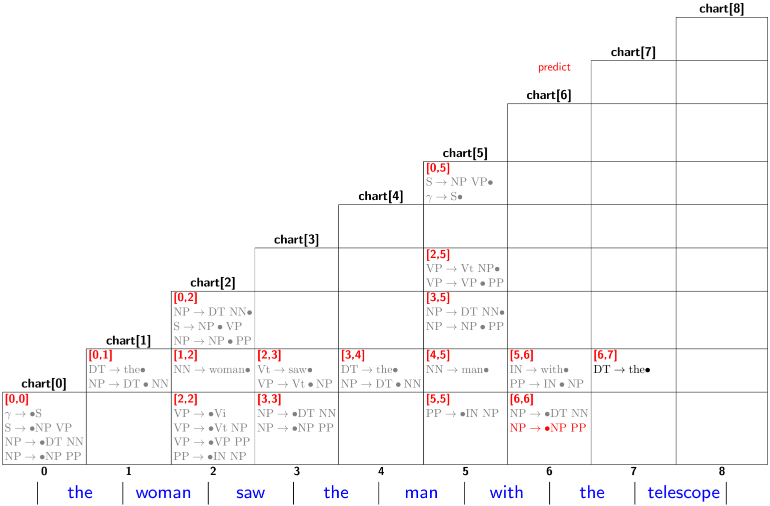
chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]	chart[7]	chart[8]
[0,0] $\gamma \rightarrow \bullet$S S $\rightarrow \bullet$NP VP NP $\rightarrow \bullet$DT NN NP $\rightarrow \bullet$NP PP	**[0,1]** DT $\rightarrow$ the$\bullet$ NP $\rightarrow$ DT $\bullet$ NN	**[0,2]** NP $\rightarrow$ DT NN$\bullet$ S $\rightarrow$ NP $\bullet$ VP NP $\rightarrow$ NP $\bullet$ PP			**[0,5]** S $\rightarrow$ NP VP$\bullet$ $\gamma \rightarrow$ S$\bullet$			
		[1,2] NN $\rightarrow$ woman$\bullet$						
		[2,2] VP $\rightarrow \bullet$Vi VP $\rightarrow \bullet$Vt NP VP $\rightarrow \bullet$VP PP PP $\rightarrow \bullet$IN NP	**[2,3]** Vt $\rightarrow$ saw$\bullet$ VP $\rightarrow$ Vt $\bullet$ NP		**[2,5]** VP $\rightarrow$ Vt NP$\bullet$ VP $\rightarrow$ VP $\bullet$ PP			
			[3,3] NP $\rightarrow \bullet$DT NN NP $\rightarrow \bullet$NP PP	**[3,4]** DT $\rightarrow$ the$\bullet$ NP $\rightarrow$ DT $\bullet$ NN	**[3,5]** NP $\rightarrow$ DT NN$\bullet$ NP $\rightarrow$ NP $\bullet$ PP			
					[4,5] NN $\rightarrow$ man$\bullet$			
					[5,5] PP $\rightarrow \bullet$IN NP	**[5,6]** IN $\rightarrow$ with$\bullet$ PP $\rightarrow$ IN $\bullet$ NP		
						[6,6] NP $\rightarrow \bullet$DT NN NP $\rightarrow \bullet$NP PP		

0 the **1** woman **2** saw **3** the **4** man **5** with **6** the **7** telescope **8**

Earley Algorithm



Earley Algorithm



Earley Algorithm

chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]	chart[7]	chart[8]
					[0,5] S → NP VP• γ → S•			
					[2,5] VP → Vt NP• VP → VP • PP			
		[0,2] NP → DT NN• S → NP • VP NP → NP • PP			[3,5] NP → DT NN• NP → NP • PP			
	[0,1] DT → the• NP → DT • NN	[1,2] NN → woman•	[2,3] Vt → saw• VP → Vt • NP	[3,4] DT → the• NP → DT • NN	[4,5] NN → man•	[5,6] IN → with• PP → IN • NP	[6,7] DT → the•	
[0,0] γ → •S S → •NP VP NP → •DT NN NP → •NP PP		[2,2] VP → •Vi VP → •Vt NP VP → •VP PP PP → •IN NP	[3,3] NP → •DT NN NP → •NP PP		[5,5] PP → •IN NP	[6,6] NP → •DT NN NP → •NP PP		

0 | the | 1 | woman | 2 | saw | 3 | the | 4 | man | 5 | with | 6 | the | 7 | telescope | 8

