# From DPLL to CDCL SAT solvers

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Logic and Algebra in Computer Science

Session 4

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Departament de Llenguatges i Sistemes Informatics

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# Overview of the session

- Conflict Analysis
  - Motivating example
  - Backjumping
  - Conflict graph
  - Lemma shortening
- Lemma removal
- Decision heuristics
- Restarts
- Efficient implemetation of UnitProp:
  - Occur lists
  - Two-watched literals
- Final remarks


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# Motivating Example

$$\emptyset \Longrightarrow$$

$\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$

$\overline{p}_{11} \vee p_{13} \vee p_{16}$

$p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$

$\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$

$p_{10} \vee \overline{p}_8 \vee p_1$

$p_{10} \vee p_3$

$\overline{p}_3 \vee p_{26}$

$p_{10} \vee \overline{p}_5$

$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$

$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$

$p_{21} \vee \overline{p}_6$

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$p_{20} \vee p_{23}$

$\overline{p}_{20} \vee p_{24}$

$p_{25}$

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$p_{25} \Longrightarrow$

$p_{25}\, \overline{p}^d_{21} \Longrightarrow$

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$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6 \Longrightarrow$

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$p_{25}\, \overline{p}^d_{21} \Longrightarrow$

$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6 \Longrightarrow$

$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17} \Longrightarrow$

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$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6 \Longrightarrow$

$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17} \Longrightarrow$

$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22} \Longrightarrow$

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$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6 \Longrightarrow$

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$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22} \Longrightarrow$

$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13} \Longrightarrow$

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$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8 \Longrightarrow$

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$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23} \Longrightarrow$

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$$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20} \Longrightarrow$$

$$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23} \Longrightarrow$$

$$\underbrace{p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}}_{M}\; p^d_{11} \Longrightarrow$$

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$\overline{p}_3 \vee p_{26}$

$p_{10} \vee \overline{p}_5$

$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$

$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$

$p_{21} \vee \overline{p}_6$

$p_{21} \vee \overline{p}_{17}$

$\overline{p}_{22} \vee \overline{p}_{13}$

$p_{13} \vee p_8$

$\overline{p}_4 \vee p_{19}$

$p_{20} \vee p_{23}$

$\overline{p}_{20} \vee p_{24}$

$p_{25}$

$$\emptyset \Longrightarrow$$

$$p_{25} \Longrightarrow$$

$$p_{25}\, \overline{p}^d_{21} \Longrightarrow$$

$$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6 \Longrightarrow$$

$$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17} \Longrightarrow$$

$$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22} \Longrightarrow$$

$$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13} \Longrightarrow$$

$$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8 \Longrightarrow$$

$$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4 \Longrightarrow$$

$$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19} \Longrightarrow$$

$$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20} \Longrightarrow$$

$$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23} \Longrightarrow$$

$$\underbrace{p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}}_{M}\, p^d_{11} \Longrightarrow$$

$$M\, p^d_{11} \Longrightarrow$$

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# Motivating Example

$\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$

$\overline{p}_{11} \vee p_{13} \vee p_{16}$

$p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$

$\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$

$p_{10} \vee \overline{p}_8 \vee p_1$

$p_{10} \vee p_3$

$\overline{p}_3 \vee p_{26}$

$p_{10} \vee \overline{p}_5$

$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$

$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$

$p_{21} \vee \overline{p}_6$

$p_{21} \vee \overline{p}_{17}$

$\overline{p}_{22} \vee \overline{p}_{13}$

$p_{13} \vee p_8$

$\overline{p}_4 \vee p_{19}$

$p_{20} \vee p_{23}$

$\overline{p}_{20} \vee p_{24}$

$p_{25}$

$\emptyset \Longrightarrow$

$p_{25} \Longrightarrow$

$p_{25}\, \overline{p}^d_{21} \Longrightarrow$

$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6 \Longrightarrow$

$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17} \Longrightarrow$

$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22} \Longrightarrow$

$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13} \Longrightarrow$

$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8 \Longrightarrow$

$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4 \Longrightarrow$

$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19} \Longrightarrow$

$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20} \Longrightarrow$

$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23} \Longrightarrow$

$\underbrace{p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}}_{M}\, p^d_{11} \Longrightarrow$

$M\, p^d_{11} \Longrightarrow$

Before we continue...Some notation:

Literal $p_{25}$ belongs to decision level 0

Literals $\overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}$ belong to decision level 1

...

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# Motivating Example (2)

$\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$

$\overline{p}_{11} \vee p_{13} \vee p_{16}$

$p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$

$\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$

$p_{10} \vee \overline{p}_8 \vee p_1$

$p_{10} \vee p_3$

$\overline{p}_3 \vee p_{26}$

$p_{10} \vee \overline{p}_5$

$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$

$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$

$p_{21} \vee \overline{p}_6$

$p_{21} \vee \overline{p}_{17}$

$\overline{p}_{22} \vee \overline{p}_{13}$

$p_{13} \vee p_8$

$\overline{p}_4 \vee p_{19}$

$p_{20} \vee p_{23}$

$\overline{p}_{20} \vee p_{24}$

$p_{25}$

Remember $M$ is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

# Motivating Example (2)

$\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$

$\overline{p}_{11} \vee p_{13} \vee p_{16}$

$p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$

$\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$

$p_{10} \vee \overline{p}_8 \vee p_1$

$p_{10} \vee p_3$

$\overline{p}_3 \vee p_{26}$

$p_{10} \vee \overline{p}_5$

$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$

$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$

$p_{21} \vee \overline{p}_6$

$p_{21} \vee \overline{p}_{17}$

$\overline{p}_{22} \vee \overline{p}_{13}$

$p_{13} \vee p_8$

$\overline{p}_4 \vee p_{19}$

$p_{20} \vee p_{23}$

$\overline{p}_{20} \vee p_{24}$

$p_{25}$

Remember $M$ is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

$M\, p^d_{11} \Longrightarrow$

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# Motivating Example (2)

$\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$

$\overline{p}_{11} \vee p_{13} \vee p_{16}$

$p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$

$\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$

$p_{10} \vee \overline{p}_8 \vee p_1$

$p_{10} \vee p_3$

$\overline{p}_3 \vee p_{26}$

$p_{10} \vee \overline{p}_5$

$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$

$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$

$p_{21} \vee \overline{p}_6$

$p_{21} \vee \overline{p}_{17}$

$\overline{p}_{22} \vee \overline{p}_{13}$

$p_{13} \vee p_8$

$\overline{p}_4 \vee p_{19}$

$p_{20} \vee p_{23}$

$\overline{p}_{20} \vee p_{24}$

$p_{25}$

Remember $M$ is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

$M\, p^d_{11} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12} \Longrightarrow$

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# Motivating Example (2)

$\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$

$\overline{p}_{11} \vee p_{13} \vee p_{16}$

$p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$

$\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$

$p_{10} \vee \overline{p}_8 \vee p_1$

$p_{10} \vee p_3$

$\overline{p}_3 \vee p_{26}$

$p_{10} \vee \overline{p}_5$

$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$

$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$

$p_{21} \vee \overline{p}_6$

$p_{21} \vee \overline{p}_{17}$

$\overline{p}_{22} \vee \overline{p}_{13}$

$p_{13} \vee p_8$

$\overline{p}_4 \vee p_{19}$

$p_{20} \vee p_{23}$

$\overline{p}_{20} \vee p_{24}$

$p_{25}$

Remember $M$ is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

$M\, p^d_{11} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16} \Longrightarrow$

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# Motivating Example (2)

$\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$

$\overline{p}_{11} \vee p_{13} \vee p_{16}$

$p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$

$\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$

$p_{10} \vee \overline{p}_8 \vee p_1$

$p_{10} \vee p_3$

$\overline{p}_3 \vee p_{26}$

$p_{10} \vee \overline{p}_5$

$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$

$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$

$p_{21} \vee \overline{p}_6$

$p_{21} \vee \overline{p}_{17}$

$\overline{p}_{22} \vee \overline{p}_{13}$

$p_{13} \vee p_8$

$\overline{p}_4 \vee p_{19}$

$p_{20} \vee p_{23}$

$\overline{p}_{20} \vee p_{24}$

$p_{25}$

Remember $M$ is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

$M\, p^d_{11} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2 \Longrightarrow$

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# Motivating Example (2)

$\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$

$\overline{p}_{11} \vee p_{13} \vee p_{16}$

$p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$

$\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$

$p_{10} \vee \overline{p}_8 \vee p_1$

$p_{10} \vee p_3$

$\overline{p}_3 \vee p_{26}$

$p_{10} \vee \overline{p}_5$

$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$

$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$

$p_{21} \vee \overline{p}_6$

$p_{21} \vee \overline{p}_{17}$

$\overline{p}_{22} \vee \overline{p}_{13}$

$p_{13} \vee p_8$

$\overline{p}_4 \vee p_{19}$

$p_{20} \vee p_{23}$

$\overline{p}_{20} \vee p_{24}$

$p_{25}$

Remember $M$ is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

$M\, p^d_{11} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2 \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10} \Longrightarrow$

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# Motivating Example (2)

$\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$

$\overline{p}_{11} \vee p_{13} \vee p_{16}$

$p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$

$\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$

$p_{10} \vee \overline{p}_8 \vee p_1$

$p_{10} \vee p_3$

$\overline{p}_3 \vee p_{26}$

$p_{10} \vee \overline{p}_5$

$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$

$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$

$p_{21} \vee \overline{p}_6$

$p_{21} \vee \overline{p}_{17}$

$\overline{p}_{22} \vee \overline{p}_{13}$

$p_{13} \vee p_8$

$\overline{p}_4 \vee p_{19}$

$p_{20} \vee p_{23}$

$\overline{p}_{20} \vee p_{24}$

$p_{25}$

Remember $M$ is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

$M\, p^d_{11} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2 \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1 \Longrightarrow$

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# Motivating Example (2)

$\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$

$\overline{p}_{11} \vee p_{13} \vee p_{16}$

$p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$

$\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$

$p_{10} \vee \overline{p}_8 \vee p_1$

$p_{10} \vee p_3$

$\overline{p}_3 \vee p_{26}$

$p_{10} \vee \overline{p}_5$

$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$

$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$

$p_{21} \vee \overline{p}_6$

$p_{21} \vee \overline{p}_{17}$

$\overline{p}_{22} \vee \overline{p}_{13}$

$p_{13} \vee p_8$

$\overline{p}_4 \vee p_{19}$

$p_{20} \vee p_{23}$

$\overline{p}_{20} \vee p_{24}$

$p_{25}$

Remember $M$ is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

$M\, p^d_{11} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2 \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1 \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3 \Longrightarrow$

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# Motivating Example (2)

$\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$

$\overline{p}_{11} \vee p_{13} \vee p_{16}$

$p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$

$\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$

$p_{10} \vee \overline{p}_8 \vee p_1$

$p_{10} \vee p_3$

$\overline{p}_3 \vee p_{26}$

$p_{10} \vee \overline{p}_5$

$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$

$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$

$p_{21} \vee \overline{p}_6$

$p_{21} \vee \overline{p}_{17}$

$\overline{p}_{22} \vee \overline{p}_{13}$

$p_{13} \vee p_8$

$\overline{p}_4 \vee p_{19}$

$p_{20} \vee p_{23}$

$\overline{p}_{20} \vee p_{24}$

$p_{25}$

Remember $M$ is $p_{25}\, \overline{p}_{21}^{d}\, \overline{p}_6\, \overline{p}_{17}\, p_{22}^{d}\, \overline{p}_{13}\, p_8\, p_4^{d}\, p_{19}\, \overline{p}_{20}^{d}\, p_{23}$

$M\, p_{11}^{d} \Longrightarrow$

$M\, p_{11}^{d}\, \overline{p}_{12} \Longrightarrow$

$M\, p_{11}^{d}\, \overline{p}_{12}\, p_{16} \Longrightarrow$

$M\, p_{11}^{d}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2 \Longrightarrow$

$M\, p_{11}^{d}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10} \Longrightarrow$

$M\, p_{11}^{d}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1 \Longrightarrow$

$M\, p_{11}^{d}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3 \Longrightarrow$

$M\, p_{11}^{d}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3\, p_{26} \Longrightarrow$

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# Motivating Example (2)

$\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$

$\overline{p}_{11} \vee p_{13} \vee p_{16}$

$p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$

$\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$

$p_{10} \vee \overline{p}_8 \vee p_1$

$p_{10} \vee p_3$

$\overline{p}_3 \vee p_{26}$

$p_{10} \vee \overline{p}_5$

$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$

$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$

$p_{21} \vee \overline{p}_6$

$p_{21} \vee \overline{p}_{17}$

$\overline{p}_{22} \vee \overline{p}_{13}$

$p_{13} \vee p_8$

$\overline{p}_4 \vee p_{19}$

$p_{20} \vee p_{23}$

$\overline{p}_{20} \vee p_{24}$

$p_{25}$

Remember $M$ is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

$M\, p^d_{11} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2 \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1 \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3 \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3\, p_{26} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3\, p_{26}\, \overline{p}_5 \Longrightarrow$

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# Motivating Example (2)

$\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$

$\overline{p}_{11} \vee p_{13} \vee p_{16}$

$p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$

$\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$

$p_{10} \vee \overline{p}_8 \vee p_1$

$p_{10} \vee p_3$

$\overline{p}_3 \vee p_{26}$

$p_{10} \vee \overline{p}_5$

$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$

$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$

$p_{21} \vee \overline{p}_6$

$p_{21} \vee \overline{p}_{17}$

$\overline{p}_{22} \vee \overline{p}_{13}$

$p_{13} \vee p_8$

$\overline{p}_4 \vee p_{19}$

$p_{20} \vee p_{23}$

$\overline{p}_{20} \vee p_{24}$

$p_{25}$

Remember $M$ is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

$M\, p^d_{11} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2 \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1 \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3 \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3\, p_{26} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3\, p_{26}\, \overline{p}_5 \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3\, p_{26}\, \overline{p}_5\, p_{18} \Longrightarrow$

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# Motivating Example (2)

$\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$

$\overline{p}_{11} \vee p_{13} \vee p_{16}$

$p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$

$\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$

$p_{10} \vee \overline{p}_8 \vee p_1$

$p_{10} \vee p_3$

$\overline{p}_3 \vee p_{26}$

$p_{10} \vee \overline{p}_5$

$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$

$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$

$p_{21} \vee \overline{p}_6$

$p_{21} \vee \overline{p}_{17}$

$\overline{p}_{22} \vee \overline{p}_{13}$

$p_{13} \vee p_8$

$\overline{p}_4 \vee p_{19}$

$p_{20} \vee p_{23}$

$\overline{p}_{20} \vee p_{24}$

$p_{25}$

Remember $M$ is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

$M\, p^d_{11} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2 \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1 \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3 \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3\, p_{26} \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3\, p_{26}\, \overline{p}_5 \Longrightarrow$

$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3\, p_{26}\, \overline{p}_5\, p_{18} \Longrightarrow$

$fail$

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# Motivating Example (2)

$\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
$\overline{p}_{11} \vee p_{13} \vee p_{16}$
$p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
$\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
$p_{10} \vee \overline{p}_8 \vee p_1$
$p_{10} \vee p_3$
$\overline{p}_3 \vee p_{26}$
$p_{10} \vee \overline{p}_5$
$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
$p_{21} \vee \overline{p}_6$
$p_{21} \vee \overline{p}_{17}$
$\overline{p}_{22} \vee \overline{p}_{13}$
$p_{13} \vee p_8$
$\overline{p}_4 \vee p_{19}$
$p_{20} \vee p_{23}$
$\overline{p}_{20} \vee p_{24}$
$p_{25}$

Remember $M$ is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

$M\, p^d_{11} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2 \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1 \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3 \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3\, p_{26} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3\, p_{26}\, \overline{p}_5 \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3\, p_{26}\, \overline{p}_5\, p_{18} \Longrightarrow$
$fail$

- Let's try to find out the causes of the conflict
- First of all we will compute, for each literal, the reason why it is true

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# Motivating Example (3)

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. $p_{25}$


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# Motivating Example (3)

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. $p_{25}$

| Literal: | $p_{11}^d$ | $\overline{p}_{12}$ | $p_{16}$ | $\overline{p}_2$ | $\overline{p}_{10}$ | $p_1$ | $p_3$ | $p_{26}$ | $\overline{p}_5$ | $p_{18}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Reason: | $\emptyset$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

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# Motivating Example (3)

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. $p_{25}$

| Literal: | $p_{11}^d$ | $\overline{p}_{12}$ | $p_{16}$ | $\overline{p}_2$ | $\overline{p}_{10}$ | $p_1$ | $p_3$ | $p_{26}$ | $\overline{p}_5$ | $p_{18}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Reason: | $\emptyset$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

Let us take the conflicting clause $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$.
The reason why $p_{18}$ is true is clause 9. Resolution gives:

$$\frac{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18} \qquad \overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_5 \vee p_{17}}$$

# Motivating Example (3)

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. $p_{25}$

| Literal: | $p_{11}^d$ | $\overline{p}_{12}$ | $p_{16}$ | $\overline{p}_2$ | $\overline{p}_{10}$ | $p_1$ | $p_3$ | $p_{26}$ | $\overline{p}_5$ | $p_{18}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Reason: | $\emptyset$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

Let us take the conflicting clause $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$.
The reason why $p_{18}$ is true is clause 9. Resolution gives:

$$\frac{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18} \qquad \overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_5 \vee p_{17}}$$

Now, last assigned lit false in the resulting clause is $p_5$.
The reason why $p_5$ is false is clause 8. Again, resolution:

$$\frac{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_5 \vee p_{17} \qquad p_{10} \vee \overline{p}_5}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}}$$

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# Motivating Example (3)

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. $p_{25}$

| Literal: | $p_{11}^d$ | $\overline{p}_{12}$ | $p_{16}$ | $\overline{p}_2$ | $\overline{p}_{10}$ | $p_1$ | $p_3$ | $p_{26}$ | $\overline{p}_5$ | $p_{18}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Reason: | $\emptyset$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

Let us take the conflicting clause $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$.
The reason why $p_{18}$ is true is clause 9. Resolution gives:

$$\frac{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18} \qquad \overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_5 \vee p_{17}}$$

Now, last assigned lit false in the resulting clause is $p_5$.
The reason why $p_5$ is false is clause 8. Again, resolution:

$$\frac{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_5 \vee p_{17} \qquad p_{10} \vee \overline{p}_5}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}}$$

The process is now iterated...