Earley Algorithm

Trees and Grammars

Constituency Parsing

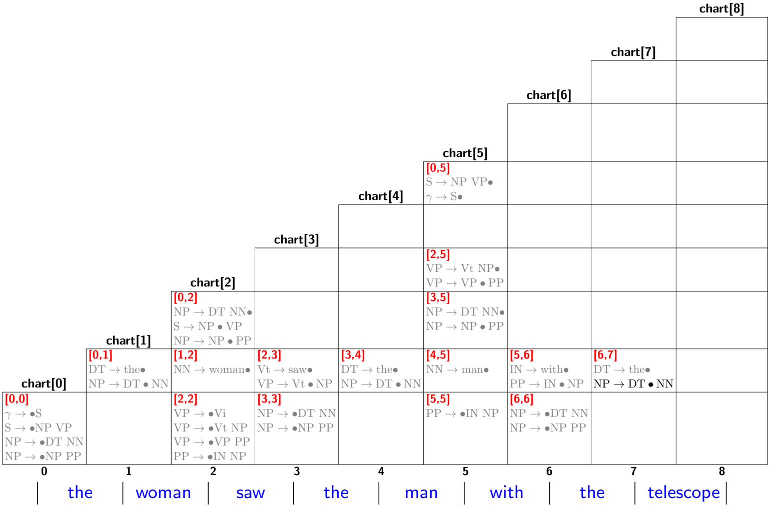
Earley Algorithm

complete

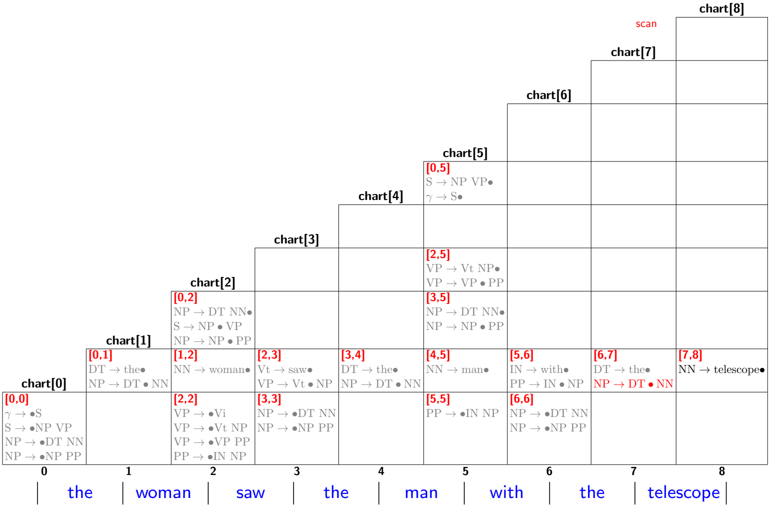
chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]	chart[7]	chart[8]
					[0,5] S $\rightarrow$ NP VP$\bullet$ $\gamma \rightarrow$ S$\bullet$			
					[2,5] VP $\rightarrow$ Vt NP$\bullet$ VP $\rightarrow$ VP $\bullet$ PP			
		[0,2] NP $\rightarrow$ DT NN$\bullet$ S $\rightarrow$ NP $\bullet$ VP NP $\rightarrow$ NP $\bullet$ PP			**[3,5]** NP $\rightarrow$ DT NN$\bullet$ NP $\rightarrow$ NP $\bullet$ PP			
	[0,1] DT $\rightarrow$ the$\bullet$ NP $\rightarrow$ DT $\bullet$ NN	**[1,2]** NN $\rightarrow$ woman$\bullet$	**[2,3]** Vt $\rightarrow$ saw$\bullet$ VP $\rightarrow$ Vt $\bullet$ NP	**[3,4]** DT $\rightarrow$ the$\bullet$ NP $\rightarrow$ DT $\bullet$ NN	**[4,5]** NN $\rightarrow$ man$\bullet$	**[5,6]** IN $\rightarrow$ with$\bullet$ PP $\rightarrow$ IN $\bullet$ NP	**[6,7]** DT $\rightarrow$ the$\bullet$ NP $\rightarrow$ DT $\bullet$ NN	
[0,0] $\gamma \rightarrow \bullet$S S $\rightarrow \bullet$NP VP NP $\rightarrow \bullet$DT NN NP $\rightarrow \bullet$NP PP		**[2,2]** VP $\rightarrow \bullet$Vi VP $\rightarrow \bullet$Vt NP VP $\rightarrow \bullet$VP PP PP $\rightarrow \bullet$IN NP	**[3,3]** NP $\rightarrow \bullet$DT NN NP $\rightarrow \bullet$NP PP		**[5,5]** PP $\rightarrow \bullet$IN NP	**[6,6]** NP $\rightarrow \bullet$DT NN NP $\rightarrow \bullet$NP PP		
0	1	2	3	4	5	6	7	8