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# Motivating Example (4)

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. $p_{25}$

Remember $M$ is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

| Literal: | $p^d_{11}$ | $\overline{p}_{12}$ | $p_{16}$ | $\overline{p}_2$ | $\overline{p}_{10}$ | $p_1$ | $p_3$ | $p_{26}$ | $\overline{p}_5$ | $p_{18}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Reason: | $\emptyset$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

# Motivating Example (4)

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. $p_{25}$

Remember $M$ is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

| Literal: | $p^d_{11}$ | $\overline{p}_{12}$ | $p_{16}$ | $\overline{p}_2$ | $\overline{p}_{10}$ | $p_1$ | $p_3$ | $p_{26}$ | $\overline{p}_5$ | $p_{18}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Reason: | $\emptyset$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

$$\dfrac{\dfrac{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18} \qquad \overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_5 \vee p_{17}} \qquad p_{10} \vee \overline{p}_5}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}}$$

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# Motivating Example (4)

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. $p_{25}$

Remember $M$ is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

| Literal: | $p^d_{11}$ | $\overline{p}_{12}$ | $p_{16}$ | $\overline{p}_2$ | $\overline{p}_{10}$ | $p_1$ | $p_3$ | $p_{26}$ | $\overline{p}_5$ | $p_{18}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Reason: | $\emptyset$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

$$\dfrac{\dfrac{\dfrac{\dfrac{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18} \qquad \overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_5 \vee p_{17}} \qquad p_{10} \vee \overline{p}_5}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}} \qquad p_{10} \vee p_3}{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}}$$

# Motivating Example (4)

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. $p_{25}$

Remember $M$ is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

| Literal: | $p^d_{11}$ | $\overline{p}_{12}$ | $p_{16}$ | $\overline{p}_2$ | $\overline{p}_{10}$ | $p_1$ | $p_3$ | $p_{26}$ | $\overline{p}_5$ | $p_{18}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Reason: | $\emptyset$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

$$\dfrac{\dfrac{\dfrac{\dfrac{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18} \qquad \overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_5 \vee p_{17}} \qquad p_{10} \vee \overline{p}_5}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}} \qquad p_{10} \vee p_3}{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}}$$

$$\dfrac{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10} \qquad p_{10} \vee \overline{p}_8 \vee p_1}{\overline{p}_{19} \vee p_{17} \vee p_{10} \vee \overline{p}_8}$$

# Motivating Example (4)

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. $p_{25}$

Remember $M$ is $p_{25}\, \overline{p}_{21}^{d}\, \overline{p}_6\, \overline{p}_{17}\, p_{22}^{d}\, \overline{p}_{13}\, p_8\, p_4^{d}\, p_{19}\, \overline{p}_{20}^{d}\, p_{23}$

| Literal: | $p_{11}^{d}$ | $\overline{p}_{12}$ | $p_{16}$ | $\overline{p}_2$ | $\overline{p}_{10}$ | $p_1$ | $p_3$ | $p_{26}$ | $\overline{p}_5$ | $p_{18}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Reason: | $\emptyset$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

$$\dfrac{\dfrac{\dfrac{\dfrac{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18} \qquad \overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_5 \vee p_{17}} \qquad p_{10} \vee \overline{p}_5}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}} \qquad p_{10} \vee p_3}{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}}$$

$$\dfrac{\dfrac{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10} \qquad p_{10} \vee \overline{p}_8 \vee p_1}{\overline{p}_{19} \vee p_{17} \vee p_{10} \vee \overline{p}_8} \qquad p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee p_2 \vee \overline{p}_4 \vee p_{20}}$$

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# Motivating Example (4)

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. $p_{25}$

Remember $M$ is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

| Literal: | $p^d_{11}$ | $\overline{p}_{12}$ | $p_{16}$ | $\overline{p}_2$ | $\overline{p}_{10}$ | $p_1$ | $p_3$ | $p_{26}$ | $\overline{p}_5$ | $p_{18}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Reason: | $\emptyset$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

$$\dfrac{\dfrac{\dfrac{\dfrac{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18} \qquad \overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_5 \vee p_{17}} \qquad p_{10} \vee \overline{p}_5}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}} \qquad p_{10} \vee p_3}{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}}$$

$$\dfrac{\dfrac{\dfrac{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10} \qquad p_{10} \vee \overline{p}_8 \vee p_1}{\overline{p}_{19} \vee p_{17} \vee p_{10} \vee \overline{p}_8} \qquad p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee p_2 \vee \overline{p}_4 \vee p_{20}} \qquad p_{12} \vee \overline{p}_{16} \vee \overline{p}_2}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{16}}$$

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# Motivating Example (4)

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. $p_{25}$

Remember $M$ is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

| Literal: | $p^d_{11}$ | $\overline{p}_{12}$ | $p_{16}$ | $\overline{p}_2$ | $\overline{p}_{10}$ | $p_1$ | $p_3$ | $p_{26}$ | $\overline{p}_5$ | $p_{18}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Reason: | $\emptyset$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

$$\dfrac{\dfrac{\dfrac{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18} \qquad \overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_5 \vee p_{17}} \qquad p_{10} \vee \overline{p}_5}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}} \qquad p_{10} \vee p_3}{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}}$$

$$\dfrac{\dfrac{\dfrac{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10} \qquad p_{10} \vee \overline{p}_8 \vee p_1}{\overline{p}_{19} \vee p_{17} \vee p_{10} \vee \overline{p}_8} \qquad p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee p_2 \vee \overline{p}_4 \vee p_{20}} \qquad p_{12} \vee \overline{p}_{16} \vee \overline{p}_2}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{16}}$$

$$\dfrac{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{16} \qquad \overline{p}_{11} \vee p_{13} \vee p_{16}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{11} \vee p_{13}}$$

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# Motivating Example (4)

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. $p_{25}$

Remember $M$ is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

| Literal: | $p^d_{11}$ | $\overline{p}_{12}$ | $p_{16}$ | $\overline{p}_2$ | $\overline{p}_{10}$ | $p_1$ | $p_3$ | $p_{26}$ | $\overline{p}_5$ | $p_{18}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Reason: | $\emptyset$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

$$\dfrac{\dfrac{\dfrac{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18} \qquad \overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_5 \vee p_{17}} \qquad p_{10} \vee \overline{p}_5}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}} \qquad p_{10} \vee p_3}{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}}$$

$$\dfrac{\dfrac{\dfrac{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10} \qquad p_{10} \vee \overline{p}_8 \vee p_1}{\overline{p}_{19} \vee p_{17} \vee p_{10} \vee \overline{p}_8} \qquad p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee p_2 \vee \overline{p}_4 \vee p_{20}} \qquad p_{12} \vee \overline{p}_{16} \vee \overline{p}_2}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{16}}$$

$$\dfrac{\dfrac{\dfrac{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{16} \qquad \overline{p}_{11} \vee p_{13} \vee p_{16}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{11} \vee p_{13}} \qquad \overline{p}_{11} \vee p_6 \vee \overline{p}_{12}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{11} \vee p_{13} \vee p_6}$$

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# Motivating Example (4)

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. $p_{25}$

Remember $M$ is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

| Literal: | $p^d_{11}$ | $\overline{p}_{12}$ | $p_{16}$ | $\overline{p}_2$ | $\overline{p}_{10}$ | $p_1$ | $p_3$ | $p_{26}$ | $\overline{p}_5$ | $p_{18}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Reason: | $\emptyset$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

$$\dfrac{\dfrac{\dfrac{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18} \qquad \overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_5 \vee p_{17}} \qquad p_{10} \vee \overline{p}_5}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}} \qquad p_{10} \vee p_3}{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}}$$

$$\dfrac{\dfrac{\dfrac{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10} \qquad p_{10} \vee \overline{p}_8 \vee p_1}{\overline{p}_{19} \vee p_{17} \vee p_{10} \vee \overline{p}_8} \qquad p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee p_2 \vee \overline{p}_4 \vee p_{20}} \qquad p_{12} \vee \overline{p}_{16} \vee \overline{p}_2}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{16}}$$

$$\dfrac{\dfrac{\dfrac{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{16} \qquad \overline{p}_{11} \vee p_{13} \vee p_{16}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{11} \vee p_{13}} \qquad \overline{p}_{11} \vee p_6 \vee \overline{p}_{12}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{11} \vee p_{13} \vee p_6}}$$

Note that process now can't continue

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# Motivating Example (4)

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. $p_{25}$

Remember $M$ is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

| Literal: | $p^d_{11}$ | $\overline{p}_{12}$ | $p_{16}$ | $\overline{p}_2$ | $\overline{p}_{10}$ | $p_1$ | $p_3$ | $p_{26}$ | $\overline{p}_5$ | $p_{18}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Reason: | $\emptyset$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

$$\dfrac{\dfrac{\dfrac{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18} \qquad \overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_5 \vee p_{17}} \qquad p_{10} \vee \overline{p}_5}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}} \qquad p_{10} \vee p_3}{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}}$$

$$\dfrac{\dfrac{\dfrac{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10} \qquad p_{10} \vee \overline{p}_8 \vee p_1}{\overline{p}_{19} \vee p_{17} \vee p_{10} \vee \overline{p}_8} \qquad p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee p_2 \vee \overline{p}_4 \vee p_{20}} \qquad p_{12} \vee \overline{p}_{16} \vee \overline{p}_2}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{16}}$$

$$\dfrac{\dfrac{\dfrac{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{16} \qquad \overline{p}_{11} \vee p_{13} \vee p_{16}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{11} \vee p_{13}} \qquad \overline{p}_{11} \vee p_6 \vee \overline{p}_{12}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{11} \vee p_{13} \vee p_6}}$$

Note also that all obtained clauses are false in assignment.

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# Motivating Example (4)

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $\overline{p}_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. $p_{25}$

Remember $M$ is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

| Literal: | $p^d_{11}$ | $\overline{p}_{12}$ | $p_{16}$ | $\overline{p}_2$ | $\overline{p}_{10}$ | $p_1$ | $p_3$ | $p_{26}$ | $\overline{p}_5$ | $p_{18}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Reason: | $\emptyset$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

$$\dfrac{\dfrac{\dfrac{\dfrac{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18} \qquad \overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_5 \vee p_{17}} \qquad p_{10} \vee \overline{p}_5}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}} \qquad p_{10} \vee p_3}{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}}$$

$$\dfrac{\dfrac{\dfrac{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10} \qquad p_{10} \vee \overline{p}_8 \vee p_1}{\overline{p}_{19} \vee p_{17} \vee p_{10} \vee \overline{p}_8} \qquad p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee p_2 \vee \overline{p}_4 \vee p_{20}} \qquad p_{12} \vee \overline{p}_{16} \vee \overline{p}_2}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{16}}$$

$$\dfrac{\dfrac{\dfrac{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{16} \qquad \overline{p}_{11} \vee p_{13} \vee p_{16}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{11} \vee p_{13}} \qquad \overline{p}_{11} \vee p_6 \vee \overline{p}_{12}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{11} \vee p_{13} \vee p_6}$$

Now, in blue lits false at the current decision level (5)

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# Motivating Example (5)

- Three clauses with only one literal assigned at the last DL (5):
  - $\overline{p}_{19} \vee p_{17} \vee p_{10} \vee \overline{p}_8$ (max DL of others:3)
  - $\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee p_2 \vee \overline{p}_4 \vee p_{20}$ (max DL of others:4)
  - $\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{11} \vee p_{13} \vee p_6$ (max DL of others:4)
- If we had had those clauses:
  - At DL. 3 we could've propagated $p_{10}$
  - At DL. 4 we could've propagated $p_2$
  - At DL. 4 we could've propagated $\overline{p}_{11}$
- In practice, procedure stops when first such clause is found, because:
  - It is the cheapest one to find
  - It can propagated lits at a lower DL

# Backjump rule

This examples motivates us to introduce the rule:

Backjump

$$M\, l^{\mathsf{d}}\, N \parallel F \Longrightarrow M\, l' \parallel F \quad \textbf{if} \begin{cases} \text{for some clause } C \vee l' : \\ \quad F \models C \vee l' \text{ and } M \models \neg C \\ \quad l' \text{ is undefined in } M \\ \quad l' \text{ or } \neg l' \text{ occurs in } F \end{cases}$$

The only thing we need is a backjump clause $C \vee l'$ such that:

1. It is a logical consequence of the rest of the clauses
2. All its literals are false at some previous decision level $d$, except one which was undefined at $d$

# Conflict Analysis

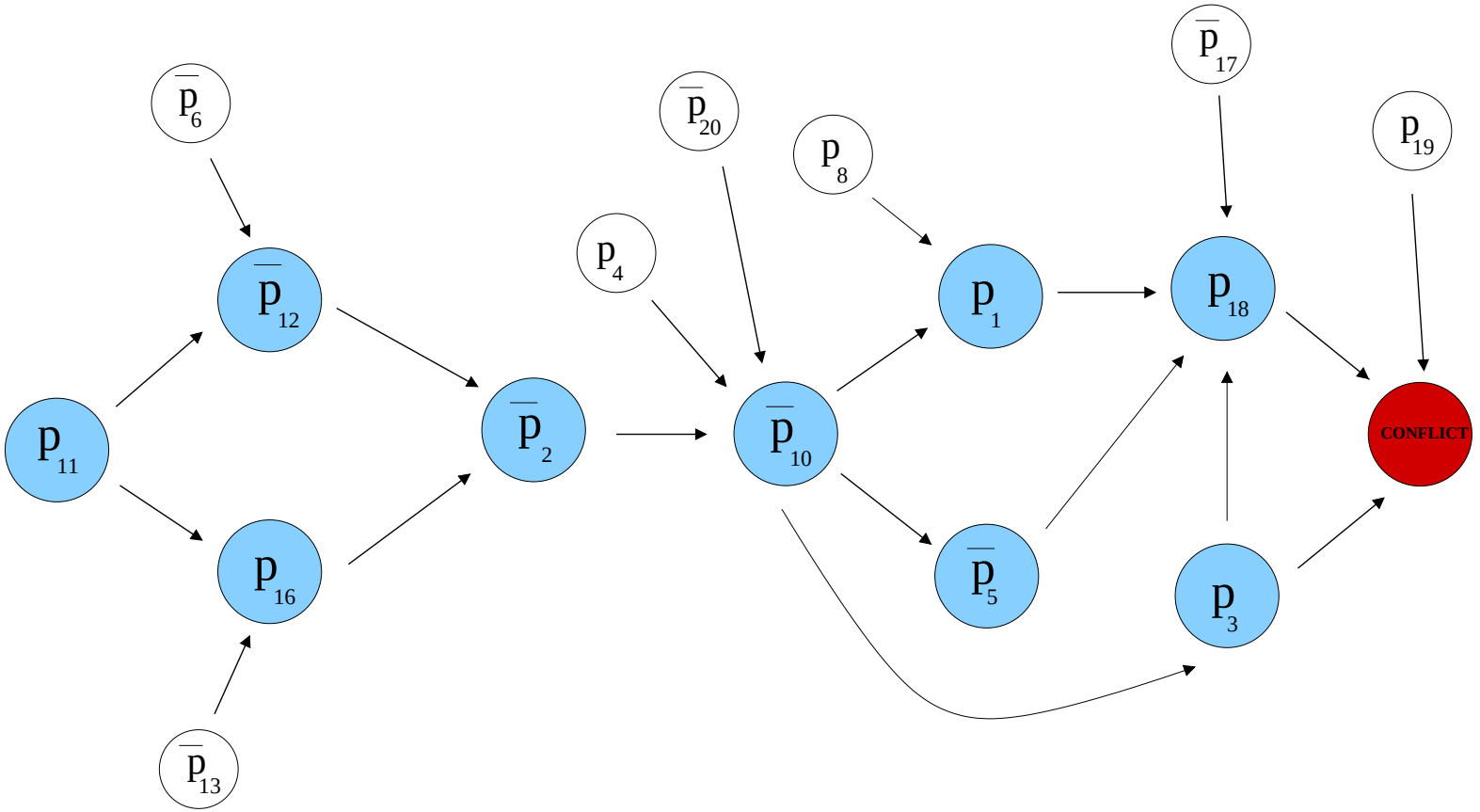
- The procedure shown in the example is called conflict analysis
- Why the obtained clause is a logical consequence of the input?
  - Because resolution is correct

# Conflict Analysis (2)

- The procedure shown in the example is called conflict analysis
- Why always a false clause with only one lit set at the last decision level ($dl$) is obtained?
  - Conflicting clause has at least two lits false at $dl$ (provided unit propagation applied before any decision)
  - Each non-decision lit $l$ false at $dl$ can be resolved away
  - $l$ is replaced by lits $l_1, \ldots, l_n$ such that
    1. All of them are false
    2. All of them have been added to the assignment before $l$ (hence their decision level is $\leq dl$)
    3. At least one was set at $dl$ (again, provided ....)
  - By 3, obtained clauses contain at least one lit false at $dl$
  - Procedure terminates because of 2. In the worst case, with last decision lit being the only set to false at $dl$

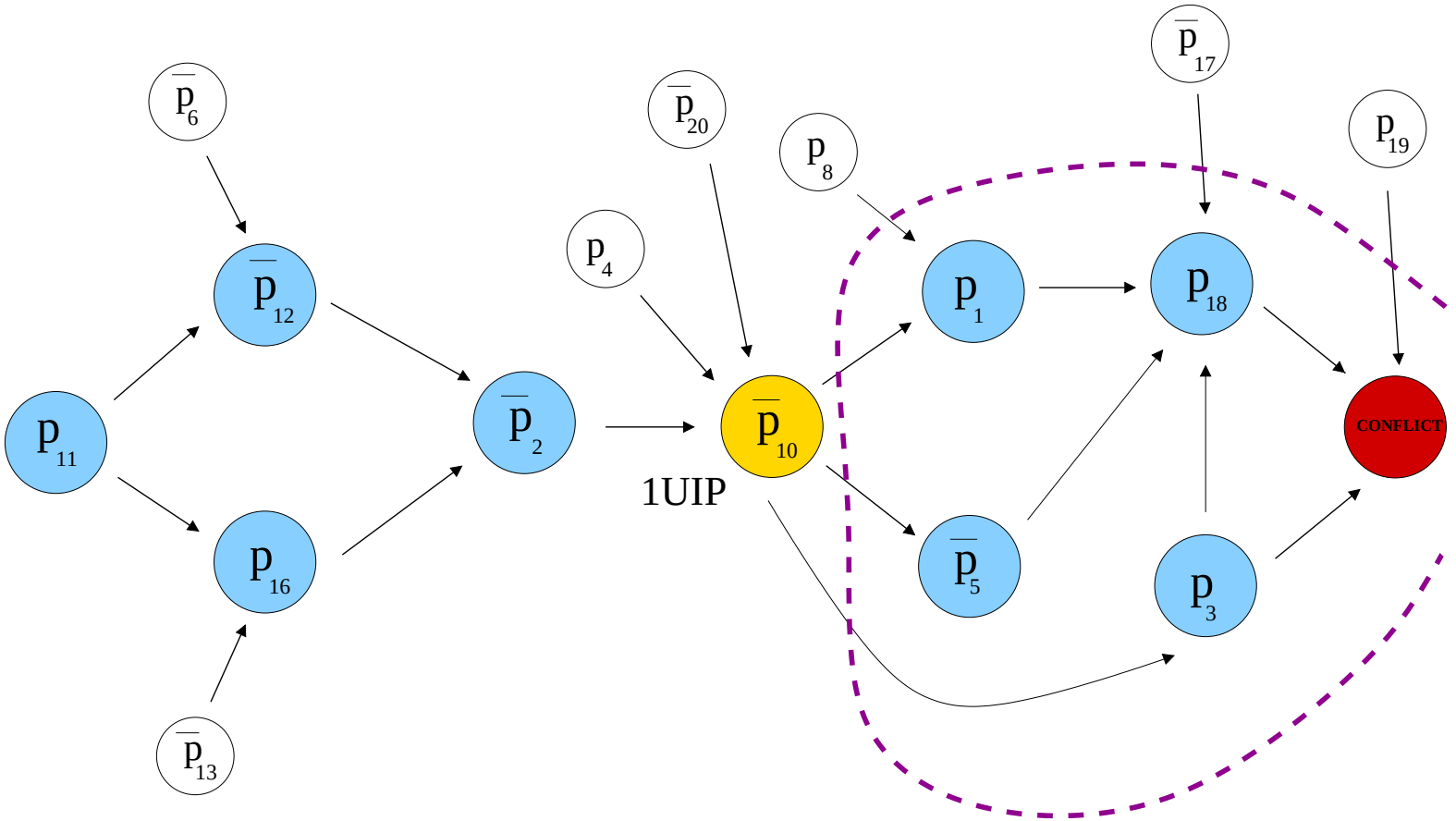
# Conflict Analysis - Conflict Graph

- Situation is best observed in the conflict graph:



# Conflict Analysis - Conflict Graph (2)

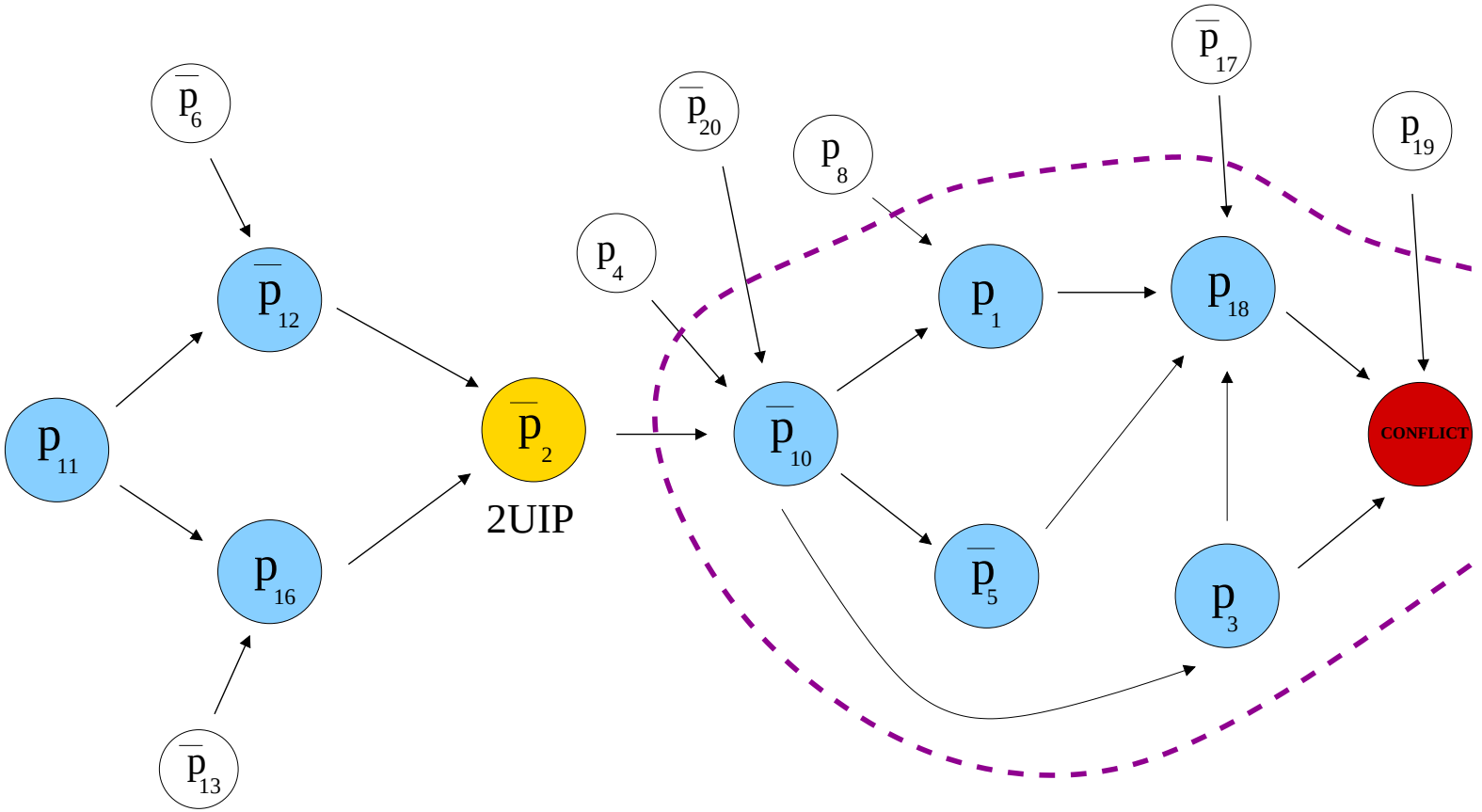
- This cut represents the 1-UIP learning scheme (Unique Implication Point)




- Clause learned is $\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee p_{10}$

# Conflict Analysis - Conflict Graph (3)

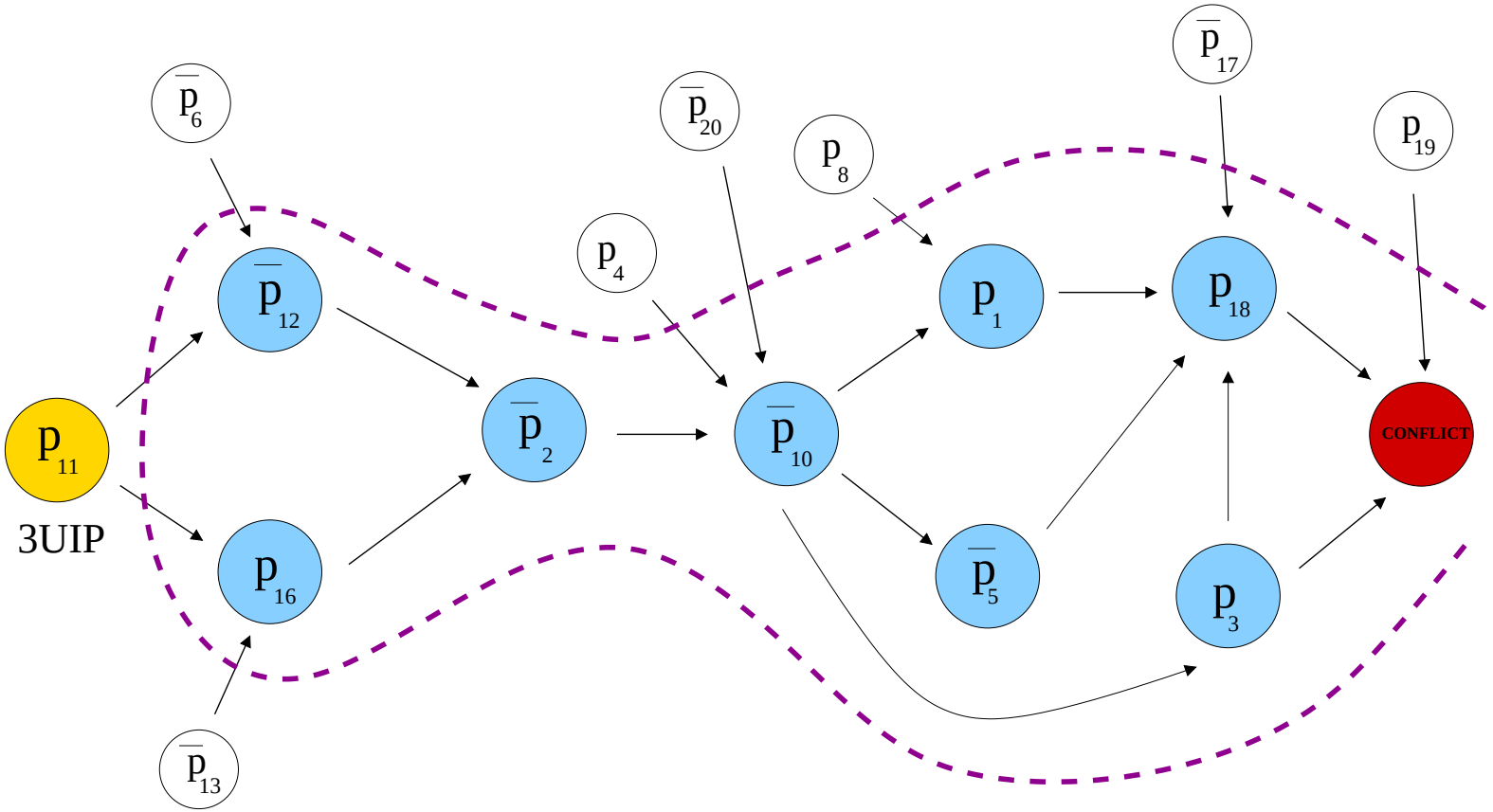
- This cut represents the 2-UIP learning scheme




- Clause learned is $\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee p_{20} \vee \overline{p}_4 \vee p_2$

# Conflict Analysis - Conflict Graph (4)

- This cut represents the 3-UIP learning scheme




- Clause learned is $\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee p_{20} \vee \overline{p}_4 \vee p_6 \vee \overline{p}_{11} \vee p_{13}$

# What is a good lemma?

- Every time a conflict is found, conflict analysis is started
- Backjump clause is added to the clause database:

  Learn

  $$M \parallel F \implies M \parallel F, C \ \textbf{if} \begin{cases} \text{all atoms of } C \text{ occur in } F \\ F \models C \end{cases}$$

- Backjump clauses are usually known as lemmas
- Learning them helps to prevent future similar conflicts
- It can easily be seen that 1UIP gives shorter clauses than 2UIP
- Also 1UIP allows one to backjump to a lower or equal DL
- However, it is difficult to assess in advance the quality of a lemma

# Lemma Shortening

- But, given a lemma $L$, any lemma $L' \subseteq L$ is clearly better.
- Given $L$, how to obtain a possible $L'$?
- **LOCAL MINIZATION:**
  - Generate lemma $L$ and mark the negation of its literals
  - Remove those non-decision literals $l \in L$ such that $\mathit{reason}(\bar{l}) \setminus \{l\}$ contains only marked literals

EXAMPLE: our 2-UIP clause was

$$\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee p_2 \vee \overline{p}_4 \vee p_{20}$$

with $\mathit{reason}(p_{19}) = \overline{p}_4 \vee p_{19}$. Hence $\overline{p}_{19}$ can be removed. Why?

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with $\mathit{reason}(p_{19}) = \overline{p}_4 \vee p_{19}$. Hence $\overline{p}_{19}$ can be removed. Why?