| the | woman | saw | the | man | with | the | telescope |

Earley Algorithm



Earley Algorithm

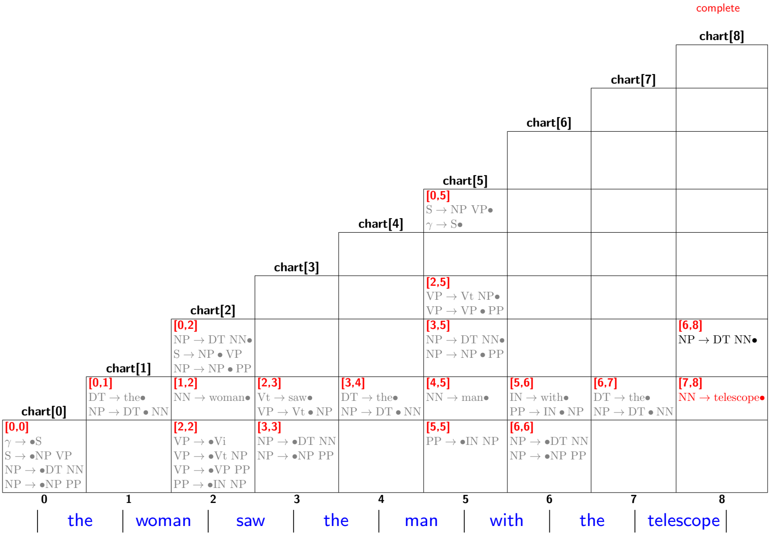


Earley Algorithm

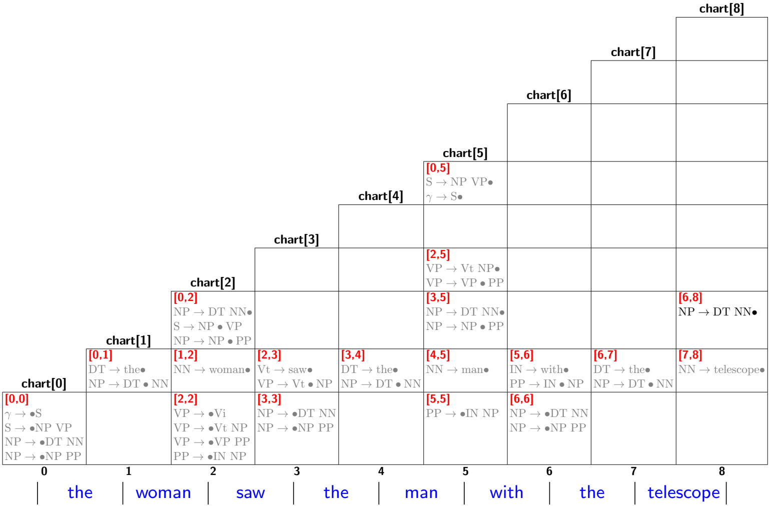
chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]	chart[7]	chart[8]
					[0,5] S → NP VP• γ → S•			
					[2,5] VP → Vt NP• VP → VP • PP			
		[0,2] NP → DT NN• S → NP • VP NP → NP • PP			[3,5] NP → DT NN• NP → NP • PP			
	[0,1] DT → the• NP → DT • NN	[1,2] NN → woman•	[2,3] Vt → saw• VP → Vt • NP	[3,4] DT → the• NP → DT • NN	[4,5] NN → man•	[5,6] IN → with• PP → IN • NP	[6,7] DT → the• NP → DT • NN	[7,8] NN → telescope•
[0,0] γ → •S S → •NP VP NP → •DT NN NP → •NP PP		[2,2] VP → •Vi VP → •Vt NP VP → •VP PP PP → •IN NP	[3,3] NP → •DT NN NP → •NP PP		[5,5] PP → •IN NP	[6,6] NP → •DT NN NP → •NP PP		
0	1	2	3	4	5	6	7	8

| the | woman | saw | the | man | with | the | telescope |

Earley Algorithm



Earley Algorithm



Earley Algorithm

complete

chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]	chart[7]	chart[8]
					[0,5] S → NP VP• γ → S•			
					[2,5] VP → Vt NP• VP → VP • PP			**[5,8]** PP → IN NP•
		[0,2] NP → DT NN• S → NP • VP NP → NP • PP			**[3,5]** NP → DT NN• NP → NP • PP			**[6,8]** NP → DT NN• NP → NP • PP
	[0,1] DT → the• NP → DT • NN	**[1,2]** NN → woman•	**[2,3]** Vt → saw• VP → Vt • NP	**[3,4]** DT → the• NP → DT • NN	**[4,5]** NN → man•	**[5,6]** IN → with• PP → IN • NP	**[6,7]** DT → the• NP → DT • NN	**[7,8]** NN → telescope•
[0,0] γ → •S S → •NP VP NP → •DT NN NP → •NP PP		**[2,2]** VP → •Vi VP → •Vt NP VP → •VP PP PP → •IN NP	**[3,3]** NP → •DT NN NP → •NP PP		**[5,5]** PP → •IN NP	**[6,6]** NP → •DT NN NP → •NP PP		
0	1	2	3	4	5	6	7	8

| the | woman | saw | the | man | with | the | telescope |

Earley Algorithm

chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]	chart[7]	chart[8]
					[0,5] S $\rightarrow$ NP VP$\bullet$ $\gamma \rightarrow$ S$\bullet$			
					[2,5] VP $\rightarrow$ Vt NP$\bullet$ VP $\rightarrow$ VP $\bullet$ PP			[5,8] PP $\rightarrow$ IN NP$\bullet$
		[0,2] NP $\rightarrow$ DT NN$\bullet$ S $\rightarrow$ NP $\bullet$ VP NP $\rightarrow$ NP $\bullet$ PP			[3,5] NP $\rightarrow$ DT NN$\bullet$ NP $\rightarrow$ NP $\bullet$ PP			[6,8] NP $\rightarrow$ DT NN$\bullet$ NP $\rightarrow$ NP $\bullet$ PP
	[0,1] DT $\rightarrow$ the$\bullet$ NP $\rightarrow$ DT $\bullet$ NN	[1,2] NN $\rightarrow$ woman$\bullet$	[2,3] Vt $\rightarrow$ saw$\bullet$ VP $\rightarrow$ Vt $\bullet$ NP	[3,4] DT $\rightarrow$ the$\bullet$ NP $\rightarrow$ DT $\bullet$ NN	[4,5] NN $\rightarrow$ man$\bullet$	[5,6] IN $\rightarrow$ with$\bullet$ PP $\rightarrow$ IN $\bullet$ NP	[6,7] DT $\rightarrow$ the$\bullet$ NP $\rightarrow$ DT $\bullet$ NN	[7,8] NN $\rightarrow$ telescope$\bullet$
[0,0] $\gamma \rightarrow \bullet$S S $\rightarrow \bullet$NP VP NP $\rightarrow \bullet$DT NN NP $\rightarrow \bullet$NP PP		[2,2] VP $\rightarrow \bullet$Vi VP $\rightarrow \bullet$Vt NP VP $\rightarrow \bullet$VP PP PP $\rightarrow \bullet$IN NP	[3,3] NP $\rightarrow \bullet$DT NN NP $\rightarrow \bullet$NP PP		[5,5] PP $\rightarrow \bullet$IN NP	[6,6] NP $\rightarrow \bullet$DT NN NP $\rightarrow \bullet$NP PP		
0	1	2	3	4	5	6	7	8

| the | woman | saw | the | man | with | the | telescope |

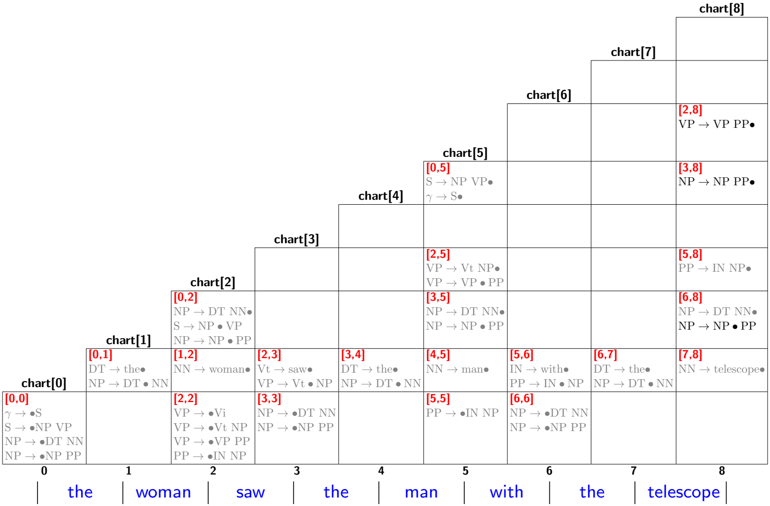
Earley Algorithm

complete

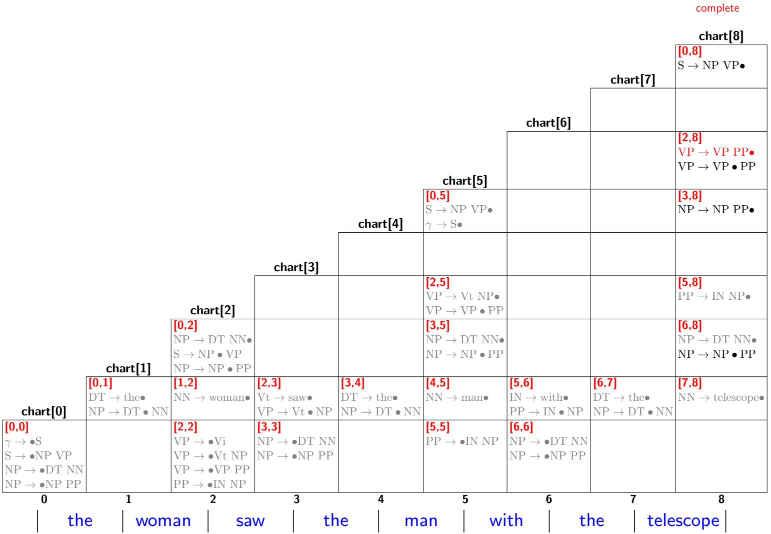
chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]	chart[7]	chart[8]
[0,0] γ → •S S → •NP VP NP → •DT NN NP → •NP PP	[0,1] DT → the• NP → DT • NN	[0,2] NP → DT NN• S → NP • VP NP → NP • PP			[0,5] S → NP VP• γ → S•			
		[1,2] NN → woman•						
		[2,2] VP → •Vi VP → •Vt NP VP → •VP PP PP → •IN NP	[2,3] Vt → saw• VP → Vt • NP		[2,5] VP → Vt NP• VP → VP • PP			[2,8] VP → VP PP•
			[3,3] NP → •DT NN NP → •NP PP	[3,4] DT → the• NP → DT • NN	[3,5] NP → DT NN• NP → NP • PP			[3,8] NP → NP PP•
					[4,5] NN → man•			
					[5,5] PP → •IN NP	[5,6] IN → with• PP → IN • NP		[5,8] PP → IN NP•
						[6,6] NP → •DT NN NP → •NP PP	[6,7] DT → the• NP → DT • NN	[6,8] NP → DT NN• NP → NP • PP
								[7,8] NN → telescope•

0 | the | 1 | woman | 2 | saw | 3 | the | 4 | man | 5 | with | 6 | the | 7 | telescope | 8

Earley Algorithm



Earley Algorithm



Earley Algorithm

chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]	chart[7]	chart[8]
								[0,8] S → NP VP•
								[2,8] VP → VP PP• VP → VP • PP
					[0,5] S → NP VP• γ → S•			**[3,8]** NP → NP PP•
					[2,5] VP → Vt NP• VP → VP • PP			**[5,8]** PP → IN NP•
		[0,2] NP → DT NN• S → NP • VP NP → NP • PP			**[3,5]** NP → DT NN• NP → NP • PP			**[6,8]** NP → DT NN• NP → NP • PP
	[0,1] DT → the• NP → DT • NN	**[1,2]** NN → woman•	**[2,3]** Vt → saw• VP → Vt • NP	**[3,4]** DT → the• NP → DT • NN	**[4,5]** NN → man•	**[5,6]** IN → with• PP → IN • NP	**[6,7]** DT → the• NP → DT • NN	**[7,8]** NN → telescope•
[0,0] γ → •S S → •NP VP NP → •DT NN NP → •NP PP		**[2,2]** VP → •Vi VP → •Vt NP VP → •VP PP PP → •IN NP	**[3,3]** NP → •DT NN NP → •NP PP		**[5,5]** PP → •IN NP	**[6,6]** NP → •DT NN NP → •NP PP		
0	1	2	3	4	5	6	7	8