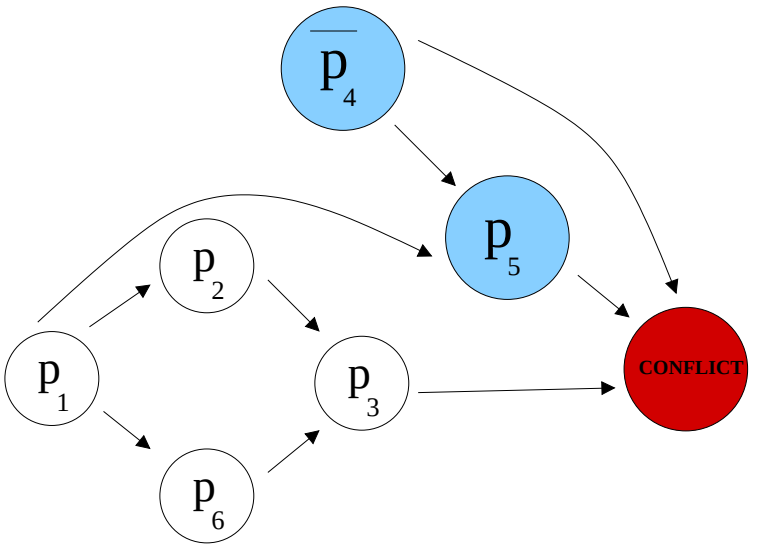
$$\frac{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee p_2 \vee \overline{p}_4 \vee p_{20} \qquad \overline{p}_4 \vee p_{19}}{p_{17} \vee \overline{p}_8 \vee p_2 \vee \overline{p}_4 \vee p_{20}}$$

# Lemma Shortening (2)

- **RECURSIVE MINIMIZATION:**
  - Generate lemma $L$ and mark the negation of its literals
  - Non-decision lits in $L$ are candidates for removal
  - $l$ is removed if search backwards from $\overline{l}$ in the implication graph ends at marked literals

1. $\overline{p}_1 \vee p_2$
2. $\overline{p}_1 \vee p_6$
3. $\overline{p}_2 \vee \overline{p}_6 \vee p_3$
4. $\overline{p}_1 \vee p_4 \vee p_5$
5. $\overline{p}_3 \vee p_4 \vee \overline{p}_5$

$$\emptyset \Longrightarrow \ldots \Longrightarrow p_1{}^{\mathsf{d}}\, p_2\, p_6\, p_3\, p_4{}^{\mathsf{d}}\, p_5$$




- 1UIP lemma is $\overline{p}_3 \vee p_4 \vee \overline{p}_1$
- $\overline{p}_3$ is clearly removable

# Overview of the session

- Conflict Analysis
  - Motivating example
  - Backjumping
  - Conflict graph
  - Lemma shortening
- Lemma removal
- Decision heuristics
- Restarts
- Efficient implemetation of UnitProp:
  - Occur lists
  - Two-watched literals
- Final remarks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# Lemma Removal

- Effects of adding lemmas:
  - \+ Reduces the search space
  - \- Space traversal slower since unit prop becomes expensive
- Hence we cannot keep all generated lemmas. We need:

  Forget

  $M \parallel F, C \Longrightarrow M \parallel F$ **if** $F \models C$
- Which lemmas to keep and which ones to forget?
  - Each lemma has a number called activity
  - Activity incremented when lemma is used in conflict analysis
  - From time to time, lemmas with low activity are removed
  - Mixed policies: short lemmas, recent lemmas kept, ...

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# Decision Heuristic

- SAT instances have thousands of variables
- We can't keep enough lemmas to store info about all vars
- Most SAT instances have clusters of variables:
  Sets of variables that are semantically linked

**GOAL:** force the SAT solver to work on one cluster at a time

- Each var/lit has an associated activity
- Each time it appears in a conflict analysis, activity incremented
- Recent activity should be given more importance:
  - Divide all activities by integer $K$ from time to time, or
  - Keep increasing the activity increment
- Decide chooses unassigned lit with highest activity
- Note that heuristic does not depend on clauses: CHEAP!

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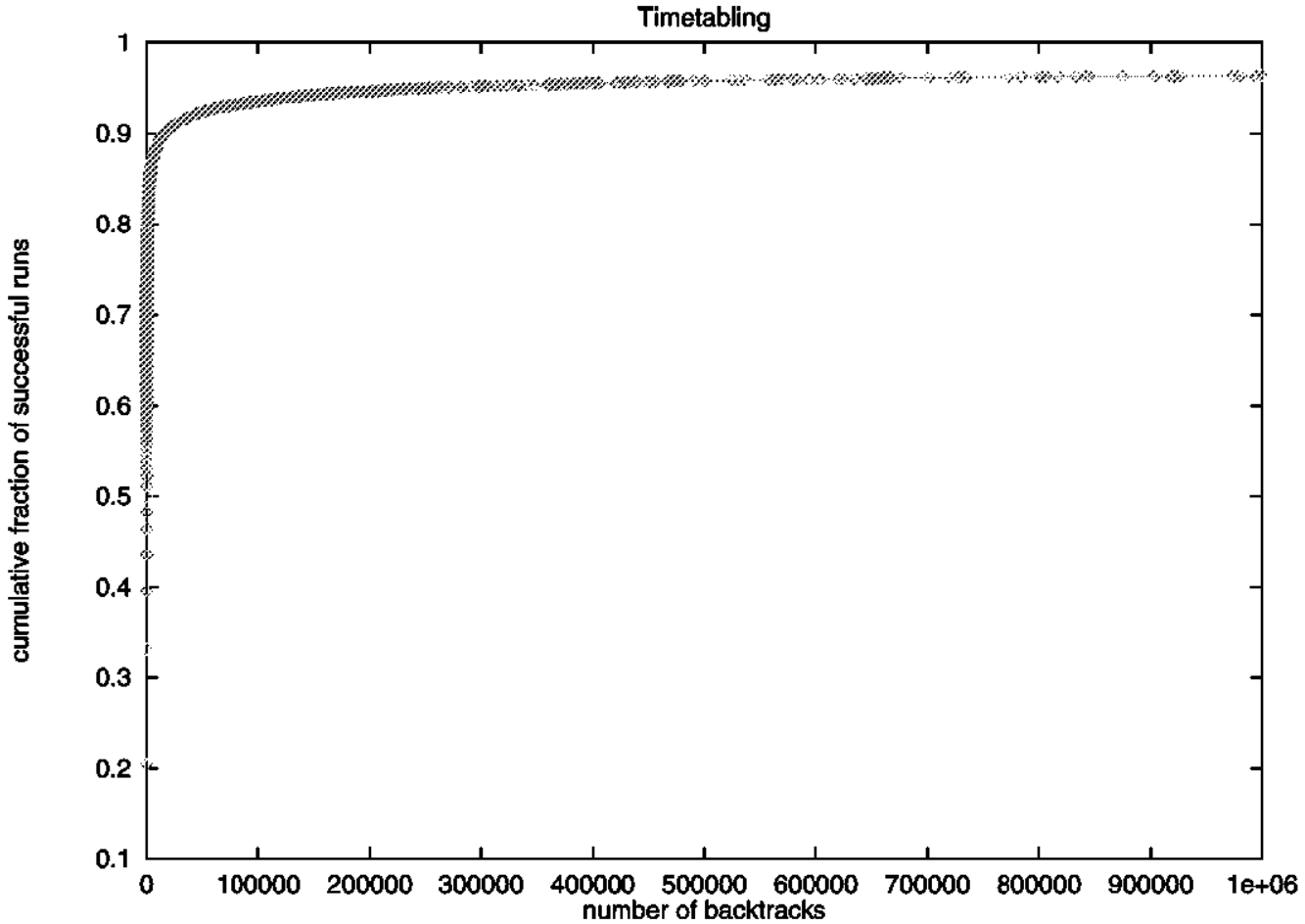
# Restarts

DPLL-based SAT solvers present a heavy tail phenomenon:



# Restarts (2)

- Assume a family of randomly created problems
- Run a SAT solver on them and count the number of backtracks
- The previous picture shows, roughly speaking, that
  - Although most problems can be solved easily....
  - there is a heavy tail that affects the mean of the distribution
    (i.e. large runtimes cannot be considered outliers)

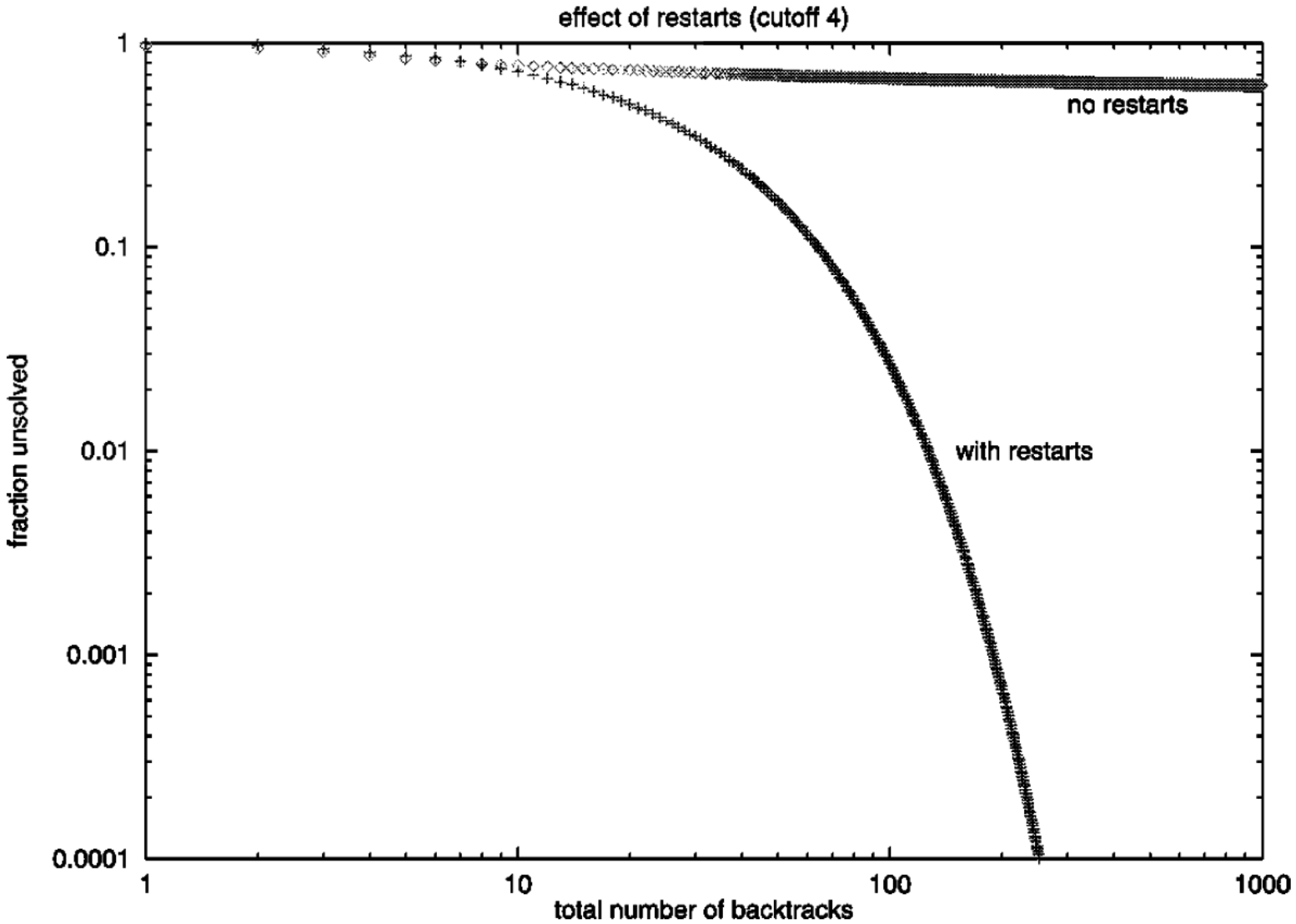
**HOW TO AVOID THIS BEHAVIOUR?**

- Introduce restarts:

  Restart

  $$M \parallel F \Longrightarrow \emptyset \parallel F$$

# Restarts (3)

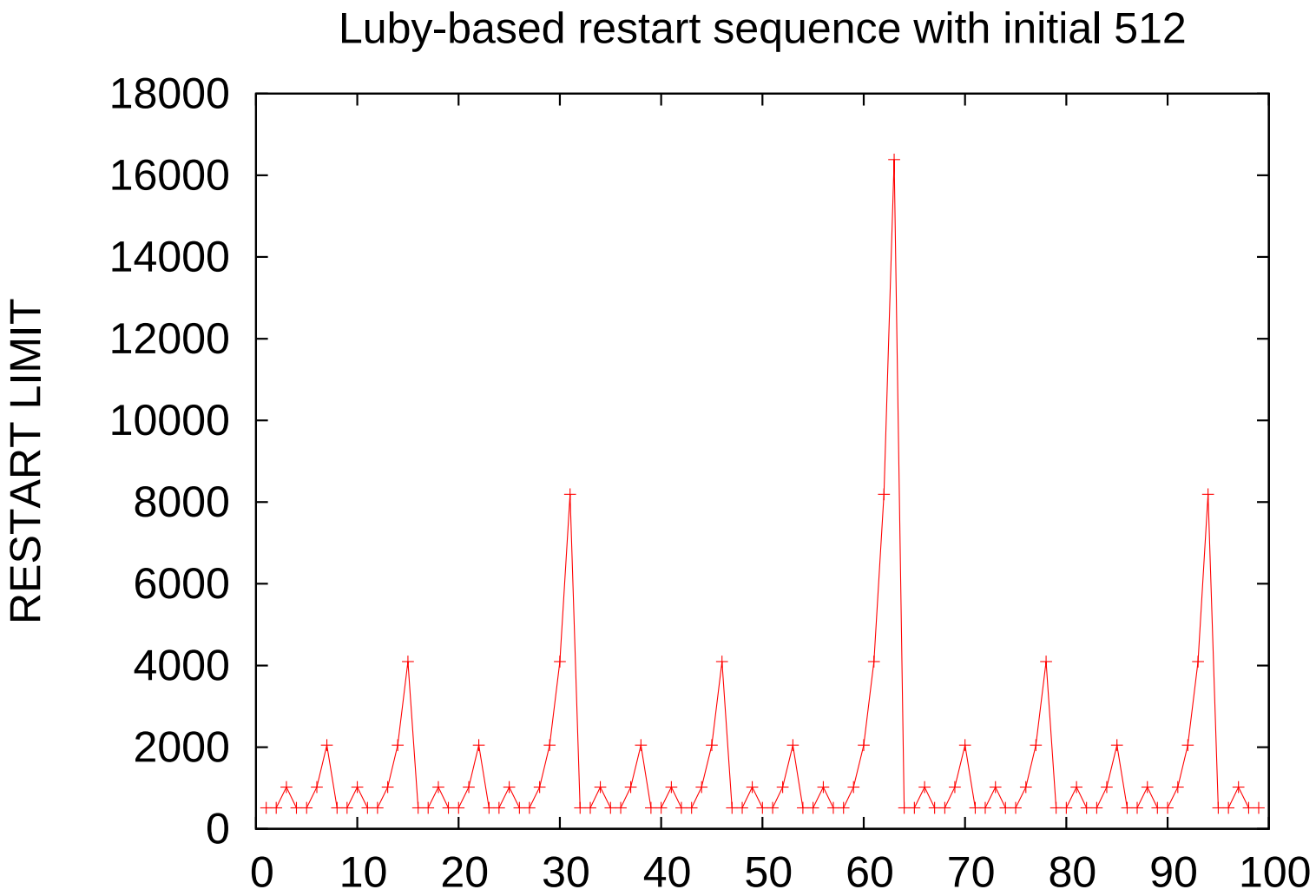

effect of restarts (cutoff 4)
fraction unsolved
1
0.1
0.01
0.001
0.0001
1
10
100
1000
total number of backtracks
no restarts
with restarts

# Restarts - Strategies

- Unrestricted application of Restart leads to incompleteness
- What is done in practice?
  - Initially set RESTART_LIMIT to N
  - After RESTART_LIMIT conflicts:
    - Increment RESTART_LIMIT
    - Apply Restart
- Other variants:
  - Let RESTART_LIMIT follow the sequence $r_i$:
    - $r_0 := N$; $r_i := N \cdot l_i$, where

$$l_i = \begin{cases} 2^{k-1} & \text{if } \exists k \text{ with } i = 2^k - 1 \\ l_{i-2^{k-1}+1} & \text{if } \exists k \text{ with } 2^{k-1} \leq i < 2^k - 1 \end{cases}$$

# Restarts - Strategies (2)

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# Restarts - Strategies (3)

Yet another possibility is an inner-outer geometric sequence:

```
int inner = 100, outer = 100;

for (;;){

  // Run SAT-solver for 'inner' conflicts

  if (inner >= outer){
    outer *= 1.1;
    inner = 100;
  }
  else
    inner*=1.1

}
```