| the | woman | saw | the | man | with | the | telescope |

Earley Algorithm

complete

	chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]	chart[7]	chart[8]
0	**[0,0]** $\gamma \to \bullet$S S $\to \bullet$NP VP NP $\to \bullet$DT NN NP $\to \bullet$NP PP	**[0,1]** DT $\to$ the$\bullet$ NP $\to$ DT $\bullet$ NN	**[0,2]** NP $\to$ DT NN$\bullet$ S $\to$ NP $\bullet$ VP NP $\to$ NP $\bullet$ PP			**[0,5]** S $\to$ NP VP$\bullet$ $\gamma \to$ S$\bullet$			**[0,8]** S $\to$ NP VP$\bullet$ $\gamma \to$ S$\bullet$
1			**[1,2]** NN $\to$ woman$\bullet$						
2			**[2,2]** VP $\to \bullet$Vi VP $\to \bullet$Vt NP VP $\to \bullet$VP PP PP $\to \bullet$IN NP	**[2,3]** Vt $\to$ saw$\bullet$ VP $\to$ Vt $\bullet$ NP		**[2,5]** VP $\to$ Vt NP$\bullet$ VP $\to$ VP $\bullet$ PP			**[2,8]** VP $\to$ VP PP$\bullet$ VP $\to$ VP $\bullet$ PP
3				**[3,3]** NP $\to \bullet$DT NN NP $\to \bullet$NP PP	**[3,4]** DT $\to$ the$\bullet$ NP $\to$ DT $\bullet$ NN	**[3,5]** NP $\to$ DT NN$\bullet$ NP $\to$ NP $\bullet$ PP			**[3,8]** NP $\to$ NP PP$\bullet$
4						**[4,5]** NN $\to$ man$\bullet$			
5						**[5,5]** PP $\to \bullet$IN NP	**[5,6]** IN $\to$ with$\bullet$ PP $\to$ IN $\bullet$ NP		**[5,8]** PP $\to$ IN NP$\bullet$
6							**[6,6]** NP $\to \bullet$DT NN NP $\to \bullet$NP PP	**[6,7]** DT $\to$ the$\bullet$ NP $\to$ DT $\bullet$ NN	**[6,8]** NP $\to$ DT NN$\bullet$ NP $\to$ NP $\bullet$ PP
7									**[7,8]** NN $\to$ telescope$\bullet$

0 | the | 1 | woman | 2 | saw | 3 | the | 4 | man | 5 | with | 6 | the | 7 | telescope | 8

Earley Algorithm

complete

	chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]	chart[7]	chart[8]
0	[0,0] γ → •S S → •NP VP NP → •DT NN NP → •NP PP	[0,1] DT → the• NP → DT • NN	[0,2] NP → DT NN• S → NP • VP NP → NP • PP			[0,5] S → NP VP• γ → S•			[0,8] S → NP VP• γ → S•
1			[1,2] NN → woman•						
2			[2,2] VP → •Vi VP → •Vt NP VP → •VP PP PP → •IN NP	[2,3] Vt → saw• VP → Vt • NP		[2,5] VP → Vt NP• VP → VP • PP			[2,8] VP → VP PP• VP → VP • PP
3				[3,3] NP → •DT NN NP → •NP PP	[3,4] DT → the• NP → DT • NN	[3,5] NP → DT NN• NP → NP • PP			[3,8] NP → NP PP•
4						[4,5] NN → man•			
5						[5,5] PP → •IN NP	[5,6] IN → with• PP → IN • NP		[5,8] PP → IN NP•
6							[6,6] NP → •DT NN NP → •NP PP	[6,7] DT → the• NP → DT • NN	[6,8] NP → DT NN• NP → NP • PP
7									[7,8] NN → telescope•
8									

0 | the | 1 | woman | 2 | saw | 3 | the | 4 | man | 5 | with | 6 | the | 7 | telescope | 8

Earley Algorithm

	chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]	chart[7]	chart[8]
0	[0,0] $\gamma \rightarrow \bullet$S S $\rightarrow \bullet$NP VP NP $\rightarrow \bullet$DT NN NP $\rightarrow \bullet$NP PP	[0,1] DT $\rightarrow$ the$\bullet$ NP $\rightarrow$ DT $\bullet$ NN	[0,2] NP $\rightarrow$ DT NN$\bullet$ S $\rightarrow$ NP $\bullet$ VP NP $\rightarrow$ NP $\bullet$ PP			[0,5] S $\rightarrow$ NP VP$\bullet$ $\gamma \rightarrow$ S$\bullet$			[0,8] S $\rightarrow$ NP VP$\bullet$ $\gamma \rightarrow$ S$\bullet$
1			[1,2] NN $\rightarrow$ woman$\bullet$						
2			[2,2] VP $\rightarrow \bullet$Vi VP $\rightarrow \bullet$Vt NP VP $\rightarrow \bullet$VP PP PP $\rightarrow \bullet$IN NP	[2,3] Vt $\rightarrow$ saw$\bullet$ VP $\rightarrow$ Vt $\bullet$ NP		[2,5] VP $\rightarrow$ Vt NP$\bullet$ VP $\rightarrow$ VP $\bullet$ PP			[2,8] VP $\rightarrow$ VP PP$\bullet$ VP $\rightarrow$ VP $\bullet$ PP
3				[3,3] NP $\rightarrow \bullet$DT NN NP $\rightarrow \bullet$NP PP	[3,4] DT $\rightarrow$ the$\bullet$ NP $\rightarrow$ DT $\bullet$ NN	[3,5] NP $\rightarrow$ DT NN$\bullet$ NP $\rightarrow$ NP $\bullet$ PP			[3,8] NP $\rightarrow$ NP PP$\bullet$
4						[4,5] NN $\rightarrow$ man$\bullet$			
5						[5,5] PP $\rightarrow \bullet$IN NP	[5,6] IN $\rightarrow$ with$\bullet$ PP $\rightarrow$ IN $\bullet$ NP		[5,8] PP $\rightarrow$ IN NP$\bullet$
6							[6,6] NP $\rightarrow \bullet$DT NN NP $\rightarrow \bullet$NP PP	[6,7] DT $\rightarrow$ the$\bullet$ NP $\rightarrow$ DT $\bullet$ NN	[6,8] NP $\rightarrow$ DT NN$\bullet$ NP $\rightarrow$ NP $\bullet$ PP
7									[7,8] NN $\rightarrow$ telescope$\bullet$

0 | the | 1 | woman | 2 | saw | 3 | the | 4 | man | 5 | with | 6 | the | 7 | telescope | 8

Earley Algorithm

predict

	chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]	chart[7]	chart[8]
start 0	[0,0] γ → •S S → •NP VP NP → •DT NN NP → •NP PP	[0,1] DT → the• NP → DT • NN	[0,2] NP → DT NN• S → NP • VP NP → NP • PP			[0,5] S → NP VP• γ → S•			[0,8] S → NP VP• γ → S•
start 1			[1,2] NN → woman•						
start 2			[2,2] VP → •Vi VP → •Vt NP VP → •VP PP PP → •IN NP	[2,3] Vt → saw• VP → Vt • NP		[2,5] VP → Vt NP• VP → VP • PP			[2,8] VP → VP PP• VP → VP • PP
start 3				[3,3] NP → •DT NN NP → •NP PP	[3,4] DT → the• NP → DT • NN	[3,5] NP → DT NN• NP → NP • PP			[3,8] NP → NP PP•
start 4						[4,5] NN → man•			
start 5						[5,5] PP → •IN NP	[5,6] IN → with• PP → IN • NP		[5,8] PP → IN NP•
start 6							[6,6] NP → •DT NN NP → •NP PP	[6,7] DT → the• NP → DT • NN	[6,8] NP → DT NN• NP → NP • PP
start 7									[7,8] NN → telescope•
start 8									[8,8] PP → •IN NP

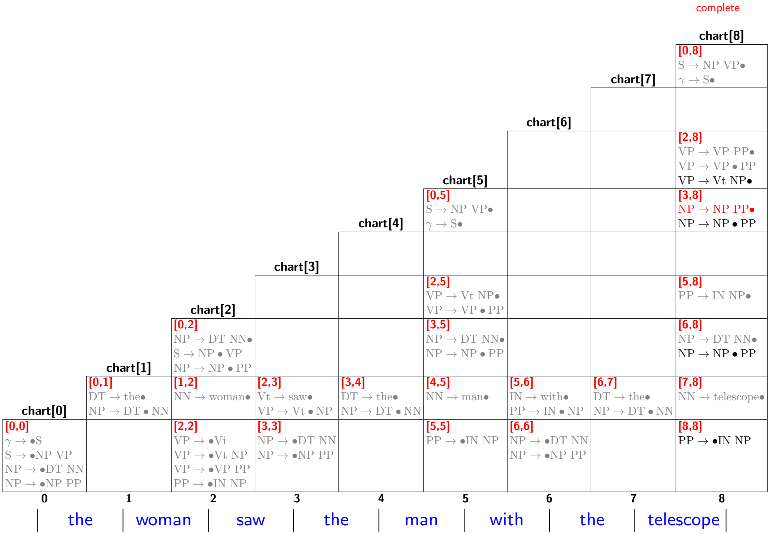
0 | the | 1 | woman | 2 | saw | 3 | the | 4 | man | 5 | with | 6 | the | 7 | telescope | 8