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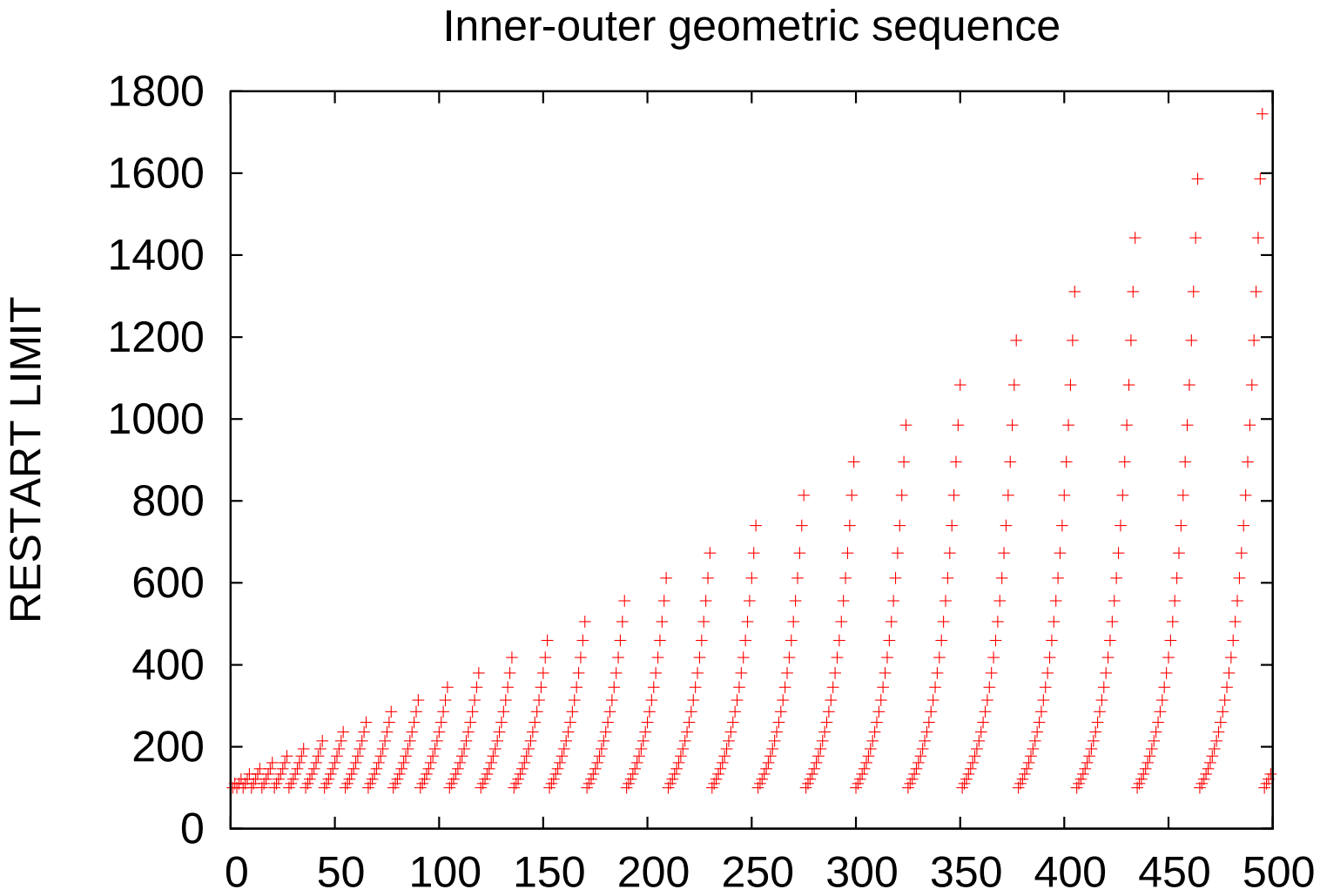
# Restarts - Strategies (4)

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# Overall CDCL algorithm

```
while(true){

    while (propagate_gives_conflict()){
        if (decision_level==0) return UNSAT;
        else analyze_conflict();
    }

    restart_if_applicable();
    remove_lemmas_if_applicable();

    if (!decide()) returns SAT; // All vars assigned
}
```

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# Performance of SAT Solvers

- The most important tasks that a SAT solver performs are:
  - Choose which variable to Decide on
  - Apply unit propagation exhaustively
  - Analyze conflicts
- When profiling a state-of-the art SAT solver we get:
  - Variable selection ($\approx 10\%$)
  - Unit propagation application ($\approx 80\%$)
  - Conflict analysis ($\approx 10\%$)
- Hence, the most important thing to optimize is unit propagation, known in the literature as BCP (Boolean Constraint Propagation)

# BCP with Occur Lists

- BCP only has to detect unit or conflicting clauses
- There is no need to detect that all clauses are true
- Instead of traversing the whole clause set again and again:
  - For each literal, store the clauses where it appears
  - Every time a new lit $l$ is added to the assignment, only clauses containing $\bar{l}$ need to be visited
- Let's see how it would work with an example

# BCP with Occur Lists

1 $p_1$ $\bar{p}_2$ $\bar{p}_6$

2 $p_2$ $\bar{p}_3$ $p_5$ $\bar{p}_1$ $\bar{p}_6$

3 $p_6$ $p_2$ $p_4$

4 $p_1$ $p_2$

5 $\bar{p}_6$ $\bar{p}_1$ $p_3$

6 $\bar{p}_5$ $p_4$ $p_2$

Current assignment: $\emptyset$

Model

| $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|
| U | U | U | U | U | U |

ToPropagate

ClausesWith

| | | | | | |
|---|---|---|---|---|---|
| $p_1$ | 1 | 4 | | | |
| $p_2$ | 2 | 3 | 4 | 6 | |
| $p_3$ | 5 | | | | |
| $p_4$ | 3 | 6 | | | |
| $p_5$ | 2 | | | | |
| $p_6$ | 3 | | | | |

| | | | |
|---|---|---|---|
| $\bar{p}_1$ | 2 | 5 | |
| $\bar{p}_2$ | 1 | | |
| $\bar{p}_3$ | 2 | | |
| $\bar{p}_4$ | | | |
| $\bar{p}_5$ | 6 | | |
| $\bar{p}_6$ | 1 | 2 | 5 |

# BCP with Occur Lists

1 $p_1$ $\overline{p}_2$ $\overline{p}_6$

2 $p_2$ $\overline{p}_3$ $p_5$ $\overline{p}_1$ $\overline{p}_6$

3 $p_6$ $p_2$ $p_4$

4 $p_1$ $p_2$

5 $\overline{p}_6$ $\overline{p}_1$ $p_3$

6 $\overline{p}_5$ $p_4$ $p_2$

Model

| $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|
| U | F | U | U | U | U |

ToPropagate

| |
|---|
| $\overline{p}_2$ |

ClausesWith

| | | | | |
|---|---|---|---|---|
| $p_1$ | 1 | 4 | | |
| $p_2$ | 2 | 3 | 4 | 6 |
| $p_3$ | 5 | | | |
| $p_4$ | 3 | 6 | | |
| $p_5$ | 2 | | | |
| $p_6$ | 3 | | | |

| | | | |
|---|---|---|---|
| $\overline{p}_1$ | 2 | 5 | |
| $\overline{p}_2$ | 1 | | |
| $\overline{p}_3$ | 2 | | |
| $\overline{p}_4$ | | | |
| $\overline{p}_5$ | 6 | | |
| $\overline{p}_6$ | 1 | 2 | 5 |

Current assignment: $\overline{p}_2{}^{\mathsf{d}}$

Now, we propagate $\overline{p}_2$ visiting ClausesWith$[p_2]$

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# BCP with Occur Lists

1 $p_1$ $\overline{p}_2$ $\overline{p}_6$

2 $p_2$ $\overline{p}_3$ $p_5$ $\overline{p}_1$ $\overline{p}_6$

3 $p_6$ $p_2$ $p_4$

4 $p_1$ $p_2$

5 $\overline{p}_6$ $\overline{p}_1$ $p_3$

6 $\overline{p}_5$ $p_4$ $p_2$

Model

| $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|
| U | F | U | U | U | U |

ToPropagate

ClausesWith

| | | | | |
|---|---|---|---|---|
| $p_1$ | 1 | 4 | | |
| $p_2$ | 2 | 3 | 4 | 6 |
| $p_3$ | 5 | | | |
| $p_4$ | 3 | 6 | | |
| $p_5$ | 2 | | | |
| $p_6$ | 3 | | | |

| | | | |
|---|---|---|---|
| $\overline{p}_1$ | 2 | 5 | |
| $\overline{p}_2$ | 1 | | |
| $\overline{p}_3$ | 2 | | |
| $\overline{p}_4$ | | | |
| $\overline{p}_5$ | 6 | | |
| $\overline{p}_6$ | 1 | 2 | 5 |

Current assignment: $\overline{p}_2{}^{\mathsf{d}}$

Literal $p_1$ has to be added to the assignment

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# BCP with Occur Lists

1 $p_1$ $\overline{p}_2$ $\overline{p}_6$

2 $p_2$ $\overline{p}_3$ $p_5$ $\overline{p}_1$ $\overline{p}_6$

3 $p_6$ $p_2$ $p_4$

4 $p_1$ $p_2$

5 $\overline{p}_6$ $\overline{p}_1$ $p_3$

6 $\overline{p}_5$ $p_4$ $p_2$

Model

| $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|
| T | F | U | U | U | U |

ToPropagate

| |
|---|
| $p_1$ |

ClausesWith

| | | |
|---|---|---|
| $p_1$ | 1 | 4 |
| $p_2$ | 2 | 3 | 4 | 6 |
| $p_3$ | 5 | | | |
| $p_4$ | 3 | 6 | | |
| $p_5$ | 2 | | | |
| $p_6$ | 3 | | | |

| | | |
|---|---|---|
| $\overline{p}_1$ | 2 | 5 |
| $\overline{p}_2$ | 1 | |
| $\overline{p}_3$ | 2 | |
| $\overline{p}_4$ | | |
| $\overline{p}_5$ | 6 | |
| $\overline{p}_6$ | 1 | 2 | 5 |

Current assignment: $\overline{p}_2$ $^{\mathsf{d}}$ $p_1$

Now, we propagate $p_1$ visiting ClausesWith[$\overline{p}_1$]

# BCP with Occur Lists

1 $p_1$ $\overline{p}_2$ $\overline{p}_6$

2 $p_2$ $\overline{p}_3$ $p_5$ $\overline{p}_1$ $\overline{p}_6$

3 $p_6$ $p_2$ $p_4$

4 $p_1$ $p_2$

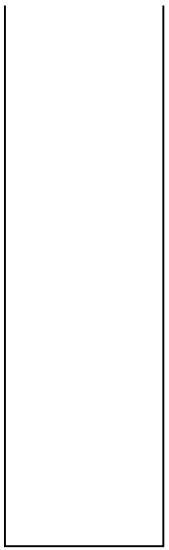
5 $\overline{p}_6$ $\overline{p}_1$ $p_3$

6 $\overline{p}_5$ $p_4$ $p_2$

Model

| $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|
| T | F | U | U | U | U |

ToPropagate



ClausesWith

| | | | | |
|---|---|---|---|---|
| $p_1$ | 1 | 4 | | |
| $p_2$ | 2 | 3 | 4 | 6 |
| $p_3$ | 5 | | | |
| $p_4$ | 3 | 6 | | |
| $p_5$ | 2 | | | |
| $p_6$ | 3 | | | |

| | | | |
|---|---|---|---|
| $\overline{p}_1$ | 2 | 5 | |
| $\overline{p}_2$ | 1 | | |
| $\overline{p}_3$ | 2 | | |
| $\overline{p}_4$ | | | |
| $\overline{p}_5$ | 6 | | |
| $\overline{p}_6$ | 1 | 2 | 5 |

Current assignment: $\overline{p}_2{}^{\mathsf{d}}\, p_1$

No lit is propagated, we have to decide

# BCP with Occur Lists

1 $p_1$ $\overline{p}_2$ $\overline{p}_6$

2 $p_2$ $\overline{p}_3$ $p_5$ $\overline{p}_1$ $\overline{p}_6$

3 $p_6$ $p_2$ $p_4$

4 $p_1$ $p_2$

5 $\overline{p}_6$ $\overline{p}_1$ $p_3$

6 $\overline{p}_5$ $p_4$ $p_2$

Model

| $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|
| T | F | U | F | U | U |

ToPropagate

| |
|---|
| $\overline{p}_4$ |

ClausesWith

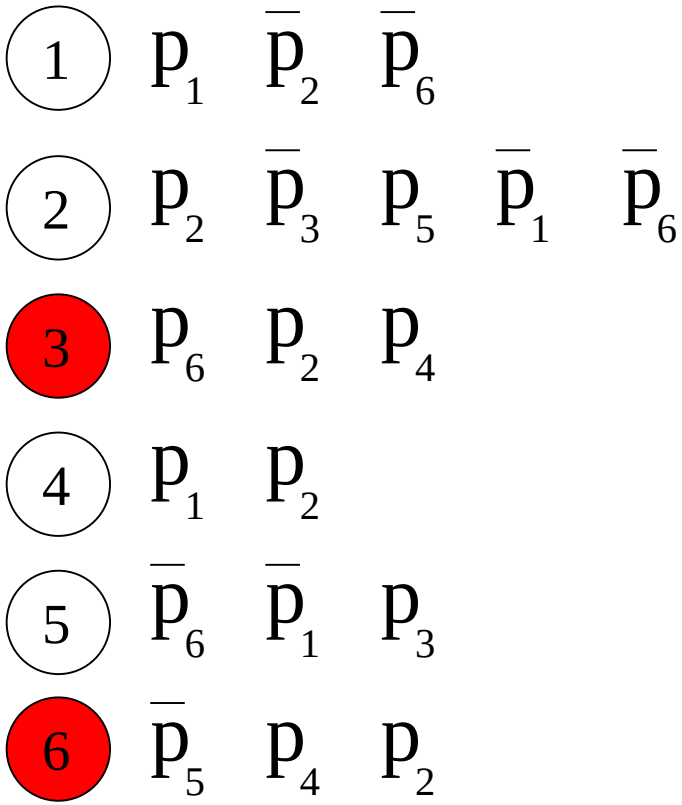
| | | | | |
|---|---|---|---|---|
| $p_1$ | 1 | 4 | | |
| $p_2$ | 2 | 3 | 4 | 6 |
| $p_3$ | 5 | | | |
| $p_4$ | 3 | 6 | | |
| $p_5$ | 2 | | | |
| $p_6$ | 3 | | | |

| | | | |
|---|---|---|---|
| $\overline{p}_1$ | 2 | 5 | |
| $\overline{p}_2$ | 1 | | |
| $\overline{p}_3$ | 2 | | |
| $\overline{p}_4$ | | | |
| $\overline{p}_5$ | 6 | | |
| $\overline{p}_6$ | 1 | 2 | 5 |

Current assignment: $\overline{p}_2{}^{d}\, p_1\, \overline{p}_4{}^{d}$

Now, we propagate $\overline{p}_4$ visiting ClauseWith[$p_4$]

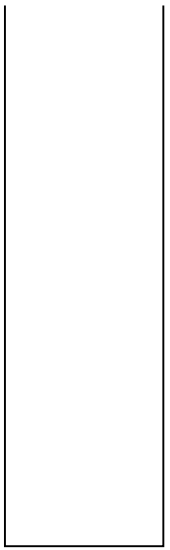
# BCP with Occur Lists




Model

| $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|
| T | F | U | F | U | U |

ToPropagate



ClausesWith

| | | | | | |
|---|---|---|---|---|---|
| $p_1$ | 1 | 4 | | | |
| $p_2$ | 2 | 3 | 4 | 6 | |
| $p_3$ | 5 | | | | |
| $p_4$ | 3 | 6 | | | |
| $p_5$ | 2 | | | | |
| $p_6$ | 3 | | | | |

| | | | |
|---|---|---|---|
| $\overline{p}_1$ | 2 | 5 | |
| $\overline{p}_2$ | 1 | | |
| $\overline{p}_3$ | 2 | | |
| $\overline{p}_4$ | | | |
| $\overline{p}_5$ | 6 | | |
| $\overline{p}_6$ | 1 | 2 | 5 |

Current assignment: $\overline{p}_2{}^{\mathsf{d}}\, p_1\, \overline{p}_4{}^{\mathsf{d}}$

Literals $p_6$, $\overline{p}_5$ have to be added to the assignment

# BCP with Occur Lists

1. $p_1 \quad \overline{p}_2 \quad \overline{p}_6$
2. $p_2 \quad \overline{p}_3 \quad p_5 \quad \overline{p}_1 \quad \overline{p}_6$
3. $p_6 \quad p_2 \quad p_4$
4. $p_1 \quad p_2$
5. $\overline{p}_6 \quad \overline{p}_1 \quad p_3$
6. $\overline{p}_5 \quad p_4 \quad p_2$

Model

| $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|
| T | F | U | F | F | T |

ToPropagate

| |
|---|
| $\overline{p}_5$ |
| $p_6$ |

ClausesWith

| | | | | |
|---|---|---|---|---|
| $p_1$ | 1 | 4 | | |
| $p_2$ | 2 | 3 | 4 | 6 |
| $p_3$ | 5 | | | |
| $p_4$ | 3 | 6 | | |
| $p_5$ | 2 | | | |
| $p_6$ | 3 | | | |

| | | | |
|---|---|---|---|
| $\overline{p}_1$ | 2 | 5 | |
| $\overline{p}_2$ | 1 | | |
| $\overline{p}_3$ | 2 | | |
| $\overline{p}_4$ | | | |
| $\overline{p}_5$ | 6 | | |
| $\overline{p}_6$ | 1 | 2 | 5 |

Current assignment: $\overline{p}_2{}^{\mathsf{d}}\, p_1\, \overline{p}_4{}^{\mathsf{d}}\, p_6\, \overline{p}_5$

Now, we propagate $\overline{p}_5$ visiting ClausesWith$[p_5]$

# BCP with Occur Lists

1 $p_1$ $\overline{p}_2$ $\overline{p}_6$

2 $p_2$ $\overline{p}_3$ $p_5$ $\overline{p}_1$ $\overline{p}_6$

3 $p_6$ $p_2$ $p_4$

4 $p_1$ $p_2$

5 $\overline{p}_6$ $\overline{p}_1$ $p_3$

6 $\overline{p}_5$ $p_4$ $p_2$

Model

| $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|
| T | F | U | F | F | T |

ToPropagate

$p_6$

ClausesWith

| | | | | | |
|---|---|---|---|---|---|
| $p_1$ | 1 | 4 | | | |
| $p_2$ | 2 | 3 | 4 | 6 | |
| $p_3$ | 5 | | | | |
| $p_4$ | 3 | 6 | | | |
| $p_5$ | 2 | | | | |
| $p_6$ | 3 | | | | |

| | | | |
|---|---|---|---|
| $\overline{p}_1$ | 2 | 5 | |
| $\overline{p}_2$ | 1 | | |
| $\overline{p}_3$ | 2 | | |
| $\overline{p}_4$ | | | |
| $\overline{p}_5$ | 6 | | |
| $\overline{p}_6$ | 1 | 2 | 5 |

Current assignment: $\overline{p}_2{}^{\mathsf{d}}\, p_1\, \overline{p}_4{}^{\mathsf{d}}\, p_6\, \overline{p}_5$

Literal $\overline{p}_3$ has to be added to the assignment

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# BCP with Occur Lists

1. $p_1 \quad \overline{p}_2 \quad \overline{p}_6$
2. $p_2 \quad \overline{p}_3 \quad p_5 \quad \overline{p}_1 \quad \overline{p}_6$
3. $p_6 \quad p_2 \quad p_4$
4. $p_1 \quad p_2$
5. $\overline{p}_6 \quad \overline{p}_1 \quad p_3$
6. $\overline{p}_5 \quad p_4 \quad p_2$

Model

| $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|
| T | F | F | F | F | T |

ToPropagate

| |
|---|
| $\overline{p}_3$ |
| $p_6$ |

ClausesWith

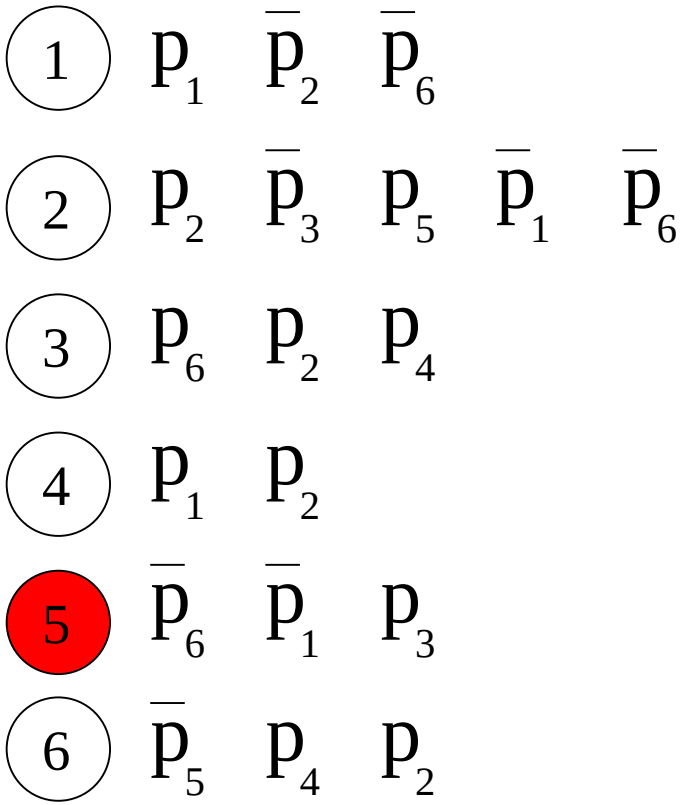
| | | | | |
|---|---|---|---|---|
| $p_1$ | 1 | 4 | | |
| $p_2$ | 2 | 3 | 4 | 6 |
| $p_3$ | 5 | | | |
| $p_4$ | 3 | 6 | | |
| $p_5$ | 2 | | | |
| $p_6$ | 3 | | | |

| | | | |
|---|---|---|---|
| $\overline{p}_1$ | 2 | 5 | |
| $\overline{p}_2$ | 1 | | |
| $\overline{p}_3$ | 2 | | |
| $\overline{p}_4$ | | | |
| $\overline{p}_5$ | 6 | | |
| $\overline{p}_6$ | 1 | 2 | 5 |

Current assignment: $\overline{p}_2{}^{\mathsf{d}}\, p_1\, \overline{p}_4{}^{\mathsf{d}}\, p_6\, \overline{p}_5\, \overline{p}_3$

Now, we propagate $\overline{p}_3$ visiting ClausesWith[$p_3$]

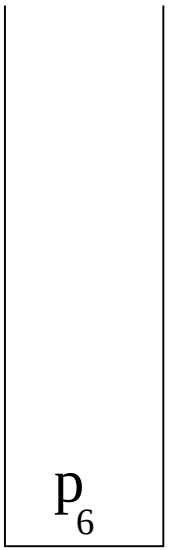
# BCP with Occur Lists




Model

| $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|
| T | F | F | F | F | T |

ToPropagate




ClausesWith

| | | | | |
|---|---|---|---|---|
| $p_1$ | 1 | 4 | | |
| $p_2$ | 2 | 3 | 4 | 6 |
| $p_3$ | 5 | | | |
| $p_4$ | 3 | 6 | | |
| $p_5$ | 2 | | | |
| $p_6$ | 3 | | | |

| | | | |
|---|---|---|---|
| $\overline{p}_1$ | 2 | 5 | |
| $\overline{p}_2$ | 1 | | |
| $\overline{p}_3$ | 2 | | |
| $\overline{p}_4$ | | | |
| $\overline{p}_5$ | 6 | | |
| $\overline{p}_6$ | 1 | 2 | 5 |

Current assignment: $\overline{p}_2{}^{\mathsf{d}}\, p_1\, \overline{p}_4{}^{\mathsf{d}}\, p_6\, \overline{p}_5\, \overline{p}_3$

Clause 5 indicates a conflict. Backtrack/backjump is called.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# BCP with Occur Lists

1 $p_1$ $\overline{p}_2$ $\overline{p}_6$

2 $p_2$ $\overline{p}_3$ $p_5$ $\overline{p}_1$ $\overline{p}_6$

3 $p_6$ $p_2$ $p_4$

4 $p_1$ $p_2$

5 $\overline{p}_6$ $\overline{p}_1$ $p_3$

6 $\overline{p}_5$ $p_4$ $p_2$

Model

| $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|
| T | F | U | T | U | U |

ToPropagate

| |
|---|
| $p_4$ |

ClausesWith

| | | | | |
|---|---|---|---|---|
| $p_1$ | 1 | 4 | | |
| $p_2$ | 2 | 3 | 4 | 6 |
| $p_3$ | 5 | | | |
| $p_4$ | 3 | 6 | | |
| $p_5$ | 2 | | | |
| $p_6$ | 3 | | | |

| | | | |
|---|---|---|---|
| $\overline{p}_1$ | 2 | 5 | |
| $\overline{p}_2$ | 1 | | |
| $\overline{p}_3$ | 2 | | |
| $\overline{p}_4$ | | | |
| $\overline{p}_5$ | 6 | | |
| $\overline{p}_6$ | 1 | 2 | 5 |

Current assignment: $\overline{p}_2{}^{\mathsf{d}}\, p_1\, p_4$

Procedure continues propagating $p_4$

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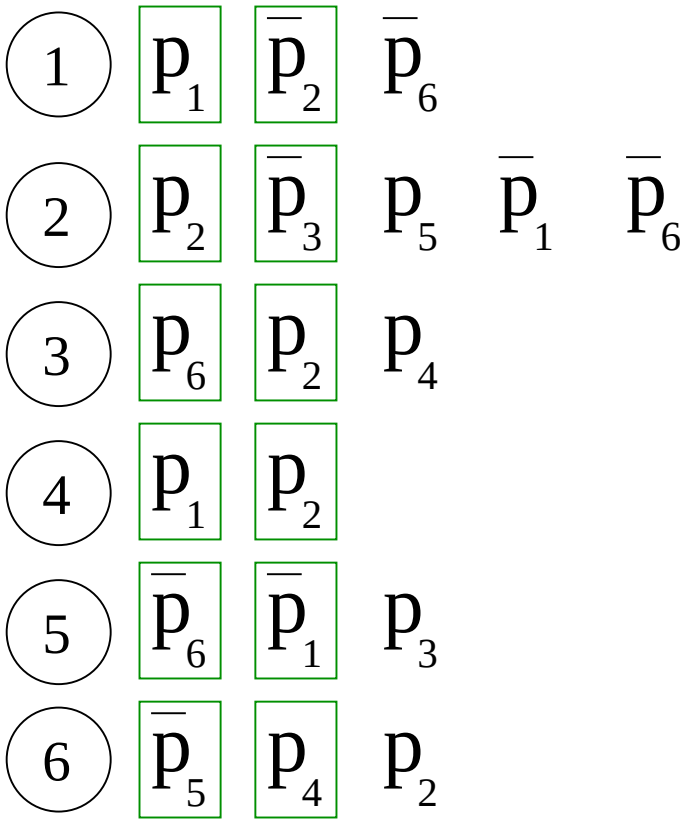
# BCP - How to improve it?

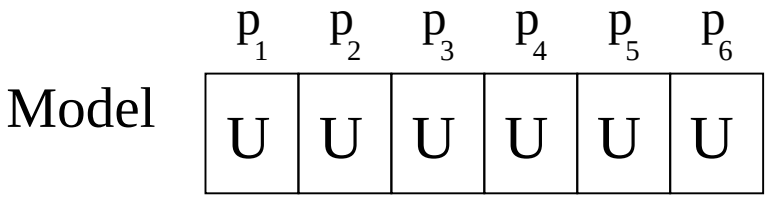
- The key observation is the following:

  **A clause with 2 non-false literals can't be unit or conflicting**

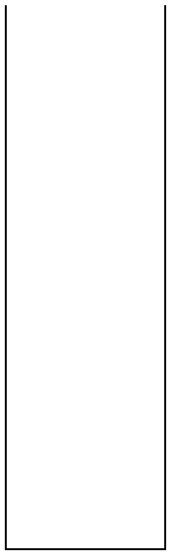
- For each clause we will try to watch two non-false literals
- Enough to visit a clause when a watched literal becomes false
- If 2 non-false literals do not exist, this is because:
  - All the lits are false (no problem, we will backtrack)
  - There is exactly one true literal $l$.
    In this case we will watch $l$ and a false literal $l'$ such that $DL(l') \geq DL(l)$ (see later why)
- This is called the two watched literals scheme

# BCP - Two Watched Literals

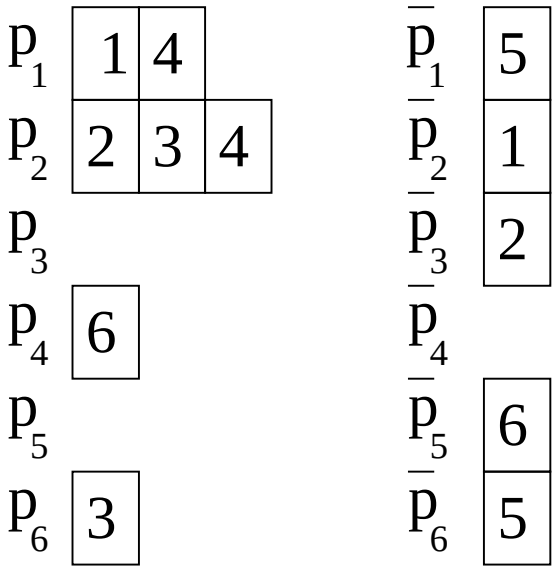



ToPropagate

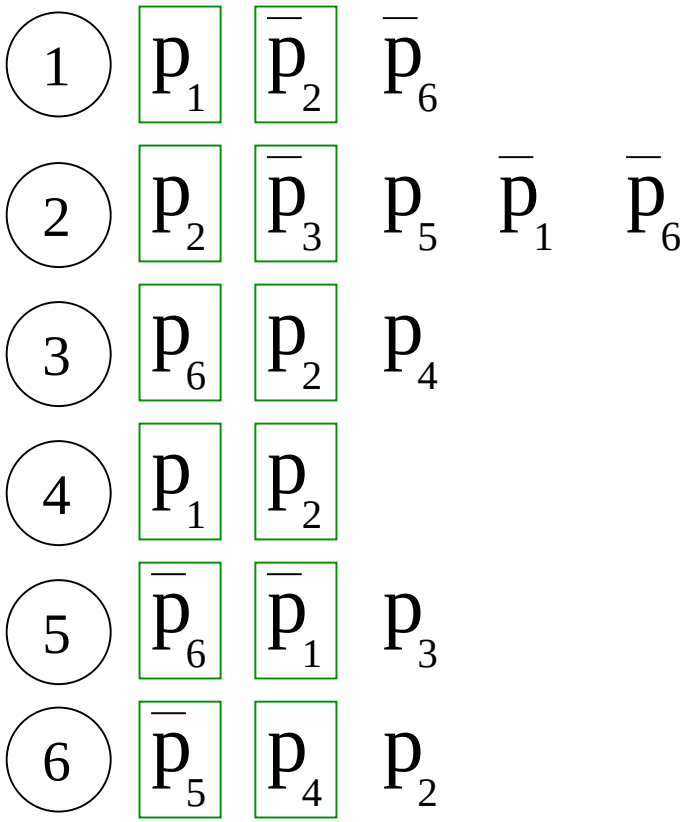


ClausesWithLitWatched




Current assignment: $\emptyset$

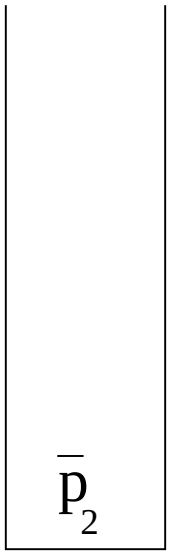
# BCP - Two Watched Literals




Model

| $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|
| U | F | U | U | U | U |

ToPropagate




ClausesWithLitWatched

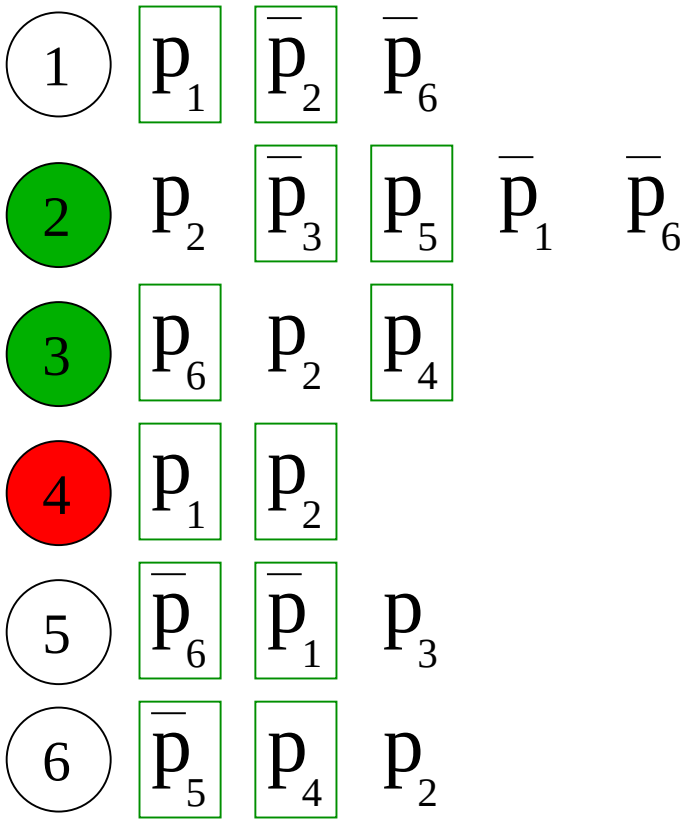
| | | | |
|---|---|---|---|
| $p_1$ | 1 | 4 | |
| $p_2$ | 2 | 3 | 4 |
| $p_3$ | | | |
| $p_4$ | 6 | | |
| $p_5$ | | | |
| $p_6$ | 3 | | |

| | |
|---|---|
| $\overline{p}_1$ | 5 |
| $\overline{p}_2$ | 1 |
| $\overline{p}_3$ | 2 |
| $\overline{p}_4$ | |
| $\overline{p}_5$ | 6 |
| $\overline{p}_6$ | 5 |

Current assignment: $\overline{p}_2$ [d]

Now, we propagate $\overline{p}_2$ visiting ClausesWithLitWatched[$p_2$]

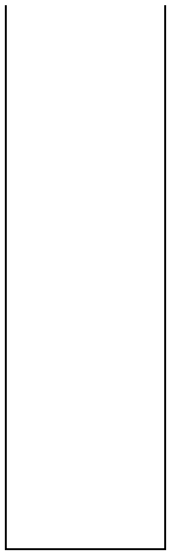
# BCP - Two Watched Literals




Model

| $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|
| U | F | U | U | U | U |

ToPropagate



ClausesWithLitWatched

| | | |
|---|---|---|
| $p_1$ | 1 | 4 |
| $p_2$ | 4 | |
| $p_3$ | | |
| $p_4$ | 6 | 3 |
| $p_5$ | 2 | |
| $p_6$ | 3 | |

| | |
|---|---|
| $\overline{p}_1$ | 5 |
| $\overline{p}_2$ | 1 |
| $\overline{p}_3$ | 2 |
| $\overline{p}_4$ | |
| $\overline{p}_5$ | 6 |
| $\overline{p}_6$ | 5 |

Current assignment: $\overline{p}_2{}^{\mathsf{d}}$

Clauses 2 and 3 are rewatched. Clause 4 can't because it is unit ($p_1$)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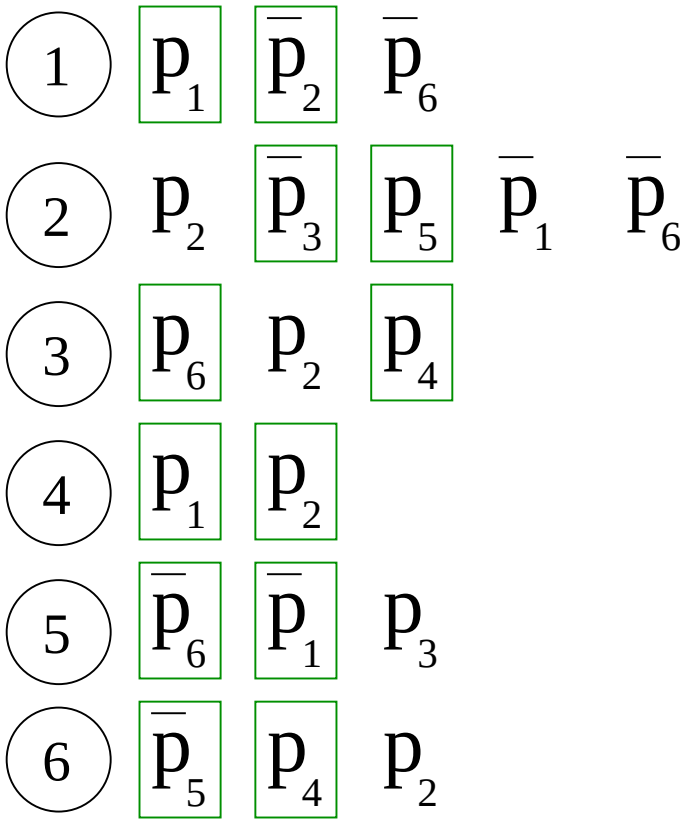
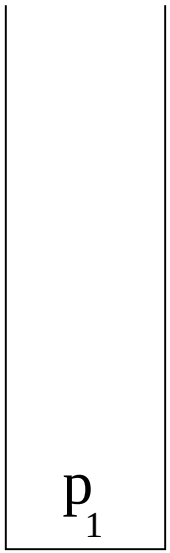
# BCP - Two Watched Literals




Model

| $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|
| T | F | U | U | U | U |

ToPropagate




ClausesWithLitWatched

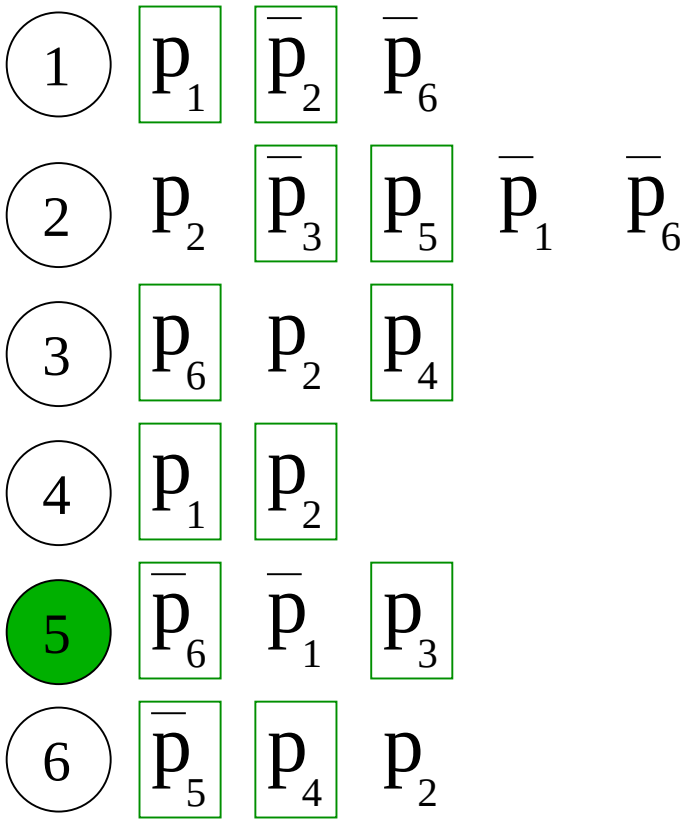
| | | |
|---|---|---|
| $p_1$ | 1 | 4 |
| $p_2$ | 4 | |
| $p_3$ | | |
| $p_4$ | 6 | 3 |
| $p_5$ | 2 | |
| $p_6$ | 3 | |

| | |
|---|---|
| $\overline{p}_1$ | 5 |
| $\overline{p}_2$ | 1 |
| $\overline{p}_3$ | 2 |
| $\overline{p}_4$ | |
| $\overline{p}_5$ | 6 |
| $\overline{p}_6$ | 5 |

Current assignment: $\overline{p}_2$ d $p_1$

Now, we propagate $p_1$ visiting ClausesWithLitWatched[$\overline{p}_1$]

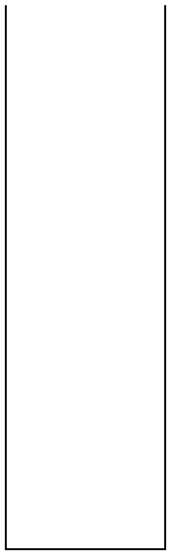
# BCP - Two Watched Literals




Model

| $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|
| T | F | U | U | U | U |

ToPropagate



ClausesWithLitWatched

| | | |
|---|---|---|
| $p_1$ | 1 | 4 |
| $p_2$ | 4 | |
| $p_3$ | 5 | |
| $p_4$ | 6 | 3 |
| $p_5$ | 2 | |
| $p_6$ | 3 | |

| | |
|---|---|
| $\overline{p}_1$ | |
| $\overline{p}_2$ | 1 |
| $\overline{p}_3$ | 2 |
| $\overline{p}_4$ | |
| $\overline{p}_5$ | 6 |
| $\overline{p}_6$ | 5 |

Current assignment: $\overline{p}_2$ d $p_1$

Clauses 5 is reselected. No lit is unit propagated. We have to decide.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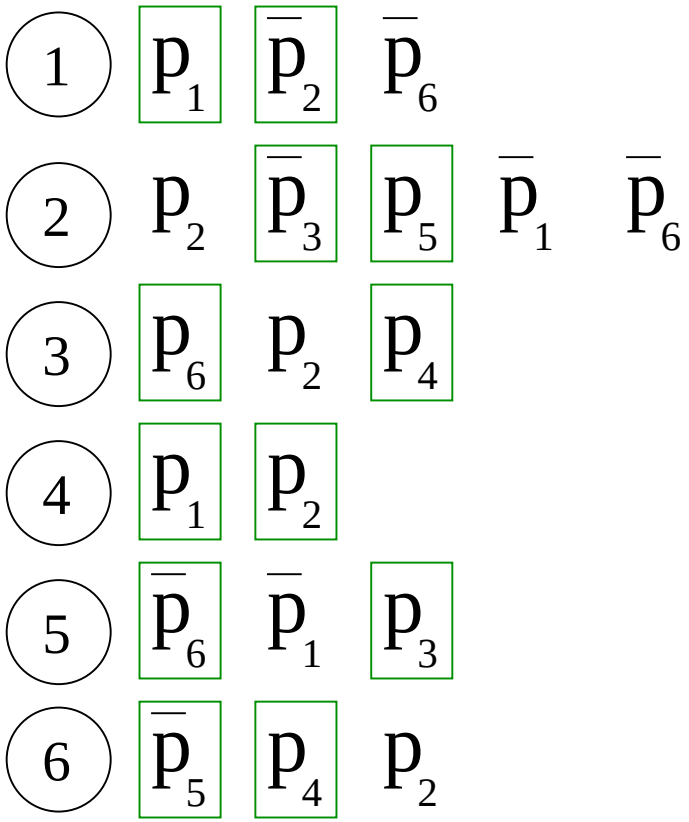
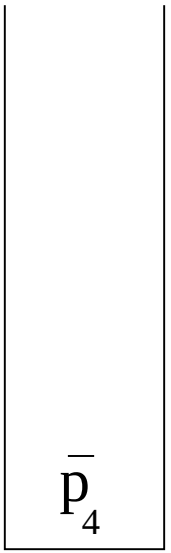
# BCP - Two Watched Literals




Model

| $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|
| T | F | U | F | U | U |

ToPropagate




ClausesWithLitWatched

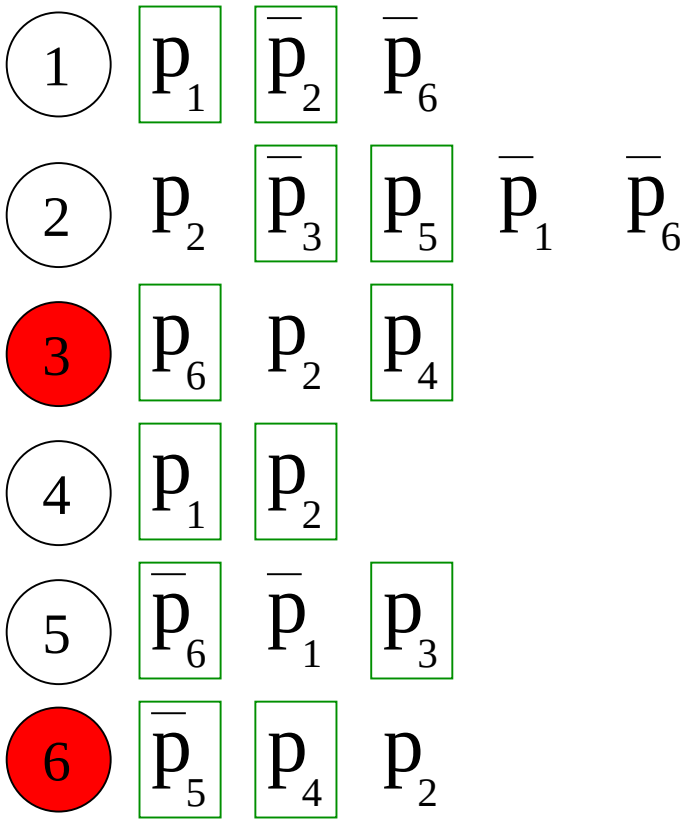
| | | |
|---|---|---|
| $p_1$ | 1 | 4 |
| $p_2$ | 4 | |
| $p_3$ | 5 | |
| $p_4$ | 6 | 3 |
| $p_5$ | 2 | |
| $p_6$ | 3 | |

| | |
|---|---|
| $\overline{p}_1$ | |
| $\overline{p}_2$ | 1 |
| $\overline{p}_3$ | 2 |
| $\overline{p}_4$ | |
| $\overline{p}_5$ | 6 |
| $\overline{p}_6$ | 5 |

Current assignment: $\overline{p}_2{}^{\mathsf{d}}\, p_1\, \overline{p}_4{}^{\mathsf{d}}$

Now, we propagate $\overline{p}_4$ visiting ClauseWithLitWatched$[p_4]$

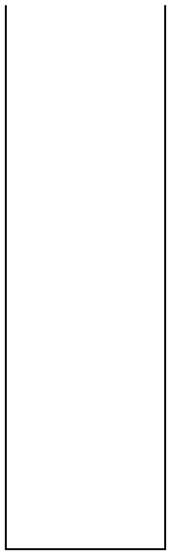
# BCP - Two Watched Literals




Model

| $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|
| T | F | U | F | U | U |

ToPropagate



ClausesWithLitWatched

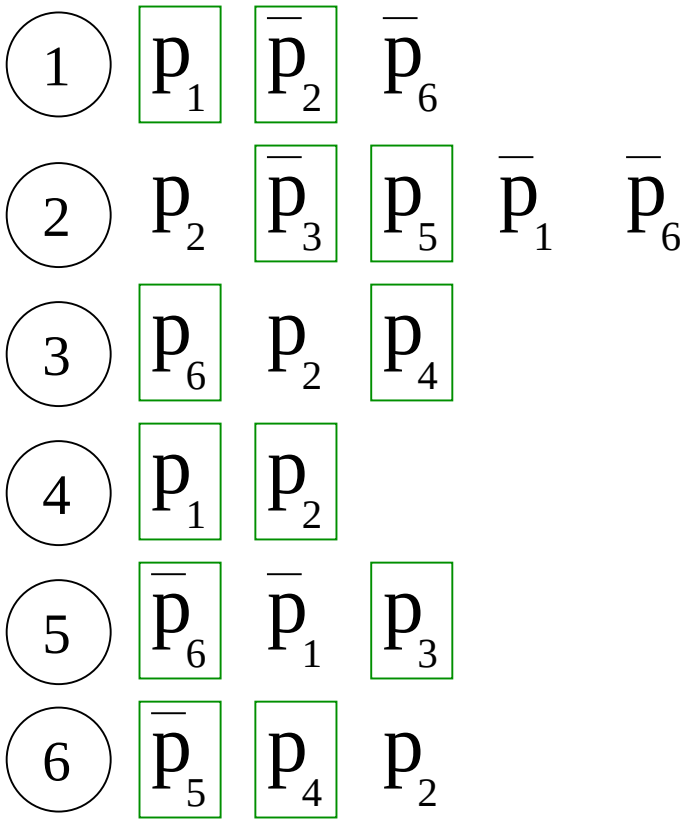
| | | |
|---|---|---|
| $p_1$ | 1 | 4 |
| $p_2$ | 4 | |
| $p_3$ | 5 | |
| $p_4$ | 6 | 3 |
| $p_5$ | 2 | |
| $p_6$ | 3 | |

| | |
|---|---|
| $\overline{p}_1$ | |
| $\overline{p}_2$ | 1 |
| $\overline{p}_3$ | 2 |
| $\overline{p}_4$ | |
| $\overline{p}_5$ | 6 |
| $\overline{p}_6$ | 5 |

Current assignment: $\overline{p}_2{}^{\mathsf{d}}\, p_1\, \overline{p}_4{}^{\mathsf{d}}$

Clause 6 unit propagates $\overline{p}_5$ and clause 3 propagates $p_6$.

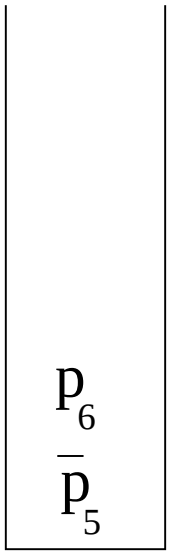
# BCP - Two Watched Literals



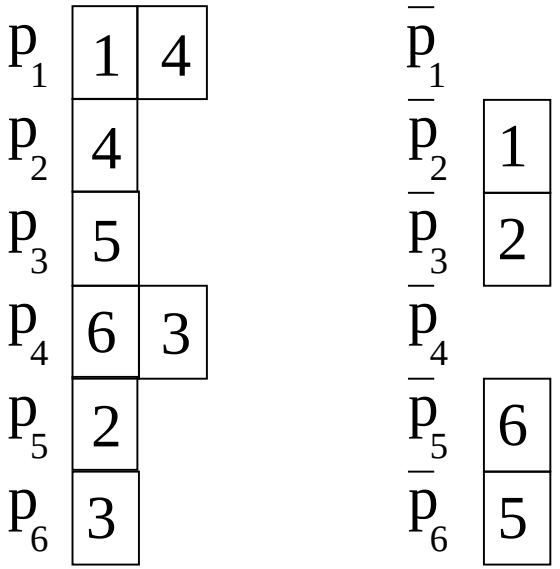

Model

| $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|
| T | F | U | F | F | T |

ToPropagate



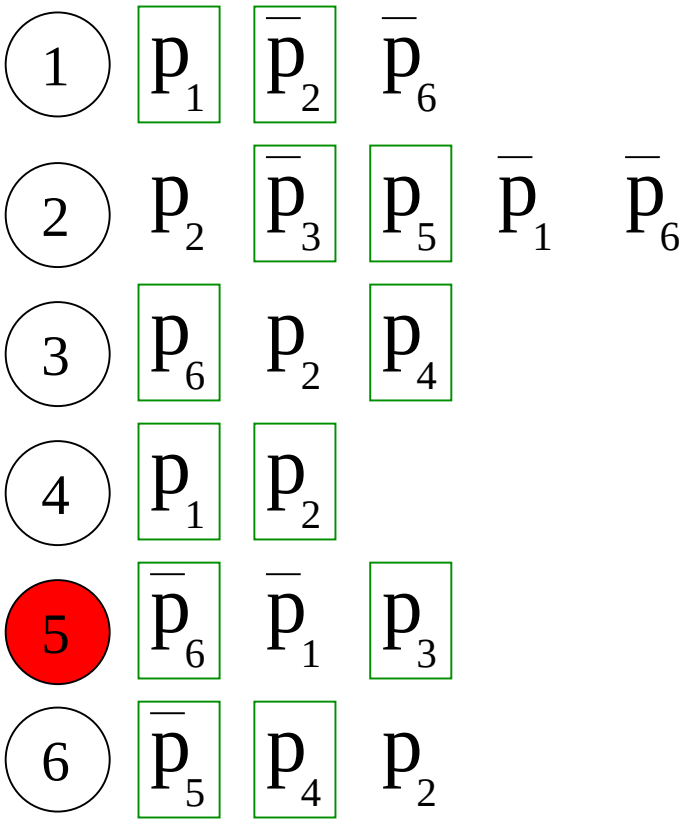

ClausesWithLitWatched




Current assignment: $\overline{p}_2{}^{\mathsf{d}}\, p_1\, \overline{p}_4{}^{\mathsf{d}}\, \overline{p}_5\, p_6$

Now, we propagate $p_6$ visiting ClauseWithLitWatched$[\overline{p}_6]$

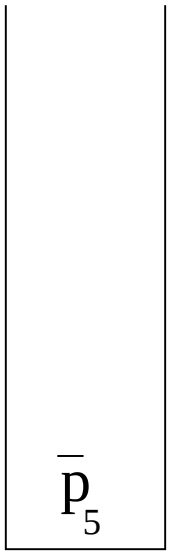
# BCP - Two Watched Literals




Model

| $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|
| T | F | U | F | F | T |

ToPropagate




ClausesWithLitWatched

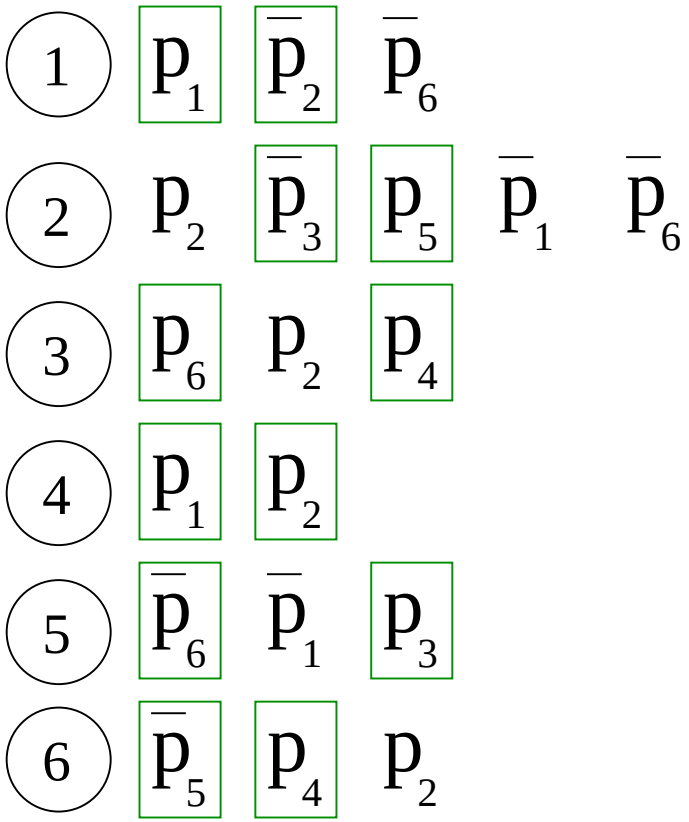
| | | |
|---|---|---|
| $p_1$ | 1 | 4 |
| $p_2$ | 4 | |
| $p_3$ | 5 | |
| $p_4$ | 6 | 3 |
| $p_5$ | 2 | |
| $p_6$ | 3 | |

| | |
|---|---|
| $\overline{p}_1$ | |
| $\overline{p}_2$ | 1 |
| $\overline{p}_3$ | 2 |
| $\overline{p}_4$ | |
| $\overline{p}_5$ | 6 |
| $\overline{p}_6$ | 5 |

Current assignment: $\overline{p}_2{}^{\mathsf{d}}\, p_1\, \overline{p}_4{}^{\mathsf{d}}\, \overline{p}_5\, p_6$

Clause 5 can't be reselected because it is unit ($p_3$).

# BCP - Two Watched Literals




Model

| $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|
| T | F | T | F | F | T |

ToPropagate

$p_3$
$\overline{p}_5$

ClausesWithLitWatched

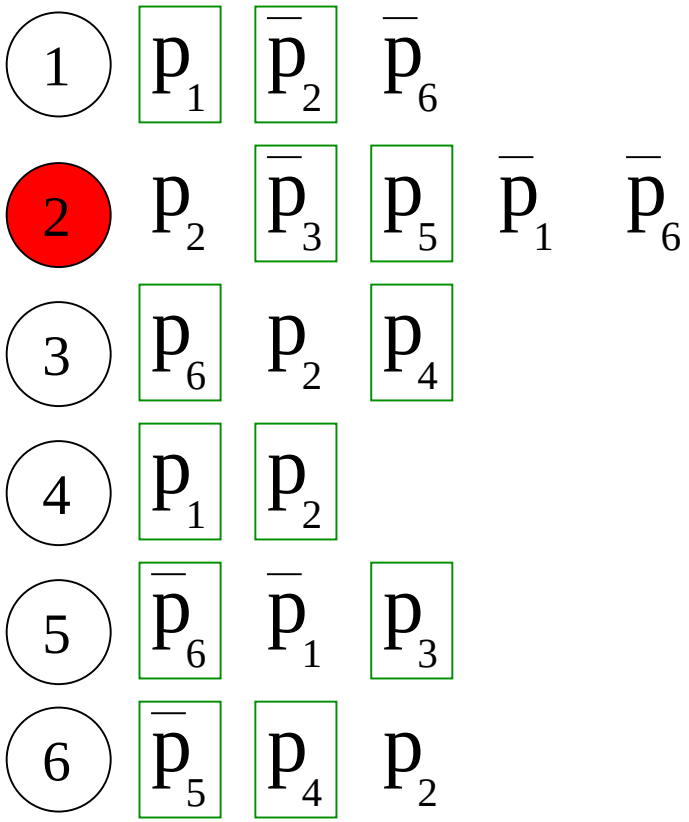
| | | |
|---|---|---|
| $p_1$ | 1 | 4 |
| $p_2$ | 4 | |
| $p_3$ | 5 | |
| $p_4$ | 6 | 3 |
| $p_5$ | 2 | |
| $p_6$ | 3 | |

| | |
|---|---|
| $\overline{p}_1$ | |
| $\overline{p}_2$ | 1 |
| $\overline{p}_3$ | 2 |
| $\overline{p}_4$ | |
| $\overline{p}_5$ | 6 |
| $\overline{p}_6$ | 5 |

Current assignment: $\overline{p}_2$ d $p_1$ $\overline{p}_4$ d $\overline{p}_5$ $p_6$ $p_3$

Now, we propagate $p_3$ visiting ClauseWithLitWatched[$\overline{p}_3$]

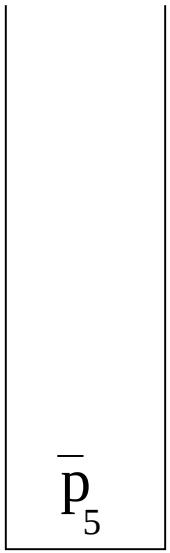
# BCP - Two Watched Literals




Model

| $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|
| T | F | T | F | F | T |

ToPropagate



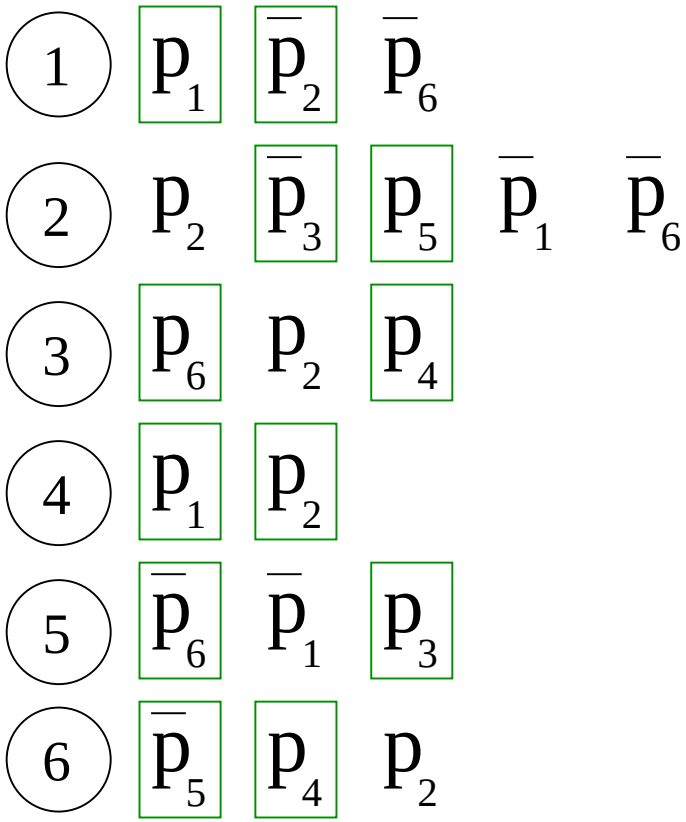

ClausesWithLitWatched

| | | |
|---|---|---|
| $p_1$ | 1 | 4 |
| $p_2$ | 4 | |
| $p_3$ | 5 | |
| $p_4$ | 6 | 3 |
| $p_5$ | 2 | |
| $p_6$ | 3 | |

| | |
|---|---|
| $\overline{p}_1$ | |
| $\overline{p}_2$ | 1 |
| $\overline{p}_3$ | 2 |
| $\overline{p}_4$ | |
| $\overline{p}_5$ | 6 |
| $\overline{p}_6$ | 5 |

Current assignment: $\overline{p}_2{}^{\mathsf{d}}\, p_1\, \overline{p}_4{}^{\mathsf{d}}\, \overline{p}_5\, p_6\, p_3$
Clause 2 indicates a conflict. Backjump/backtrack is called.

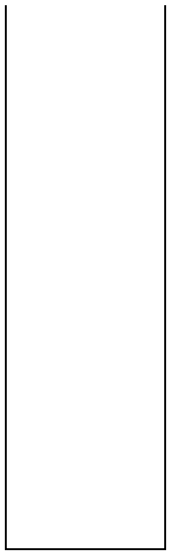
# BCP - Two Watched Literals




Model

| $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|
| T | F | U | U | U | U |

ToPropagate



ClausesWithLitWatched

| | | |
|---|---|---|
| $p_1$ | 1 | 4 |
| $p_2$ | 4 | |
| $p_3$ | 5 | |
| $p_4$ | 6 | 3 |
| $p_5$ | 2 | |
| $p_6$ | 3 | |

| | |
|---|---|
| $\overline{p}_1$ | |
| $\overline{p}_2$ | 1 |
| $\overline{p}_3$ | 2 |
| $\overline{p}_4$ | |
| $\overline{p}_5$ | 6 |
| $\overline{p}_6$ | 5 |

Current assignment: $\overline{p}_2$ $^{\mathrm{d}}$ $p_1$ (lit $p_4$ not yet added)

After backtrack watches are properly placed!

# Two watched literals - Analysis

- Each clause is visited far less often
- Upon backtrack, nothing has to be done
- Inactive literals tend to be watched, hence further reducing the number of clauses to be visited
- Very effective for long clauses (e.g. lemmas)
- For binary clauses specialized data structures are used

# Two watched literals - Analysis (2)

Why if 2 non-false literals do not exist, we have to watch a true literal $l$ and a false lit $l'$ with $DL(l') \geq DL(l)$?

- Assume we have a conflict at DL 3 and lemma generated is $p_1 \vee p_2 \vee p_3$ such that
  - $p_1$ is the UIP (set to true after BT to DL 2)
  - $p_2$ false at DL 2
  - $p_3$ false at DL 1
  - We watch $p_1$ and $p_3$
- Later on we backtrack to DL 1
- Now literal $\overline{p}_2$ is added to the assignment
- We will not detect that $p_1$ should be propagated!!!

# Overview of the session

- Conflict Analysis
  - Motivating example
  - Backjumping
  - Conflict graph
  - Lemma shortening
- Lemma removal
- Decision heuristics
- Restarts
- Efficient implemetation of UnitProp:
  - Occur lists
  - Two-watched literals
- Final remarks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# Why is DPLL really good?

Three key ingredients that only work if used TOGETHER:

- Learn at each conflict the backjump clause as a lemma:
  - makes UnitProp more powerful
  - prevents future similar conflicts
- Decide on the variable with most occurrences in recent conflicts:
  - so-called activity-based heuristics
  - idea: work off clusters of tightly related (by many clauses) vars
- Forget from time to time low-activity lemmas:
  - crucial to keep UnitProp fast and afford memory usage
  - idea: lemmas from worked off clusters no longer needed!

# A final view of CDCL SAT solvers

CDCL (Conflict Driven Clause Learning) SAT solvers are:

**A set of heuristics indicating necessary resolution steps**

# Bibliography - Some further reading


- Matthew W. Moskewicz, Conor F. Madigan, Ying Zhao, Lintao Zhang, Sharad Malik. *Chaff: Engineering an Efficient SAT Solver.* DAC 2001: 530-535

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- Robert Nieuwenhuis, Albert Oliveras, Cesare Tinelli. *Solving SAT and SAT Modulo Theories: From an abstract Davis–Putnam–Logemann–Loveland procedure to DPLL(T).* J. ACM 53(6): 937-977 (2006)