Earley Algorithm

	chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]	chart[7]	chart[8]
									[0,8] S $\rightarrow$ NP VP$\bullet$ $\gamma \rightarrow$ S$\bullet$
									[2,8] VP $\rightarrow$ VP PP$\bullet$ VP $\rightarrow$ VP $\bullet$ PP
						[0,5] S $\rightarrow$ NP VP$\bullet$ $\gamma \rightarrow$ S$\bullet$			[3,8] NP $\rightarrow$ NP PP$\bullet$
						[2,5] VP $\rightarrow$ Vt NP$\bullet$ VP $\rightarrow$ VP $\bullet$ PP			[5,8] PP $\rightarrow$ IN NP$\bullet$
			[0,2] NP $\rightarrow$ DT NN$\bullet$ S $\rightarrow$ NP $\bullet$ VP NP $\rightarrow$ NP $\bullet$ PP			[3,5] NP $\rightarrow$ DT NN$\bullet$ NP $\rightarrow$ NP $\bullet$ PP			[6,8] NP $\rightarrow$ DT NN$\bullet$ NP $\rightarrow$ NP $\bullet$ PP
		[0,1] DT $\rightarrow$ the$\bullet$ NP $\rightarrow$ DT $\bullet$ NN	[1,2] NN $\rightarrow$ woman$\bullet$	[2,3] Vt $\rightarrow$ saw$\bullet$ VP $\rightarrow$ Vt $\bullet$ NP	[3,4] DT $\rightarrow$ the$\bullet$ NP $\rightarrow$ DT $\bullet$ NN	[4,5] NN $\rightarrow$ man$\bullet$	[5,6] IN $\rightarrow$ with$\bullet$ PP $\rightarrow$ IN $\bullet$ NP	[6,7] DT $\rightarrow$ the$\bullet$ NP $\rightarrow$ DT $\bullet$ NN	[7,8] NN $\rightarrow$ telescope$\bullet$
	[0,0] $\gamma \rightarrow$ $\bullet$S S $\rightarrow$ $\bullet$NP VP NP $\rightarrow$ $\bullet$DT NN NP $\rightarrow$ $\bullet$NP PP		[2,2] VP $\rightarrow$ $\bullet$Vi VP $\rightarrow$ $\bullet$Vt NP VP $\rightarrow$ $\bullet$VP PP PP $\rightarrow$ $\bullet$IN NP	[3,3] NP $\rightarrow$ $\bullet$DT NN NP $\rightarrow$ $\bullet$NP PP		[5,5] PP $\rightarrow$ $\bullet$IN NP	[6,6] NP $\rightarrow$ $\bullet$DT NN NP $\rightarrow$ $\bullet$NP PP		[8,8] PP $\rightarrow$ $\bullet$IN NP
	0	1	2	3	4	5	6	7	8

| the | woman | saw | the | man | with | the | telescope |

Earley Algorithm



Earley Algorithm

complete

start	chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]	chart[7]	chart[8]
0	**[0,0]** γ → •S S → •NP VP NP → •DT NN NP → •NP PP	**[0,1]** DT → the• NP → DT • NN	**[0,2]** NP → DT NN• S → NP • VP NP → NP • PP			**[0,5]** S → NP VP• γ → S•			**[0,8]** S → NP VP• γ → S•
1			**[1,2]** NN → woman•						
2			**[2,2]** VP → •Vi VP → •Vt NP VP → •VP PP PP → •IN NP	**[2,3]** Vt → saw• VP → Vt • NP		**[2,5]** VP → Vt NP• VP → VP • PP			**[2,8]** VP → VP PP• VP → VP • PP VP → Vt NP•
3				**[3,3]** NP → •DT NN NP → •NP PP	**[3,4]** DT → the• NP → DT • NN	**[3,5]** NP → DT NN• NP → NP • PP			**[3,8]** NP → NP PP• NP → NP • PP
4						**[4,5]** NN → man•			
5						**[5,5]** PP → •IN NP	**[5,6]** IN → with• PP → IN • NP		**[5,8]** PP → IN NP•
6							**[6,6]** NP → •DT NN NP → •NP PP	**[6,7]** DT → the• NP → DT • NN	**[6,8]** NP → DT NN• NP → NP • PP
7									**[7,8]** NN → telescope•
8									**[8,8]** PP → •IN NP

0 | the | 1 | woman | 2 | saw | 3 | the | 4 | man | 5 | with | 6 | the | 7 | telescope | 8

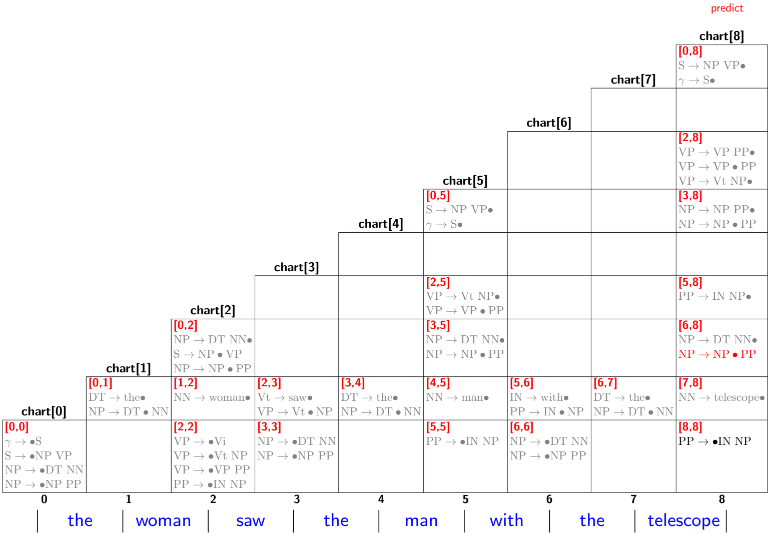
Earley Algorithm

predict

chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]	chart[7]	chart[8]
								[0,8] S → NP VP• γ → S•
								[2,8] VP → VP PP• VP → VP • PP VP → Vt NP•
					[0,5] S → NP VP• γ → S•			[3,8] NP → NP PP• NP → NP • PP
					[2,5] VP → Vt NP• VP → VP • PP			[5,8] PP → IN NP•
		[0,2] NP → DT NN• S → NP • VP NP → NP • PP			[3,5] NP → DT NN• NP → NP • PP			[6,8] NP → DT NN• NP → NP • PP
	[0,1] DT → the• NP → DT • NN	[1,2] NN → woman•	[2,3] Vt → saw• VP → Vt • NP	[3,4] DT → the• NP → DT • NN	[4,5] NN → man•	[5,6] IN → with• PP → IN • NP	[6,7] DT → the• NP → DT • NN	[7,8] NN → telescope•
[0,0] γ → •S S → •NP VP NP → •DT NN NP → •NP PP		[2,2] VP → •Vi VP → •Vt NP VP → •VP PP PP → •IN NP	[3,3] NP → •DT NN NP → •NP PP		[5,5] PP → •IN NP	[6,6] NP → •DT NN NP → •NP PP		[8,8] PP → •IN NP

0 | the | 1 | woman | 2 | saw | 3 | the | 4 | man | 5 | with | 6 | the | 7 | telescope | 8

Earley Algorithm



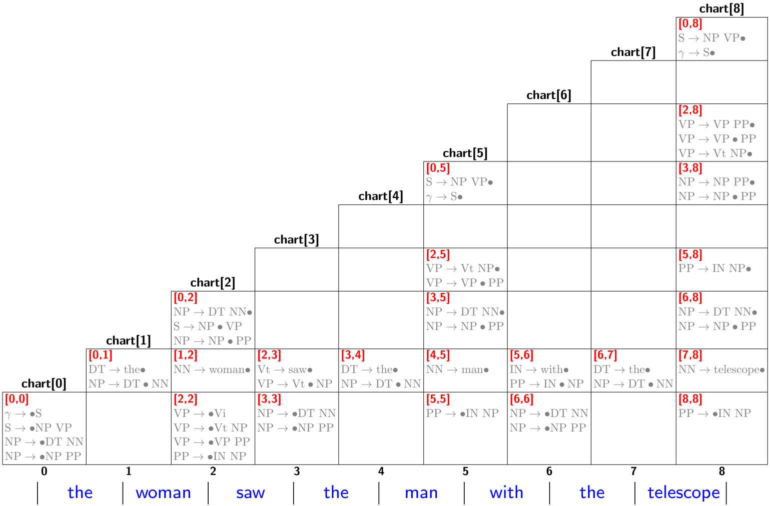
Earley Algorithm

scan

chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]	chart[7]	chart[8]
								[0,8] S → NP VP• γ → S•
								[2,8] VP → VP PP• VP → VP • PP VP → Vt NP•
					[0,5] S → NP VP• γ → S•			[3,8] NP → NP PP• NP → NP • PP
					[2,5] VP → Vt NP• VP → VP • PP			[5,8] PP → IN NP•
		[0,2] NP → DT NN• S → NP • VP NP → NP • PP			[3,5] NP → DT NN• NP → NP • PP			[6,8] NP → DT NN• NP → NP • PP
	[0,1] DT → the• NP → DT • NN	[1,2] NN → woman•	[2,3] Vt → saw• VP → Vt • NP	[3,4] DT → the• NP → DT • NN	[4,5] NN → man•	[5,6] IN → with• PP → IN • NP	[6,7] DT → the• NP → DT • NN	[7,8] NN → telescope•
[0,0] γ → •S S → •NP VP NP → •DT NN NP → •NP PP		[2,2] VP → •Vi VP → •Vt NP VP → •VP PP PP → •IN NP	[3,3] NP → •DT NN NP → •NP PP		[5,5] PP → •IN NP	[6,6] NP → •DT NN NP → •NP PP		[8,8] PP → •IN NP
0	1	2	3	4	5	6	7	8

| the | woman | saw | the | man | with | the | telescope |

Earley Algorithm



CKY *vs* Earley

CKY

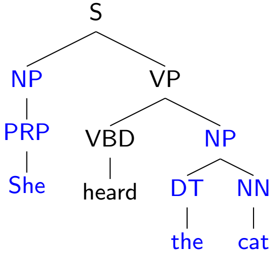
- Bottom-up
- Requires CNF
- Can compute all trees
- $\mathcal{O}(n^3)$
- Straightforward probabilistic version

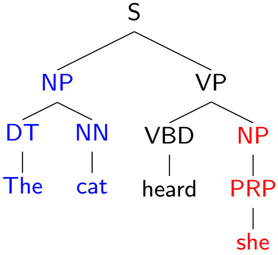
Earley

- Top-down
- Any CFG can be used, no need for CNF
- Can compute all trees
- $\mathcal{O}(n^3)$
- Not so straightforward probabilistic version

Why *context-free* ?

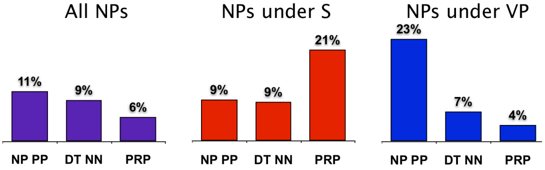
- Context-free means *independent of the context*, i.e, assumes that any expansion of a non-terminal is applicable, regardless of the context in which it occurs.





Natural Language is not Context-Free

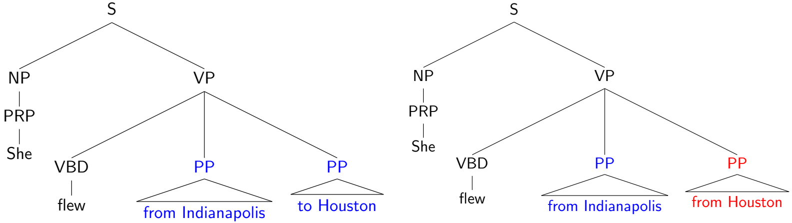
- NP expansion (for instance) is highly dependent on the parent of the NP



- Complete context independence is a too strong independence assumption for natural language.

Natural Language is not Context-Free

- The application of a rule may affect the applicability of others



Natural Language is not Context-Free

- May contain non-projective structures: