Syntactic Connectivity

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Abstract

Type logical grammar presents a paradigm of linguistic description based on what we may refer to as a Lambek-van Benthem correspondence: (logical) formulas as (linguistic) categories. Lexical signs are classified by category formulas, and the language model projected by a lexicon is determined by the consequence relation induced on category formulas by their interpretation.

In this logical model of language, (logical) proofs correspond to (linguistic) derivations, but such syntax serves just to calculate what is generated, not to define it. Although syntax plays no definitional role linguistically, from a computational linguistic point of view we are interested in the process of grammatical reasoning, and we propose to reinstate syntactic structure as the trace of such processing. Addressing the question 'What is the essential structure of the relevant kinds of proofs?' yields a new answer to the question 'What is syntactic structure?' under the slogan *proof nets as syntactic structures.* This provides a particularly vivid realisation of the notion of categorial syntactic connection of Ajdukiewicz (1935) as a harmonic mutual connectivity of the valencies of the words making up a sentence. We offer a general methodology for the development of proof nets for partially commutative categorial logics.

Keywords: categorial grammar, categorial logic, linear logic, proof nets, type logical grammar

1 Introduction

Whereas phrase structure grammar models language as a formal system, i.e. a set of strings, categorial grammar models language as a communicative system, i.e. a set of signs (form-meaning associations). Parse trees for CFG are concrete structures defining the equivalence classes of string rewriting derivations. Corresponding structures for categorial grammar must be deeper, since they incorporate also semantics. Here we investigate the idea that those structures are proof nets, that proof nets are for categorial grammar what parse trees are for CFG, hence our paradigmatic slogan: *proof nets as syntactic structures.*

The syntactic calculus **L** of Lambek (1958) provides a logical model of language which presents formulas as categories and proofs as derivations. The calculus, now recognizable as a multiplicative fragment of non-commutative intuitionistic linear logic (Girard 1987), has a sequent calculus free of structural rules, and a proof net syntax which is more geometrical than that of linear logic, for the proof nets are *planar* (Roorda 1991; Abrusci 1995).

Computationally, the proof nets provide the essential structure of derivations. They proffer no "spurious ambiguity" and support, for example, a prenormalisation allowing parsing to normal form semantic output without on-line λ-conversion (Morrill 1997: 25–30), and memoisation (Morrill 1996), something prohibitive under the shifting premises of hypothetical reasoning in other forms of categorial proof syntax.

1.1 Associative Lambek calculus

Let us recall the (associative) Lambek calculus **L**. The *category formulas* $\mathcal{F}$ are constructed from atomic category formulas $\mathcal{A}$ (*atoms*) by a product operator $\bullet$ and two directional divisors, \ ("under"), and / ("over"), as follows:

$$\mathcal{F} ::= \mathcal{A} \mid \mathcal{F}\bullet\mathcal{F} \mid \mathcal{F}\backslash\mathcal{F} \mid \mathcal{F}/\mathcal{F} \tag{1.1}$$

Lambek (1958, 1988) gives an *algebraic* interpretation in a semigroup $(L, +)$, a set L closed under an associative binary operation $+$ (we may think of the set of strings over some vocabulary, and the operation of concatenation). Formulas are interpreted as subsets of L. Given an interpretation $[\![P]\!]$ for each atom P, each category formula A receives an interpretation $[\![A]\!]$ thus:

$$\begin{array}{lcl} [\![A\backslash B]\!] & = & \{s \mid \forall s' \in [\![A]\!], s'{+}s \in [\![B]\!]\} \\ [\![B/A]\!] & = & \{s \mid \forall s' \in [\![A]\!], s{+}s' \in [\![B]\!]\} \\ [\![A\bullet B]\!] & = & \{s_1{+}s_2 \mid s_1 \in [\![A]\!] \;\&\; s_2 \in [\![B]\!]\} \end{array} \tag{1.2}$$

Van Benthem (1991) gives a *relational* interpretation in a set V (we may think of the starting and ending moments of utterances). Formulas are interpreted as binary relations, i.e. as subsets of $V \times V$. Given an interpretation $[\![P]\!]$ for each atom P, each category formula A receives an interpretation $[\![A]\!]$ thus:

$$\begin{array}{lcl} [\![A\backslash B]\!] & = & \{\langle v_2, v_3\rangle \mid \forall v_1, \langle v_1, v_2\rangle \in [\![A]\!] \to \langle v_1, v_3\rangle \in [\![B]\!]\} \\ [\![B/A]\!] & = & \{\langle v_1, v_2\rangle \mid \forall v_3, \langle v_2, v_3\rangle \in [\![A]\!] \to \langle v_1, v_3\rangle \in [\![B]\!]\} \\ [\![A\bullet B]\!] & = & \{\langle v_1, v_3\rangle \mid \exists v_2, \langle v_1, v_2\rangle \in [\![A]\!] \;\&\; \langle v_2, v_3\rangle \in [\![B]\!]\} \end{array} \tag{1.3}$$

A sequent $\Gamma \Rightarrow A$ comprises a succedent category formula A and an antecedent configuration Γ which is a non-empty finite sequence of category formulas. A sequent $A_1, \ldots, A_n \Rightarrow A$ asserts that for all algebraic interpretations, for all $s_1, \ldots, s_n \in L$, if $s_i \in [\![A_i]\!], 1{\leq}i{\leq}n$ then $s_1{+}\cdots{+}s_n \in [\![A]\!]$, and that for all relational interpretations, for all $v_0, \ldots, v_n \in V$, if $\langle v_{i-1}, v_i\rangle \in [\![A_i]\!], 1{\leq}i{\leq}n$ then $\langle v_0, v_n\rangle \in [\![A]\!]$. The valid sequents are those generated by the following sequent calculus ($\Gamma(\Delta)$ indicates a configuration Γ with a distinguished subconfiguration Δ):

(1.4)

a. $A \Rightarrow A$ id $\qquad \dfrac{\Gamma \Rightarrow A \qquad \Delta(A) \Rightarrow B}{\Delta(\Gamma) \Rightarrow B}\text{Cut}$

b. $\dfrac{\Gamma \Rightarrow A \qquad \Delta(B) \Rightarrow C}{\Delta(\Gamma, A\backslash B) \Rightarrow C}\backslash\text{L} \qquad \dfrac{A, \Gamma \Rightarrow B}{\Gamma \Rightarrow A\backslash B}\backslash\text{R}$

c. $\dfrac{\Gamma \Rightarrow A \qquad \Delta(B) \Rightarrow C}{\Delta(B/A, \Gamma) \Rightarrow C}/\text{L} \qquad \dfrac{\Gamma, A \Rightarrow B}{\Gamma \Rightarrow B/A}/\text{R}$

d. $\dfrac{\Gamma(A, B) \Rightarrow C}{\Gamma(A\bullet B) \Rightarrow C}\bullet\text{L} \qquad \dfrac{\Gamma \Rightarrow A \qquad \Delta \Rightarrow B}{\Gamma, \Delta \Rightarrow A\bullet B}\bullet\text{R}$

Each connective has a rule of *use* in which it appears in the antecedent of the conclusion sequent, and a rule of *proof* in which it appears in the succedent of the conclusion sequent; in every instance of these *logical* rule schemata there is exactly one more connective occurrence in the conclusion than in the premises so that backward chaining proof steps involving these rules are complexity-reducing: trying to prove conclusions by proving the premises generates strictly simpler subgoals. The *identity* rule schemata id and Cut reflect respectively the reflexivity and transitivity of set containment. The id rule schema has zero premises, i.e. it is an axiom schema; the instances where A is a compound formula are derivable by the other rules from atomic instances, hence id can be restricted to apply to atoms without altering the set of theorems generated. In the Cut rule schema the Cut formula A is duplicated in the premises and the rule fails to be complexity-reducing in the sense of the logical rules. However, the calculus enjoys Cut-elimination: for every proof there is an equivalent Cut-free proof. This means that naive Cut-free backward chaining proof search constitutes a decision procedure for theoremhood. The Cut-elimination result has as a corollary the *subformula property* that every theorem has a proof containing only its subformulas — namely any Cut-free proof.

The calculus of Lambek (1988) adds to that of Lambek (1958) the empty string, ε, the empty configuration, Λ, and the product unit, I. The definition (1.1) of category formulas becomes (1.5).

$$\mathcal{F} ::= \mathcal{A} \mid \mathcal{F}\bullet\mathcal{F} \mid \mathcal{F}\backslash\mathcal{F} \mid \mathcal{F}/\mathcal{F} \mid I \tag{1.5}$$

The product unit is interpreted algebraically as the set comprising the empty string, $[\![I]\!] = \{\varepsilon\}$, and relationally as the identity relation, $[\![I]\!] = \{\langle v_1, v_2\rangle \mid v_1 = v_2\}$. The sequent rules for I are thus:

$$\Rightarrow I \quad I\mathrm{R} \qquad \frac{\Gamma_1, \Gamma_2 \Rightarrow A}{\Gamma_1, I, \Gamma_2 \Rightarrow A} I\mathrm{L} \tag{1.6}$$

Lambek calculus provides a classificatory framework for subcategorization which synchronizes naturally with Fregean semantics of incompleteness and compositionality. It provides for some proper treatment of quantification, and for some action-at-a-distance. Still, from a linguistic point of view the possibilities of the Lambek calculus are extremely limited since it is a logic of only *concatenation*. Various applications have been targeted by formulating corresponding logic of discontinuity, including medial extraction, subject-oriented reflexivisation, object-oriented reflexivisation, quantification, wrapping, gapping, pied piping, comparative subdeletion, plurals and VP ellipsis (Moortgat 1988 pt. 3.3, 1990, 1991/96, 1996; Solias 1992; Morrill and Solias 1993; Morrill 1994 chs. 4–5, 1995; Moortgat and Oehrle 1994; Calcagno 1995; Hendriks 1995; Morrill and Merenciano 1996; Carpenter 1998; Jäger 1997).

1.2 Discontinuity

By way of examples of discontinuity beyond the reach of **L** we consider extraction, and in situ binding. In (1.7) the relative pronoun binds a position which is medial in the relative clause.

(the dog) that$_i$ John gave t_i to Mary (1.7)

Defining the relative pronoun as R/(N\S) or R/(S/N) (where R is CN\CN) allows it to bind only left or right peripheral positions: (1.7) is not generated. To deal with such cases, Moortgat (1988: 110) defines as follows a binary operator which we write $\uparrow_e$:

$$[\![B{\uparrow_e}A]\!] = \{s_1{+}s_2 | \forall s \in [\![A]\!], s_1{+}s{+}s_2 \in [\![B]\!]\} \tag{1.8}$$

Assigning the relative pronoun to category R/(S$\uparrow_e$N) allows both medial and (assuming ε) peripheral extraction, via the rule of proof (1.9).

$$\frac{\Gamma_1, A, \Gamma_2 \Rightarrow B}{\Gamma_1, \Gamma_2 \Rightarrow B{\uparrow_e}A}\uparrow_e\mathrm{R} \tag{1.9}$$

Morrill (1992: 13–14) notes that such a treatment potentially accommodates obligatory extraction valencies:

a. (the man) that$_i$ John assured Mary t_i to be reliable (1.10)
b. ∗John assured Mary Bill to be reliable.

If the extraction valency of "assured" is marked by $\uparrow_e$, a sequent corresponding to (1.10a) is valid while that for (1.10b) is invalid, as required. But a satisfactory rule of use cannot be formulated, as observed by Moortgat (121–2), and, as pointed out by I. Sag (p.c.), in the absence of a rule of use it is impossible to actually derive all cases like (1.10a) since when the obligatory extraction valency verb is subordinate to some functor, one needs to *make use* of the operator in the course of the derivation.

Regarding in situ binding, in (1.11) the quantifier phrase and reflexive are in situ binders, taking scope respectively at the sentence and the verb phrase levels.

a. John bought someone Fido. (1.11)
b. John bought himself Fido.

Moortgat (1991/96) introduces a ternary operator Q for which Morrill (1992: 15) offers the interpretation:

$$[\![Q(B,A,C)]\!] = \{s | \forall s_1, s_3, [\forall s_2 \in [\![A]\!], s_1{+}s_2{+}s_3 \in [\![B]\!]] \rightarrow s_1{+}s{+}s_3 \in [\![C]\!]\} \tag{1.12}$$

Moortgat categorises quantifier phrases and reflexives as sentence and verb phrase in situ binders: Q(S, N, S) and Q(N\S, N, N\S) respectively. Cases such as (1.11) are generated by means of the rule of use (1.13).

$$\frac{\Gamma(A) \Rightarrow B \qquad \Delta(C) \Rightarrow D}{\Delta(\Gamma(Q(B,A,C))) \Rightarrow D}Q\mathrm{L} \tag{1.13}$$

However, this time no satisfactory rule of proof can be given. Therefore, as pointed out by H. Hendriks (p.c.), a valid sequent such as (1.14), showing that a sentence in situ binder is also a verb phrase in situ binder, cannot actually be derived.

$$Q(\mathrm{S}, \mathrm{N}, \mathrm{S}) \Rightarrow Q(\mathrm{N}\backslash\mathrm{S}, \mathrm{N}, \mathrm{N}\backslash\mathrm{S}) \tag{1.14}$$

We make the following contributions. In section 2 we give proof nets for $\mathbf{L}{+}\{\uparrow_e, Q\}$. In section 3 we define a pure sequent calculus (free of structural rules) for sorted discontinuity calculus in which $\uparrow_e$ and Q are defined operators. In section 4 we give proof nets for this discontinuity calculus in general.

2 Proof nets

In the following two subsections we describe classical linear logic, and proof nets for classical linear logic. Subsections 2.3 and 2.4 review proof nets for the Lambek-van Benthem categorial calculus, and for the Lambek calculus. Subsection 2.5 considers proof nets for the product unit; subsection 2.6 introduces proof nets for the medial divisor; and subsection 2.7 introduces proof nets for the in situ bindor.

2.1 Classical linear logic

Consider formulas defined as follows.

$$\mathcal{F} ::= \mathcal{A} \mid \mathcal{F} \otimes \mathcal{F} \mid \mathcal{F} \wp \mathcal{F} \mid \mathcal{F} \multimap \mathcal{F} \mid \mathcal{F}^{\perp} \tag{2.1}$$

In the sequent calculus (2.2), sequents are of the form $\Gamma \Rightarrow \Delta$ where configurations Γ and Δ are finite sequences of formulas.

$$\text{a.}\quad \frac{}{A \Rightarrow A}\text{id} \qquad \frac{\Gamma_1 \Rightarrow \Delta_1, A \qquad A, \Gamma_2 \Rightarrow \Delta_2}{\Gamma_1, \Gamma_2 \Rightarrow \Delta_1, \Delta_2}\text{Cut} \tag{2.2}$$

$$\text{b.}\quad \frac{\Gamma_1, A, B, \Gamma_2 \Rightarrow \Delta}{\Gamma_1, B, A, \Gamma_2 \Rightarrow \Delta}\text{P}_L \qquad \frac{\Gamma \Rightarrow \Delta_1, A, B, \Delta_2}{\Gamma \Rightarrow \Delta_1, B, A, \Delta_2}\text{P}_R$$

$$\text{c.}\quad \frac{\Gamma, A, B \Rightarrow \Delta}{\Gamma, A \otimes B \Rightarrow \Delta}\otimes\text{L} \qquad \frac{\Gamma_1 \Rightarrow A, \Delta_1 \qquad \Gamma_2 \Rightarrow B, \Delta_2}{\Gamma_1, \Gamma_2 \Rightarrow A \otimes B, \Delta_1, \Delta_2}\otimes\text{R}$$

$$\text{d.}\quad \frac{A, \Gamma_1 \Rightarrow \Delta_1 \qquad B, \Gamma_2 \Rightarrow \Delta_2}{A \wp B, \Gamma_1, \Gamma_2 \Rightarrow \Delta_1, \Delta_2}\wp\text{L} \qquad \frac{\Gamma \Rightarrow \Delta, A, B}{\Gamma \Rightarrow \Delta, A \wp B}\wp\text{R}$$

$$\text{e.}\quad \frac{\Gamma_1 \Rightarrow A, \Delta_1 \qquad B, \Gamma_2 \Rightarrow \Delta_2}{\Gamma_1, A \multimap B, \Gamma_2 \Rightarrow \Delta_1, \Delta_2}\multimap\text{L} \qquad \frac{\Gamma, A \Rightarrow B, \Delta}{\Gamma \Rightarrow A \multimap B, \Delta}\multimap\text{R}$$

$$\text{f.}\quad \frac{\Gamma \Rightarrow A, \Delta}{\Gamma, A^{\perp} \Rightarrow \Delta}{}^{\perp}\text{L} \qquad \frac{\Gamma, A \Rightarrow \Delta}{\Gamma \Rightarrow A^{\perp}, \Delta}{}^{\perp}\text{R}$$

We recognize for $\otimes$ ("times"), $\wp$ ("par"), $\multimap$ ("linear implication"), and $^{\perp}$ ("perp") classical sequent rules for conjunction, disjunction, implication and negation respectively. Indeed, the only difference with respect to classical logic is that the structural rules of contraction and weakening are not included. This calculus, *multiplicative classical linear logic*, enjoys Cut-elimination.

Those properties of classical logic which do not depend on contraction and weakening are inherited by classical linear logic. For example, the negation is involutive, $A^{\perp\perp} \Leftrightarrow A$:

$$\text{a.}\quad \dfrac{\dfrac{A \Rightarrow A}{A, A^{\perp} \Rightarrow}{}^{\perp}\text{L}}{A \Rightarrow A^{\perp\perp}}{}^{\perp}\text{R} \qquad \text{b.}\quad \dfrac{\dfrac{A \Rightarrow A}{\Rightarrow A^{\perp}, A}{}^{\perp}\text{R}}{A^{\perp\perp} \Rightarrow A}{}^{\perp}\text{L} \tag{2.3}$$

And there are the following proofs of the two sides of the de Morgan law $(A \otimes B)^{\perp} \Leftrightarrow A^{\perp} \wp B^{\perp}$:

$$
\text{a.}\quad
\dfrac{\dfrac{\dfrac{\dfrac{A \Rightarrow A}{\Rightarrow A, A^{\perp}}{\scriptstyle \perp R} \quad \dfrac{B \Rightarrow B}{\Rightarrow B, B^{\perp}}{\scriptstyle \perp L}}{\Rightarrow A \otimes B, A^{\perp}, B^{\perp}}{\scriptstyle \otimes R}}{\Rightarrow A \otimes B, A^{\perp} \wp B^{\perp}}{\scriptstyle \wp R}}{(A \otimes B)^{\perp} \Rightarrow A^{\perp} \wp B^{\perp}}{\scriptstyle \perp L}
\qquad
\text{b.}\quad
\dfrac{\dfrac{\dfrac{\dfrac{A \Rightarrow A}{A^{\perp}, A \Rightarrow}{\scriptstyle \perp L} \quad \dfrac{B \Rightarrow B}{B^{\perp}, B \Rightarrow}{\scriptstyle \perp L}}{A^{\perp} \wp B^{\perp}, A, B \Rightarrow}{\scriptstyle \wp L}}{A^{\perp} \wp B^{\perp}, A \otimes B \Rightarrow}{\scriptstyle \otimes L}}{A^{\perp} \wp B^{\perp} \Rightarrow (A \otimes B)^{\perp}}{\scriptstyle \perp R}
\qquad (2.4)
$$

The other de Morgan law, $(A \wp B)^{\perp} \Leftrightarrow A^{\perp} \otimes B^{\perp}$, is obtained similarly, and also the equivalence $A \multimap B \Leftrightarrow A^{\perp} \wp B$. Consequently, all formulas have a negation normal form for which they may be regarded as metalinguistic abbreviations; that is the way classical linear logic is usually presented but, for expository reasons, we do otherwise here.[1]

2.2 Proof nets for classical linear logic

In sequent calculus each formula is situated with respect to an opposition, antecedent-succedent. In proof nets, each formula A will be correspondingly situated by signing it as of either input polarity, $A^{\bullet}$, or as of output polarity, A°. In order to define proof nets we first define a class of proof structures of which they are a subset. A proof structure is a connected graph with nodes labelled by signed formulas, assembled out of the *proof links* given in figure 1; in the identity links, X and $\overline{X}$ are $A^{\bullet}$ and A° (in either order). Each formula in a proof link (and a proof structure) is also labelled implicitly as either a premise or a conclusion, or else as internal. We draw edges in such a way that premises always look upwards and conclusions always look downwards; the logical links each have two premises and one conclusion; the id axiom link has two conclusions and no premises, the Cut link two premises and no conclusions.[2]

We define a *signed formula tree* to be a finite tree with leaves labelled by signed atoms, each local tree of which is a logical link. A *proof frame* is a finite sequence[3] of signed formula trees. A *proof structure* is obtained from a proof frame by connecting complementary leaves with axiom links, and complementary roots with Cut links, in such a way that each leaf is connected to exactly one other, and each root to at most one other. Alternatively viewed, proof structures are assembled by identifying premises and conclusions of proof links which are of the same signed formula; see figure 2.

A proof structure with input conclusions ${A_1}^{\bullet}, \ldots, {A_n}^{\bullet}$ and output conclusions ${B_1}^{\circ}, \ldots, {B_m}^{\circ}$ is read as asserting that $A_1, \ldots, A_n \Rightarrow B_1, \ldots, B_m$ is valid. Thus, the proof structure of figure 2 asserts $\mathrm{N} \Rightarrow (\mathrm{N} \multimap \mathrm{S}) \multimap \mathrm{S}$, which is in fact true, but not all proof structures are correct; indeed $\otimes$ and $\wp$ are not distinguished!

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[1]Furthermore, since e.g. $\Gamma, A \Rightarrow \Delta$ if and only if $\Gamma \Rightarrow A^{\perp}, \Delta$ one may convert every sequent to an equivalent one-sided sequent, and work with a one-sided calculus but, for expository reasons, we retain the (more cumbersome) two-sided view.

[2]We consider the premises of our proof links to be *ordered*, left and right, in the way they are drawn, even though this is only needed in relation to planarity, considered later; to maintain a purely graph-theoretic view we should say that there is an implicit directed edge between the premises of logical links.

[3]Again, regarding this ordering see the previous note.

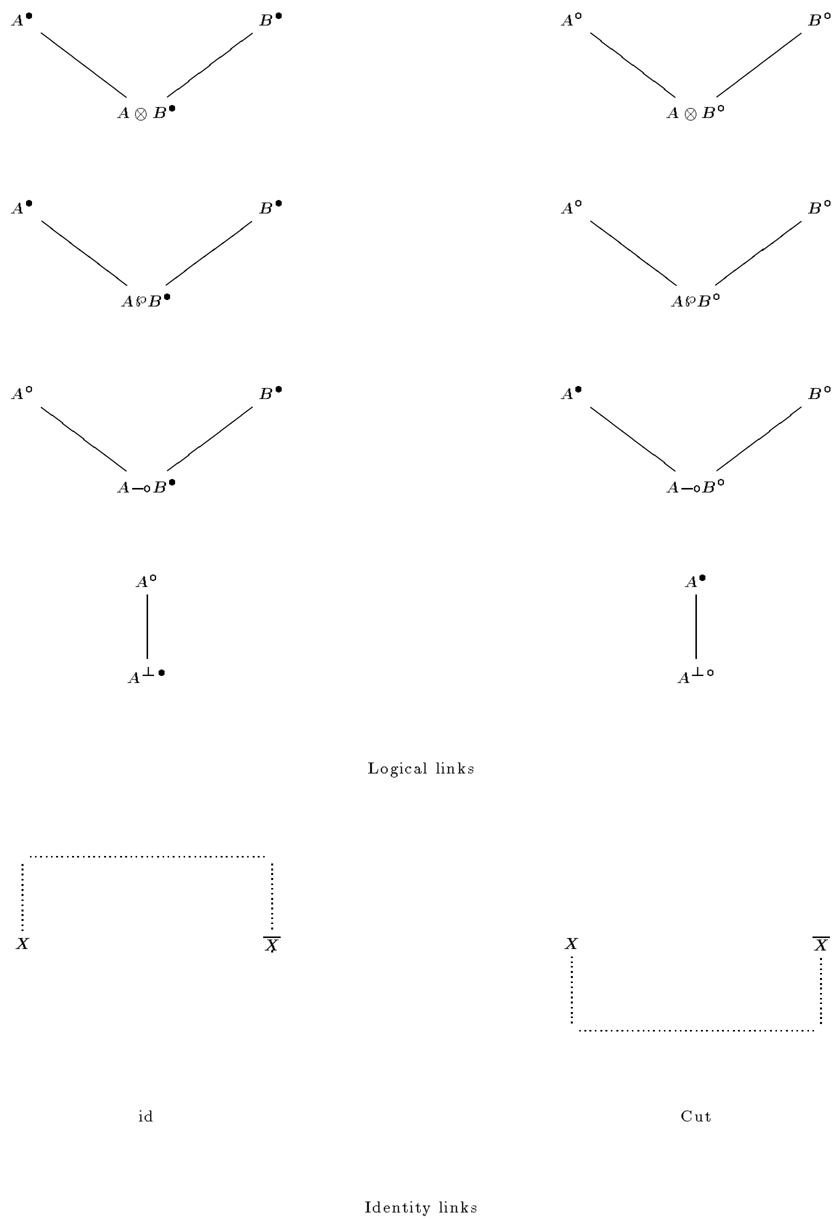

FIG. 1. Proof links of classical linear logic

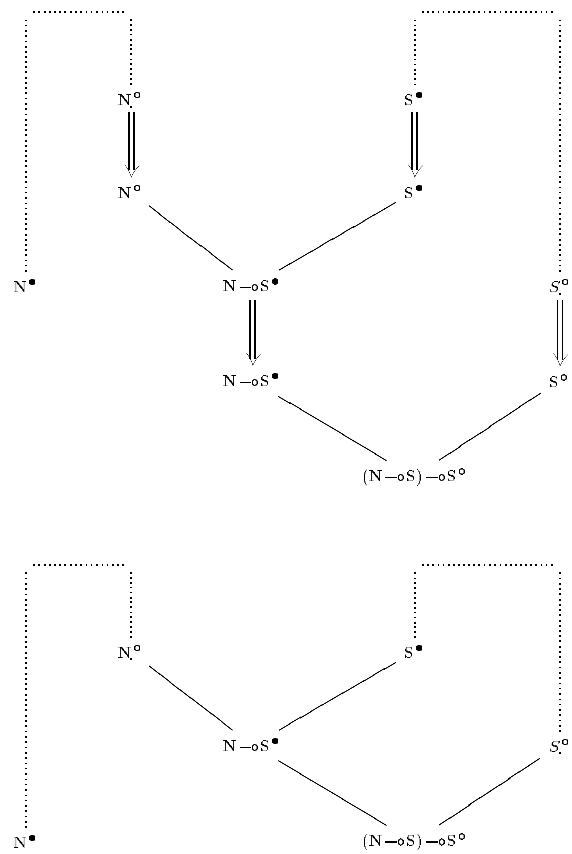

FIG. 2. Assembly of a proof structure

We shall define correctness of proof structures in terms of what we call *expanded* proof links/frames/structures. The criterion is easily applied to proof structures themselves in virtue of their (unique) expansion, but reference to the expanded level allows for a more elegant statement. The links which alter on expansion are given in figure 3. In the $\otimes$- and $\wp$-output links the central node is the principal connective of the conclusion. In the $\otimes$- and $\wp$-input links the central node is the de Morgan dual of the principal connective of the conclusion; this is because we regard input polarity as *negating*.[4] In the $\multimap$-output link we see the disjunction and polarity propagation of the equivalence $A \multimap B \Leftrightarrow A^{\perp} \wp B$, and in the $\multimap$-input link we see the conjunction and polarity propagation of the equivalence $(A \multimap B)^{\perp} \Leftrightarrow A \otimes B^{\perp}$.

The original correctness criterion of Girard (1987), the *long trip condition*, is as follows. Each $\otimes$- and $\wp$-fork in an expanded proof structure is considered a switch which determines travel instructions according to which of two states it is in: open to the left (and closed to the right) or open to the right (and closed to the left).

[4]That is, we adopt the point of view of one-sided sequents in which the antecedent is empty, which is the usual perspective of linear logic; but one could equally adopt the point of view of one-sided sequents in which the succedent is empty, which is the usual point of view of refutation, in which case we would regard output polarity as negating.

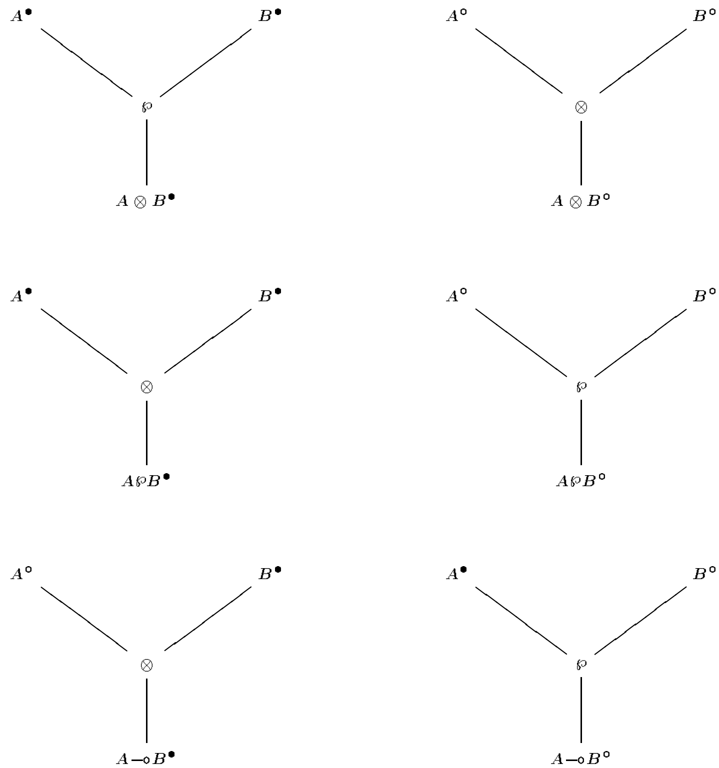

FIG. 3. Expanded logical links of classical linear logic

Entering an open premise, we always exit through the conclusion, but the other two cases depend on the connective. Entering the closed premise of $\otimes$ we exit through the other (open) premise, but entering the closed premise of ⅋ we *bounce*, returning immediately out of the same (closed) premise back the way we came. Entering the conclusion of $\otimes$ we go out through the closed premise, but entering the conclusion of ⅋ we go out through the open premise. Finally, when we arrive at a conclusion, we also bounce, returning immediately in the direction from which we just came.

A trip is a path through a proof structure according to a switching; note that once begun a trip extends deterministically. A trip is *long* if and only if it returns to its starting point having traversed each edge exactly once in each direction. A switching defines a long trip if and only if there is some long trip for the switching; in view of determinism and periodicity, a switching defines some long trip if and only if starting anywhere results in a long trip. A proof structure is correct, that is it is a *proof net*, if and only if every switching defines a long trip. A sequent $\Gamma \Rightarrow \Delta$ is a theorem of the sequent calculus iff there is a proof net with input conclusions Γ and output conclusions Δ.

The proof nets, like the sequent calculus, enjoy Cut-elimination: for every proof net there is an equivalent Cut-free proof net. This means that there is the following decision procedure for determining theoremhood via proof nets. Given a sequent

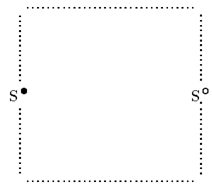

FIG. 4. Minimal circularity

$A_1, \ldots, A_n \Rightarrow B_1, \ldots, B_m$, construct the proof frame with conclusions ${A_1}^\bullet, \ldots, {A_n}^\bullet$, ${B_1}^\circ, \ldots, {B_m}^\circ$ comprising the sequence of signed formula trees given by the following recursive unfolding:

$$\frac{A^\bullet \quad B^\bullet}{A \otimes B^\bullet} \qquad \frac{A^\circ \quad B^\circ}{A \otimes B^\circ} \qquad \frac{A^\bullet \quad B^\bullet}{A \wp B^\bullet} \qquad \frac{A^\circ \quad B^\circ}{A \wp B^\circ} \tag{2.5}$$

$$\frac{A^\circ \quad B^\bullet}{A \multimap B^\bullet} \qquad \frac{A^\bullet \quad B^\circ}{A \multimap B^\circ} \qquad \frac{A^\circ}{A^{\perp\bullet}} \qquad \frac{A^\bullet}{A^{\perp\circ}}$$

Then test whether the long trip condition is satisfied for some Cut-free proof structure (there are a finite number) that can be built by putting axiom links on the proof frame.

Testing the long trip condition as it stands is not attractive computationally since in a proof structure with i $\wp$-links and j $\otimes$-links there are 2^{i+j} switchings to be tried. The situation is improved with the correctness criterion as formulated by Danos and Regnier (1989), which considers only switchings of $\wp$-links. For any given switching, a certain graph results by removing from an expanded proof net the edges between each $\wp$-conclusion and its closed premise. The result of Danos and Regnier is that a proof structure is a proof net if and only if for every switching of $\wp$-links, the result of removing these edges is acyclic and connected. A direct application of this simplified criterion requires only 2^i switchings to be tried.

The acyclicity part of the condition corresponds to the requirement of the binary rules Cut, and $\otimes$R, $\wp$L and $\multimap$L (i.e. those with $\otimes$ as the central node of expanded links) that their premises be in *different* subproofs, forbidding circularity such as that of figure 4. The connectedness condition corresponds to the requirement of the unary rules $\otimes$L, $\wp$R and $\multimap$R (i.e. those with $\wp$ as the central node of expanded links) that their premises be in the *same* subproofs.[5] The Mix rule (2.6) allows that different classical proofs can always be combined into one.

$$\frac{\Gamma_1 \Rightarrow \Delta_1 \qquad \Gamma_2 \Rightarrow \Delta_2}{\Gamma_1, \Gamma_2 \Rightarrow \Delta_1, \Delta_2}\text{Mix} \tag{2.6}$$

Mix has characteristic axiom $A \otimes B \Rightarrow A \wp B$. If we admit Mix, we drop the connectedness requirement from the correctness criterion, and require just acyclicity for every $\wp$-switching.

[5] These intuitions regarding acyclicity and binary rules and connectedness and unary rules are attributed by P. de Groote (p.c.) to J. Gallier.

That every $\wp$-switching is acyclic can be expressed in the following simple manner (adapted from Lecomte and Retoré 1995). Let us say that a *vicious circle* is a cyclic path which never crosses in immediate succession the two premises of a $\wp$-link; then every $\wp$-switching of a proof structure is acyclic iff the proof structure contains no vicious circle, i.e. if Mix is admitted:

$$\text{A proof structure is a proof net iff it contains no vicious circle.} \tag{2.7}$$

Testing whether a proof structure contains a vicious circle is of polynomial time complexity. Furthermore, the criterion can be employed incrementally: it is sufficient to check with the addition of each successive axiom link just that no vicious circle is created through this new link. However, although checking is polynomial, the search through alternative axiom linkings has no efficient solution for classical linear logic, for which the problem of validation is NP-complete (Lincoln, Mitchell, Scedrov and Shankar 1992).

2.3 Lambek-van Benthem calculus

Consider formulas defined as follows.

$$\mathcal{F} ::= \mathcal{A} \mid \mathcal{F} \otimes \mathcal{F} \mid \mathcal{F} \multimap \mathcal{F} \tag{2.8}$$

In the calculus (2.9) sequents are of the form $\Gamma \Rightarrow A$ where the antecedent configuration is a sequence of formulas as before, but the succedent comprises exactly one formula.

$$\begin{array}{ll}
\text{a.} & \dfrac{}{A \Rightarrow A}\text{id} \qquad \dfrac{\Gamma_1 \Rightarrow A \quad A, \Gamma_2 \Rightarrow B}{\Gamma_1, \Gamma_2 \Rightarrow B}\text{Cut} \\[2ex]
\text{b.} & \dfrac{\Gamma_1, A, B, \Gamma_2 \Rightarrow C}{\Gamma_1, B, A, \Gamma_2 \Rightarrow C}\text{P} \\[2ex]
\text{c.} & \dfrac{\Gamma, A, B \Rightarrow C}{\Gamma, A \otimes B \Rightarrow C}\otimes\text{L} \qquad \dfrac{\Gamma_1 \Rightarrow A \quad \Gamma_2 \Rightarrow B}{\Gamma_1, \Gamma_2 \Rightarrow A \otimes B}\otimes\text{R} \\[2ex]
\text{d.} & \dfrac{\Gamma_1 \Rightarrow A \quad B, \Gamma_2 \Rightarrow C}{\Gamma_1, A \multimap B, \Gamma_2 \Rightarrow C}\multimap\text{L} \qquad \dfrac{\Gamma, A \Rightarrow B}{\Gamma \Rightarrow A \multimap B}\multimap\text{R}
\end{array} \tag{2.9}$$

We recognize positive intuitionistic sequent rules for conjunction and implication; indeed, the only difference with respect to positive intuitionistic logic is that the structural rules of contraction and weakening are not included. This is the Lambek-van Benthem categorial calculus **LP**: a multiplicative fragment of intuitionistic linear logic; it enjoys Cut-elimination. Compared to classical linear logic, we see that there is now only one (left-sided) permutation rule, since there are never two formulas in the succedent to which a right permutation rule could apply. All the rules are instances of rules of the classical calculus, so every intuitionistic linear theorem is also a classical linear theorem; in fact, an intuitionistic sequent is an intuitionistic theorem if and only

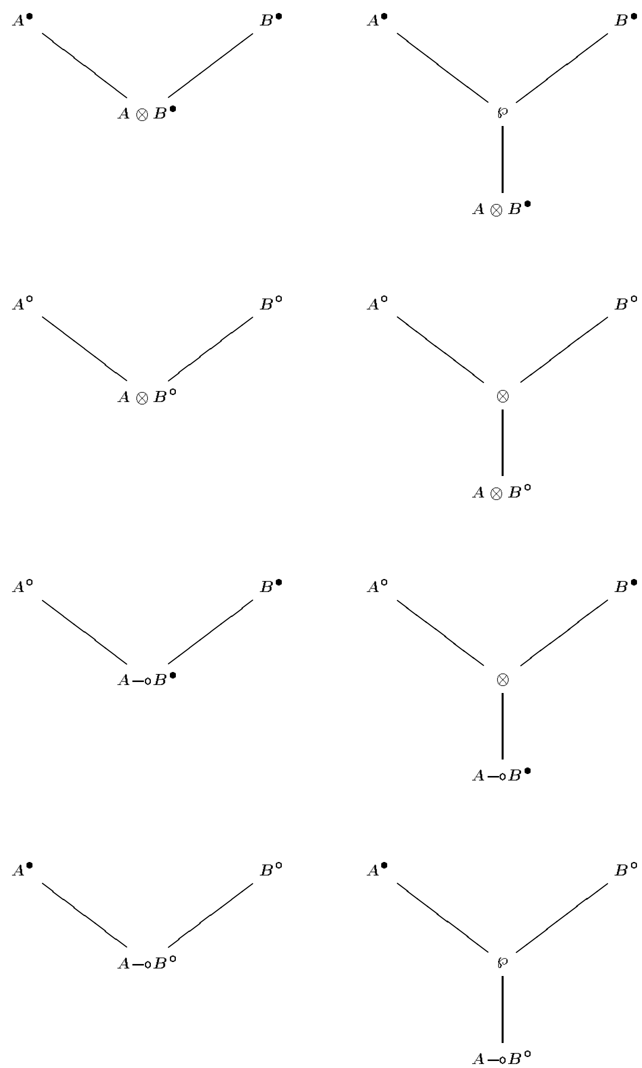

FIG. 5. Logical proof links of **LP** and their expansions

if it is a classical theorem (Johnson 1996). This means we can define intuitionistic proof nets as a special case of classical proof nets. We give the proof links in figure 5. An **LP** signed formula tree is a finite tree with atomic (signed) leaves each local tree of which is an **LP** logical link. An **LP** proof frame is a finite sequence of **LP** signed formula trees. An **LP** proof structure is obtained by connecting complementary leaves with axiom links and complementary roots with Cut links in such a way that each leaf is connected to exactly one other and each root is connected to at most one other, and which has exactly one conclusion of output polarity. A proof structure with input conclusions Γ and output conclusion A is read as asserting that $\Gamma \Rightarrow A$ is valid.

As a correctness criterion, it is sufficient just to check that there is no vicious circle, for the following reasons. In the classical system, axioms contain exactly one succedent formula, and if premise succedents contain at least one formula, then i) the conclusion succedent of Mix contains more than one formula, and ii) the left rules of permutation, times and linear implication each have the property that if some premise succedent contains more than one formula, then the conclusion succedent contains more than one formula. Consequently, every theorem of intuitionistic formulas proved using Mix contains more than one succedent formula, i.e. an intuitionistic sequent is an intuitionistic theorem if and only if it is a theorem of the classical system plus Mix. But this means we can forget about the connectedness requirement in the correctness condition. An **LP** proof structure is a proof net if and only if its expansion contains no vicious circle, and an **LP** sequent $\Gamma \Rightarrow A$ is a theorem of the sequent calculus iff there is a proof net with input conclusions Γ and output conclusion A.

The **LP** proof nets enjoy Cut-elimination, thus there is the following decision procedure for determining **LP** theoremhood by searching for Cut-free proof nets. Given a sequent $A_1, \ldots, A_n \Rightarrow A$ construct the proof frame with conclusions ${A_1}^\bullet, \ldots, {A_n}^\bullet, A^\circ$ comprising the sequence of signed formula trees given by the following recursive unfolding:

$$\frac{A^\bullet \quad B^\bullet}{A \otimes B^\bullet} \qquad \frac{A^\circ \quad B^\circ}{A \otimes B^\circ} \qquad \frac{A^\circ \quad B^\bullet}{A \multimap B^\bullet} \qquad \frac{A^\bullet \quad B^\circ}{A \multimap B^\circ} \tag{2.10}$$

Then test whether there is some proof structure that can be built by putting axiom links on the proof frame without creating any vicious circle.

Since **LP** is a restriction of intuitionistic logic, each proof can be read as an intuitionistic proof. The intuitionistic natural deduction proof, encoded as a linear term of λ-calculus with function and pair types, is extracted from a proof net as follows. First, one associates distinct variables with each output implication link and distinct constants with each input conclusion. Then, one starts travelling upwards at the unique output conclusion: going up into an output division (i.e. implication) link, λ-abstract over the associated variable the result of going up into the output premise; going up into an output product (i.e. conjunction) link, pair the result of going up into the premise for the first subformula with the result of going up into the premise for the second subformula; going up into one premise of an id link, go down into the other premise; going down into one conclusion of a Cut link, go up into the other conclusion; going down into an input division link, functionally apply the result of going down into its conclusion to the result of going up into the other premise; going down into the premise for the first subformula of an input product link, take the first projection of the result of going down into its conclusion; going down into the premise for the second subformula of an input product link, take the second projection of the result of going down into its conclusion; going down into an output division link, return the associated variable; and going down into an input conclusion, return the associated constant. This extraction procedure is the same for all categorial products and divisions, and we shall see examples in the context of linguistic application.

2.4 Lambek calculus

The (associative) Lambek calculus **L**, a multiplicative fragment of intuitionistic non-commutative linear logic, has the formulas and sequent calculus of (1.1) and (1.4). When we read $\bullet$ as $\otimes$ and both $A\backslash B$ and B/A as $A \multimap B$, each rule is seen to be an instance of an **LP** rule, so every theorem of **L** is also a theorem of **LP** when read in this way, and for a proof structure to be a proof net it is necessary that there be no vicious circle in the sense before. But this is no longer sufficient since in the absence of permutation, order must be taken into account.

Roorda (1991) addresses the ordering component in terms of a directional balance by specifying that in output logical links the subformulas of the conclusion appear with their left/right ordering switched in the premises. Then proof structures are required to be *planar*, and a (planar) proof structure is a proof net iff it satisfies the long trip condition in the usual manner. Here, however, we will be concerned with partial commutativity; it is not obvious how to systematically generalise the notion of planarity to combine commutative and non-commutative systems, and we consider instead an alternative correctness criterion based on unifiability (Morrill 1996). We will maintain the order switching of output unfolding, but do not require proof structures to be planar. Rather, our aim is for both planarity (for the non-commutative connectives), and satisfaction of the long trip condition, to be entailed by satisfaction of a *resolution criterion.*

In order to construe **L** in a manner uniform with subsequent extensions, consider the interpretation of **L** formulas that results from combining the algebraic and relational models. Interpretation takes place with respect to a semigroup $(L, +)$ and a set V. Formulas are interpreted as subsets of $L \times V \times V$. Given an interpretation $[\![P]\!]$ for each atom P, each category formula A receives an interpretation $[\![A]\!]$ thus:

$$\begin{array}{lcl} [\![A\backslash B]\!] & = & \{\langle s, v_2, v_3\rangle \mid \forall s', v_1, \langle s', v_1, v_2\rangle \in [\![A]\!] \to \langle s'{+}s, v_1, v_3\rangle \in [\![B]\!]\} \\ [\![B/A]\!] & = & \{\langle s, v_1, v_2\rangle \mid \forall s', v_3, \langle s', v_2, v_3\rangle \in [\![A]\!] \to \langle s{+}s', v_1, v_3\rangle \in [\![B]\!]\} \\ [\![A\bullet B]\!] & = & \{\langle s_1{+}s_2, v_1, v_3\rangle \mid \exists v_2, \langle s_1, v_1, v_2\rangle \in [\![A]\!] \;\&\; \langle s_2, v_2, v_3\rangle \in [\![B]\!]\} \end{array} \tag{2.11}$$

The expansion of proof links will reflect the binary relational quantificational structure. Each node labelled by a formula will have two incident dashed edges referred to as its start and its end parameter edges. For an input formula the start comes on the left and the end comes on the right; for an output formula this is reversed:

$$\text{start } A^\bullet \text{ end} \qquad \text{end } A^\circ \text{ start} \tag{2.12}$$

These parameter edges are connected to quantifiers in the expanded proof structures which bind the parameters of formulas regarded as binary predicates. The proof links of **L** are given in figures 6 and 7. Just as before, an **L** signed formula tree is a finite tree with atomic (signed) leaves each local tree of which is an **L** logical link. An **L** proof frame is a finite sequence of **L** signed formula trees and an **L** proof structure is the result of connecting complementary leaves with axiom links and complementary roots with Cut links in such a way that each leaf is connected to exactly one other, each root is connected to at most one other, and there is exactly one unconnected root of output polarity. An *expanded* proof structure has the annotation of figure 8 on its conclusions ${A_1}^\bullet, \ldots, {A_n}^\bullet, A^\circ$ in the proof frame. This corresponds to the meaning of a sequent $A_1, \ldots, A_n \Rightarrow A$ with respect to binary relational interpretation: for all $v_0, \ldots, v_n \in V$, if $\langle v_{i-1}, v_i\rangle \in [\![A_i]\!]$, $1{\leq}i{\leq}n$ then $\langle v_0, v_n\rangle \in [\![A]\!]$.

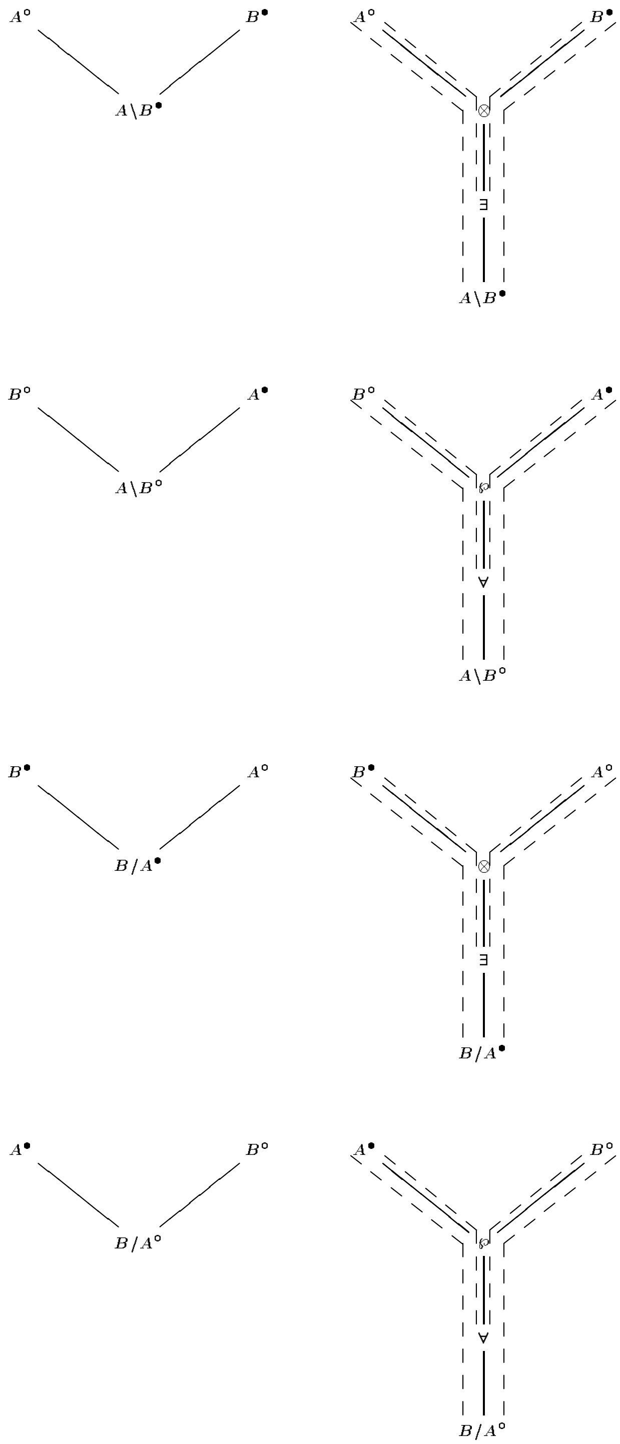

FIG. 6. Proof links of **L** and their expansions, I

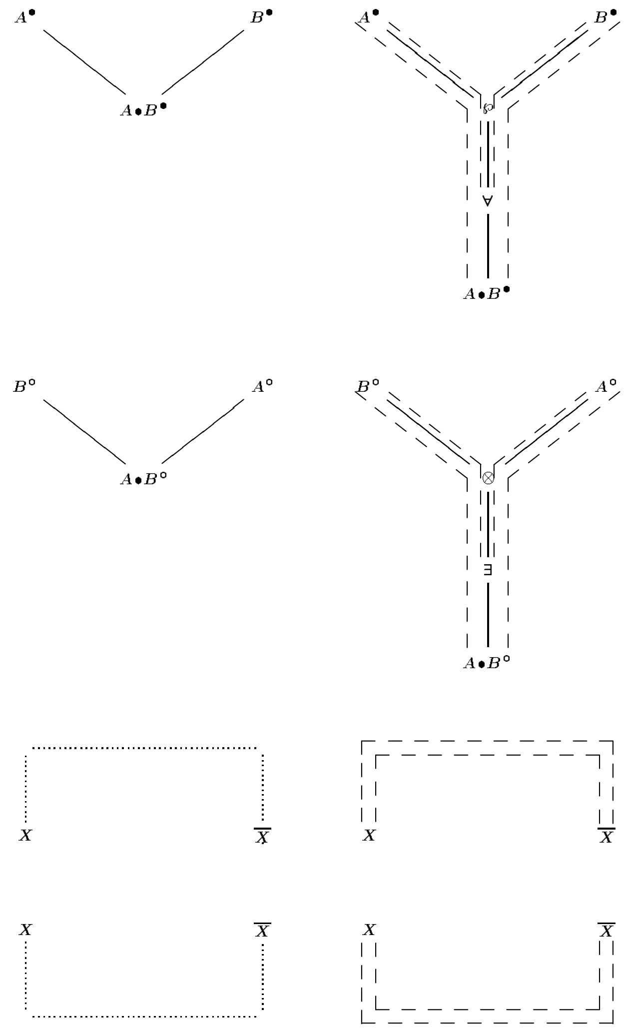

FIG. 7. Proof links of **L** and their expansions, II

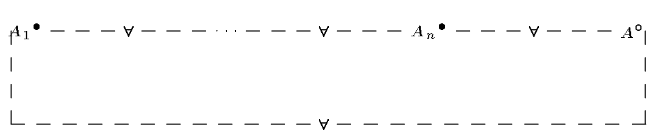

FIG. 8. Parameter expansion of conclusions

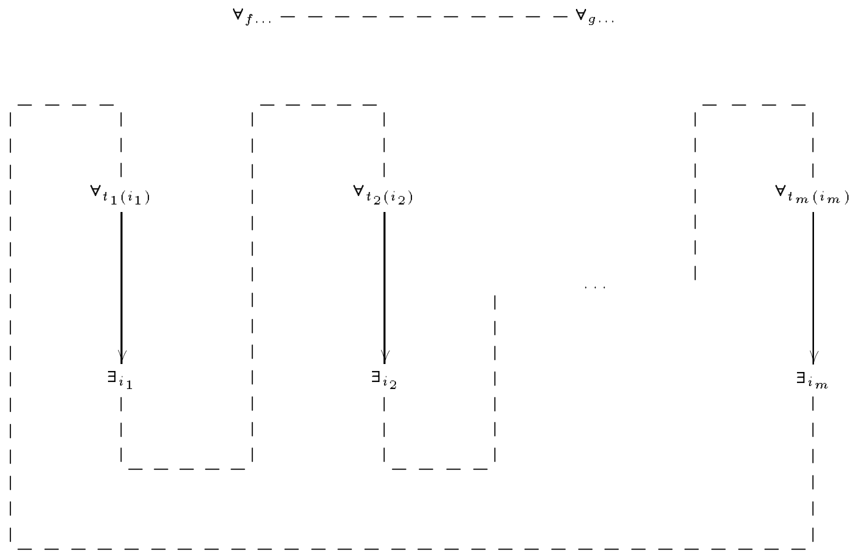

FIG. 9. Clash check and occurrence check violations

The complementary atoms linked by axioms in proof nets can be seen as the counterparts of the complementary pairs in a (non-clausal) resolution proof. This gives rise to the following correctness criterion on parameter paths in proof structures. First, to each existential quantifier we associate a new free variable, and to each universal quantifier we associate a Skolem term; note that polarities are the opposite of what is usual since resolution proofs are refutations, i.e. negate succedent formulas, whereas proof nets negate antecedent formulas. A Skolem term is a new constant in the case that the universal quantifier is not dominated by any existential; otherwise it comprises a new n-place function symbol with arguments the n variables of the n dominating existentials. Each axiom link requires the start and end parameters of its two atoms to be unified, and for a proof structure to be correct as a whole, the unification problem defined by its axiom linkings must be solvable.

We can show that the quantificational structure of a proof net is correct by exhibiting a unifier, but we do not need to insist on such a constructive proof of unifiability: the criterion only requires than such a unifier exists. Unification fails in two cases, *clash*: if we attempt to match a constant to a different constant, or to match a structured term to a structured term with a different function symbol, or to a constant, or *occurrence*: if we attempt to match a variable to a structured term containing this variable. Let us define a $\forall\exists$-cycle as a cyclic path alternating between universals and dominating existentials as shown in figure 9 (the directionality, shown explicitly, is from premise to conclusion); thus we can test correctness of expanded proof structures by the following purely graph-theoretic *resolution criterion*:

$$\text{No two distinct universals are connected by parameter edges (clash check) and there is no } \forall\exists\text{-cycle (occurrence check).} \tag{2.13}$$

The idea is that the clash check and occurrence check together take the place of planarity and acyclicity requirements (in particular the notion of ∀∃-cycle is highly similar to that of AE-cycle in Lecomte and Retoré 1995, though the rational is entirely different) so that we can show that a proof structure is incorrect by identifying either a clash check violation or an occurrence check violation.

That the resolution criterion is *necessary* is immediate if for a proof structure to be correct, it must be correct as a non-clausal resolution proof of classical logic. The question arises as to whether the resolution criterion is also *sufficient*. If it is not one must do more to show correctness than just assure solvability of the unification problem defined by a proof structure, but we continue on the assumption that a proof structure is a proof net iff it satisfies the resolution criterion.

Given Cut-elimination for **L** proof nets, there is the following algorithm for deciding the validity of an **L** sequent $A_1, \ldots, A_n \Rightarrow A$. Construct the proof frame with conclusions ${A_1}^\bullet, \ldots, {A_n}^\bullet, A^\circ$ comprising the sequence of signed formula trees given by the following recursive unfolding:

$$\frac{A^\circ \quad B^\bullet}{A\backslash B^\bullet} \qquad \frac{B^\circ \quad A^\bullet}{A\backslash B^\circ} \qquad \frac{B^\bullet \quad A^\circ}{B/A^\bullet} \qquad \frac{A^\bullet \quad B^\circ}{B/A^\circ} \tag{2.14}$$

$$\frac{A^\bullet \quad B^\bullet}{A{\bullet}B^\bullet} \qquad \frac{B^\circ \quad A^\circ}{A{\bullet}B^\circ}$$

Then test whether some proof structure can be built be adding axiom links which complies with the resolution criterion.

In figure 10 we give an expanded proof net for the valid sequent $\mathrm{N} \Rightarrow \mathrm{S}/(\mathrm{N}\backslash\mathrm{S})$, a lifting theorem. It defines the unification problem $\{0 = i, 1 = 1, i = 0, 2 = 2\}$ which has solution $\{0/i\}$. In figure 11 we give an expanded proof structure for the invalid lowering sequent $\mathrm{S}/(\mathrm{N}\backslash\mathrm{S}) \Rightarrow \mathrm{N}$; there is a clash check violation on the outer parameter edges. Figure 12 shows a partial proof structure for the invalid sequent $\Rightarrow (\mathrm{S}\backslash(\mathrm{N}\backslash\mathrm{N})){\bullet}\mathrm{S}$, in which the only parameter edge explicitly marked participates in a ∀∃-cycle completed by the two directed edges.

A categorial derivation defines a semantic construction, expressed by the typed λ-term extracted as for **LP** proofs, giving the semantics of the expression derived in terms of the semantics of its lexical signs. In the lifting example of figure 10, the semantic traversal yields the term $\lambda x(x\ a)$ where a is the semantics associated with the $\mathrm{N}^\bullet$ conclusion.

A categorial derivation also defines a prosodic construction giving the word order of the composite expression in terms of its lexical expressions. This is recovered from the parameter edges reflecting relational interpretation thus: begin travelling up at the start parameter of the unique output conclusion; this arrives at the start parameter of the first lexical expression making up the composite; continue travelling up at the end parameter of this input conclusion; this arrives at the start parameter of the second lexical expression making up the composite; continue travelling up at the end parameter of this input conclusion, and so on; the process ends by returning to the end parameter of the unique output conclusion. In the lifting example of figure 10, the prosodic traversal begins at the start parameter of the output conclusion and follows the right outermost parameter edge round to the existential and the left outermost parameter edge round to the start parameter of $\mathrm{N}^\bullet$; travelling up at the end parameter

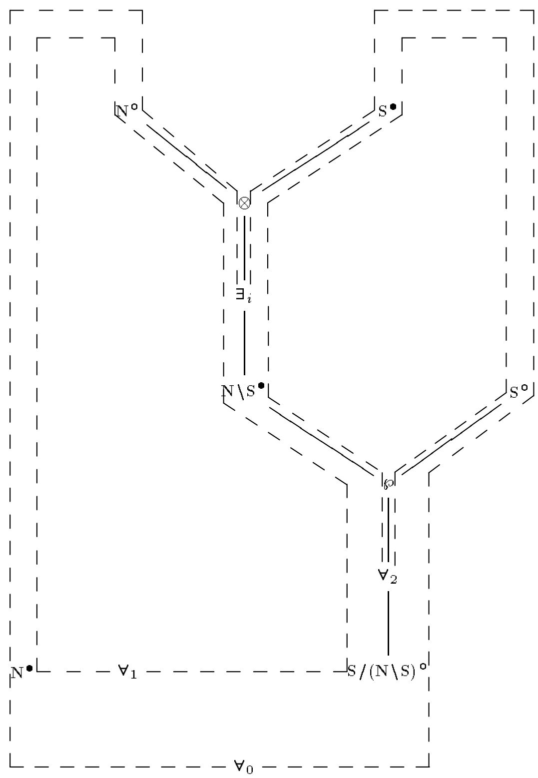

FIG. 10. Proof net for lifting

of $N^\bullet$ we return down to the end parameter of the output conclusion. In fact we write proof nets on the page in such a way that in general this traversal visits the input conclusions in left-to-right order in the case of continuity; but in the case of discontinuity input conclusions will be revisited.

2.5 Product unit

In the combined models the product unit is interpreted thus:

$$[\![I]\!] \quad = \quad \{\langle \varepsilon, v_1, v_2\rangle \mid v_1 = v_2\} \tag{2.15}$$

In proof structures I nodes, as nullary connectives, are left untouched, not connected by axiom links; in the expanded proof links for I in figure 13 the start and end parameters are connected (identified). In general this means that there may be more than one quantifier on parameter edges in proof frames. When this happens, it is the *outermost* quantifier which is relevant to axiom linking regulation; inner quantifications are inert because their restriction is identity with the outer one. Thus term labels for unification are supplied to outermost quantifiers on the parameter paths of a proof frame. Semantically the product unit is interpreted by a singleton $\{1\}$.

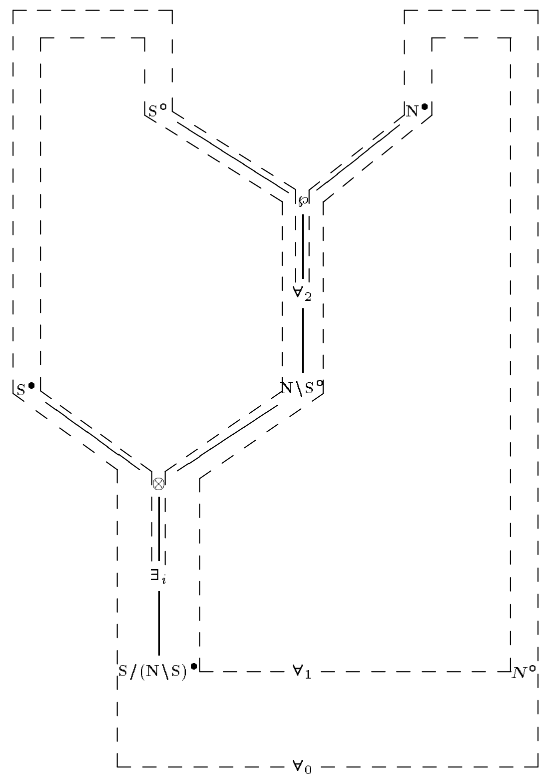

FIG. 11. Proof structure for lowering, with clash check violation

2.6 Medial divisor

In the combined models the medial divisor $\uparrow_e$ is interpreted:

$$[\![B\uparrow_e A]\!] = \{\langle s_1{+}s_2, v_1, v_2\rangle |\ \exists v \forall s, \langle s, v, v\rangle \in [\![A]\!] \rightarrow \langle s_1{+}s{+}s_2, v_1, v_2\rangle \in [\![B]\!]\} \quad (2.16)$$

Proof links for the medial divisor are shown in figure 14. Observe that the expansion of the output link is a systematic reflection of the propositional and relational quantificational structure of the interpretation, and that in the input link we find the de Morgan dual. For reasons of uniformity we continue the convention of switching the order of subformulas in output links, but the medial divisor will have non-planar proof nets.

In figure 15 we give the expanded proof net (abbreviating N\S to VP) for the medial

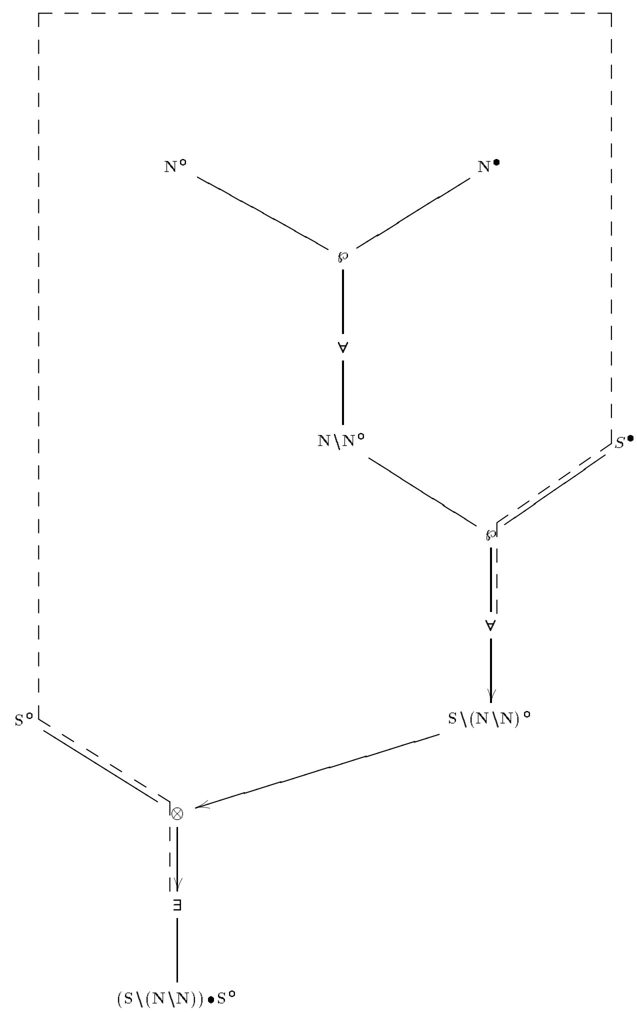

Fig. 12. Partial proof structure with ∀∃-cycle

Fig. 13. Expanded proof links for the product unit

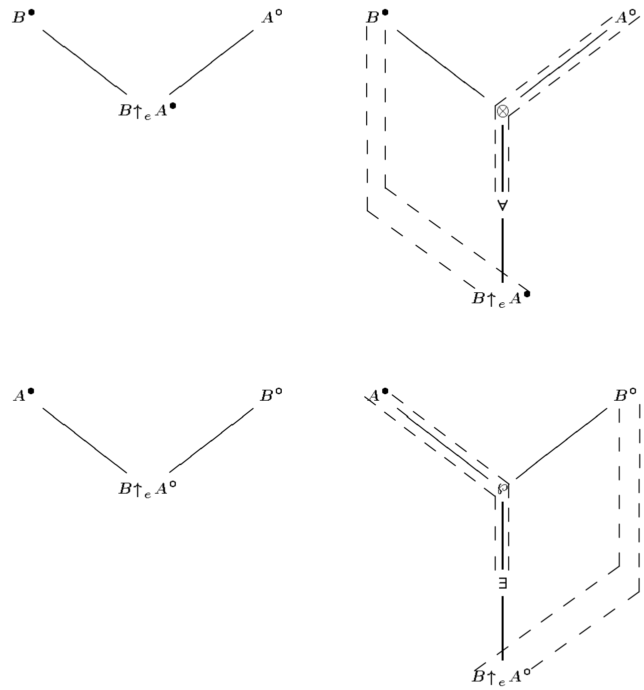

FIG. 14. Proof links for the medial divisor and their expansions

extraction (1.7) on the basis of the following assignments:

that	–	*intersect*	(2.17)
	:	$\mathrm{R}/(\mathrm{S}{\uparrow_e}\mathrm{N})$	
John	–	j	
	:	N	
gave	–	*give*	
	:	$((\mathrm{N}\backslash\mathrm{S})/\mathrm{PP})/\mathrm{N}$	
to+Mary	–	m	
	:	PP	

The unification problem defined (omitting repetitions) is $\{0 = 0, i = 4, j = 3, j = k, i = l, 1 = m, 2 = 2, l = 4, k = 3\}$ which has solution $\{4/i, 3/j, 3/k, 4/l, 1/m\}$.

The edges of successive prosodic traversal are labelled $0, 1, 2, 3, 4$: beginning travelling up at the start of the unique output conclusion, five 0-lines lead to the start of the type for 'that', which is the first word; going up at the end of this type, twelve 1-lines lead to the start of the type for 'John', and so on, yielding in order the words 'gave' and 'to Mary'; hence the prosodic form of the sign is **that+John+gave+to+Mary**.

Arrows mark the directions of semantic traversal; starting with the axiom link going from the outermost right to the outermost left, successive stages of semantic

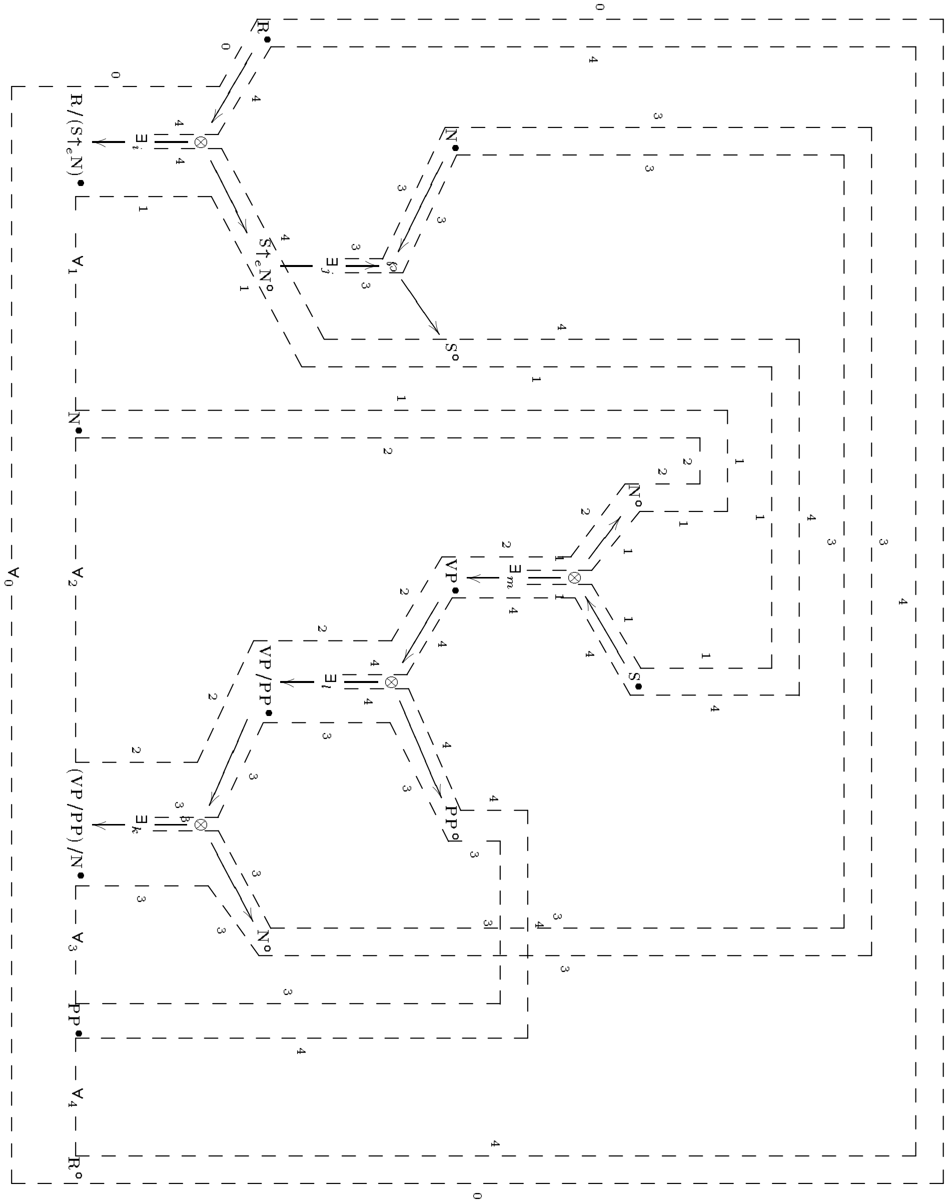

Fig. 15. Proof net for '(the dog) that John gave to Mary' via the medial divisor

form extraction are as follows:

$$\begin{array}{l}
(\cdot\ \cdot) \\
(intersect\ \cdot) \\
(intersect\ \lambda x_j \cdot) \\
(intersect\ \lambda x_j(\cdot\ \cdot)) \\
(intersect\ \lambda x_j(\cdot\ \ \cdot\ \ j)) \\
(intersect\ \lambda x_j(\cdot\ \ \cdot\ \ m\ j)) \\
(intersect\ \lambda x_j(give\ x_j\ \ m\ j))
\end{array} \tag{2.18}$$

Hence the semantic form of the sign is $(intersect\ \lambda x_j(give\ x_j\ \ m\ j))$.

In figure 16 we give the expanded proof net (abbreviating (N\S)/VP to XVP) for the obligatory extraction (1.10a), assuming (additional) type assignments:

$$\begin{array}{lll}
\textbf{assures} & - & assure \\
 & : & (((N\backslash S)/VP)\uparrow_e N)/N \\
\textbf{Mary} & - & m \\
 & : & N \\
\textbf{to+be+reliable} & - & reliable \\
 & : & VP
\end{array} \tag{2.19}$$

The unification problem defined (omitting repetitions and equations of identical terms) is $\{i = 5, j = 6(k), i = l, 1 = m, l = 5, k = 4\}$ which has solution $\{5/i, 5/l, 4/k, 6(4)/j, 1/m\}$. The sign generated has prosodic form **that+John+-assures+Mary+to+be+reliable** and the semantic form extracted is $(intersect$ - $\lambda x_j(assure\ m\ x_j\ \ reliable\ j)$.

A partial proof structure for the ungrammatical (1.10b) is given in figure 17; the only parameter edge explicitly marked mediates a clash between two universals.

2.7 In situ bindor

In the combined models, the in situ bindor Q is interpreted:

$$\begin{array}{rcl}
[\![Q(A,B,C)]\!] & = & \{\langle s, v_2, v_3\rangle |\ \forall s_1, s_3, v_1, v_4, \\
 & & [\forall s_2, \langle s_2, v_2, v_3\rangle \in [\![A]\!] \rightarrow \langle s_1{+}s_2{+}s_3, v_1, v_4\rangle \in [\![B]\!]] \rightarrow \\
 & & \langle s_1{+}s{+}s_3, v_1, v_4\rangle \in [\![C]\!]\}
\end{array} \tag{2.20}$$

The proof links for the in situ bindor are shown in figure 18. Again, the expansions are a systematic reflection of the interpretation, and for uniformity orderings of polar opposites are mirror images.

In figure 19 we give the (expanded) proof net (abbreviating (N\S)/N to TV) for the in situ binding (1.11a) assuming type assignments (2.21).

$$\begin{array}{lll}
\textbf{bought} & - & buy \\
 & : & ((N\backslash S)/N)/N \\
\textbf{someone} & - & \lambda x \exists y[(person\ y) \wedge (x\ y)] \\
 & : & Q(S, N, S) \\
\textbf{Fido} & - & f \\
 & : & N
\end{array} \tag{2.21}$$

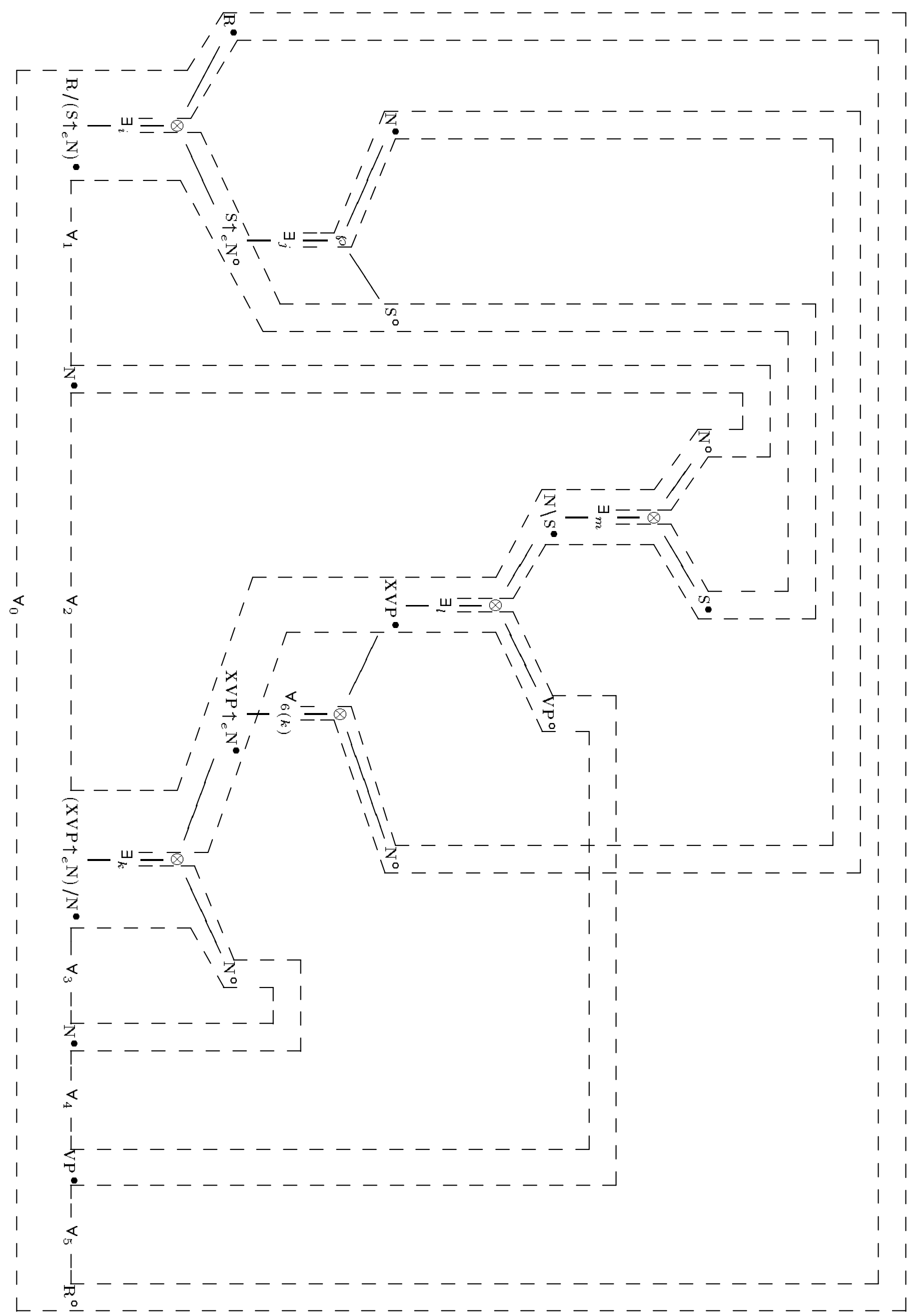

Fig. 16: Proof net for '(the man) that John assured Mary to be reliable' via the medial divisor

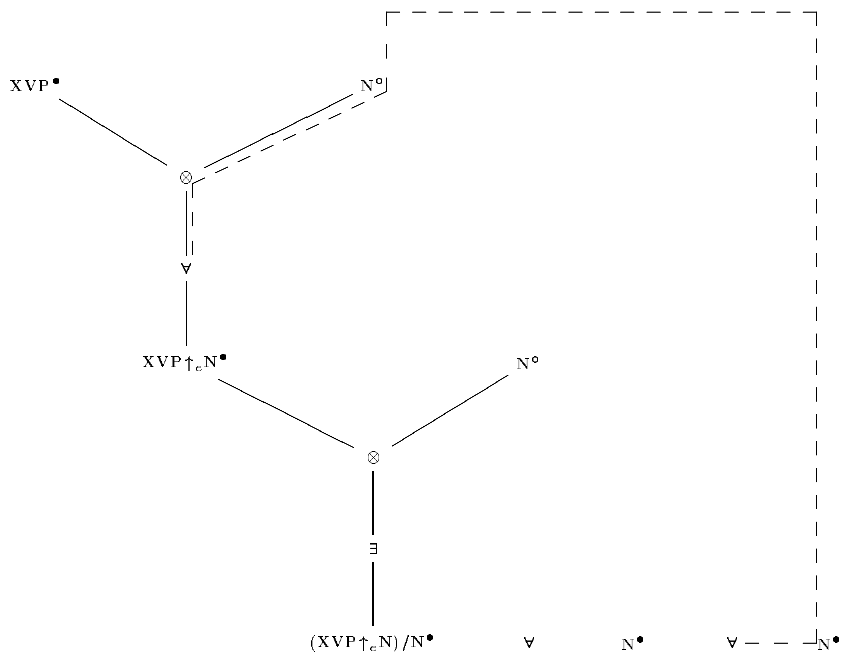

FIG. 17: Partial proof structure for 'John assured Mary Bill to be reliable' via the medial divisor, with clash check violation

The unification problem defined by the linking is $\{0 = k, k = m, j = l, j = 4, i = 3, m = 0, l = 4\}$ which has solution $\{0/k, 0/m, 4/l, 4/j, 3/i\}$. The result of semantic traversal is $(someone\ \lambda x(buy\ x\ f\ j))$ which on substitution of lexical semantics simplifies to $\exists y[(person\ y) \wedge (buy\ y\ f\ j)]$.

The reader may check the proof net constructions showing that the assignment (2.22) yields the semantics $(buy\ j\ f\ j)$ for (1.11b), and showing (1.14).

$$\begin{array}{lll} \textbf{himself} & - & \lambda x \lambda y(x\ y\ y) \\ & : & Q(\mathrm{N\backslash S, N, N\backslash S}) \end{array} \qquad (2.22)$$

3 Sorted discontinuity calculus

Based on considerations in Morrill and Solias (1993), Morrill (1994, chs. 4–5; 1995) presents an (unsorted) discontinuity calculus and Morrill (1995, app.) and Morrill and Merenciano (1996) a sorted discontinuity calculus. The former has a pure sequent calculus, but does not fully solve the problems alluded to in our introduction. The latter has a *labelled* sequent calculus, and can solve these problems, treating $\uparrow_e$ and Q as defined operators. In a labelled sequent calculus a wider class of sequents is generated by rules for formulas which is then filtered by conditions on labels. However, it would be even more satisfactory to have a one-stage characterisation in the spirit of pure sequent calculus.

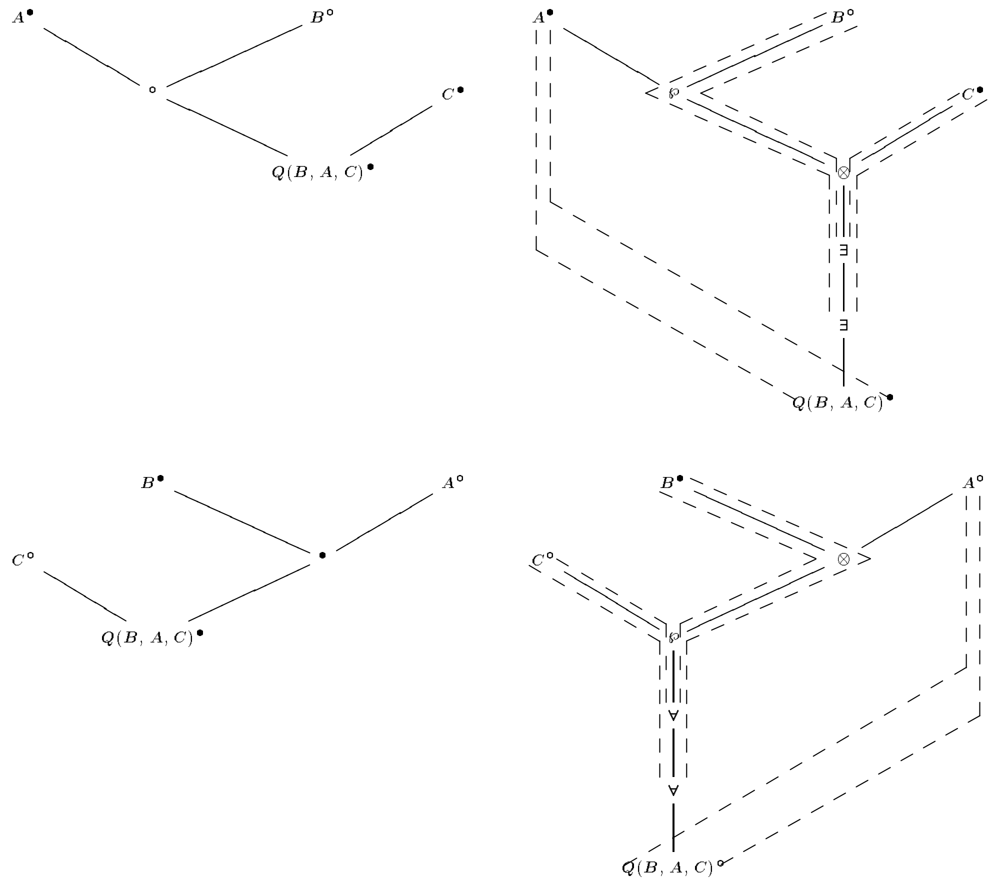

FIG. 18. Proof links for the in situ bindor and their expansions

In this section we provide such a pure sequent calculus for sorted discontinuity and show how the issues raised in our introduction are resolved. We show in the next section how to give proof nets for the full sorted discontinuity calculus.

In the sorted discontinuity calculus, category formulas fall into two sorts: those $\mathcal{F}$ of sort string, interpreted algebraically as subsets of L (and relationally as binary relations) , and those $\mathcal{F}^2$ of sort split string, interpreted algebraically as subsets of L^2 (and relationally as quaternary relations). Our definition (1.5) of category formulas becomes (3.1).

$$\begin{array}{lll} \mathcal{F} & ::= & \mathcal{A} \mid \mathcal{F}\bullet\mathcal{F} \mid \mathcal{F}\backslash\mathcal{F} \mid \mathcal{F}/\mathcal{F} \mid I \mid \mathcal{F}^2\odot\mathcal{F} \mid \mathcal{F}^2\downarrow\mathcal{F} \\ \mathcal{F}^2 & ::= & \mathcal{F}\uparrow\mathcal{F} \end{array} \tag{3.1}$$

The discontinuous product operator $\odot$ and the divisors $\downarrow$ ("infix") and $\uparrow$ ("extract") are interpreted by "residuation" with respect to an interpolation adjunction W of functionality $L^2, L \rightarrow L$, defined by $\langle s_1, s_2\rangle Ws = s_1{+}s{+}s_2$, in exactly the same way that the continuity operators are interpreted by residuation with respect to a

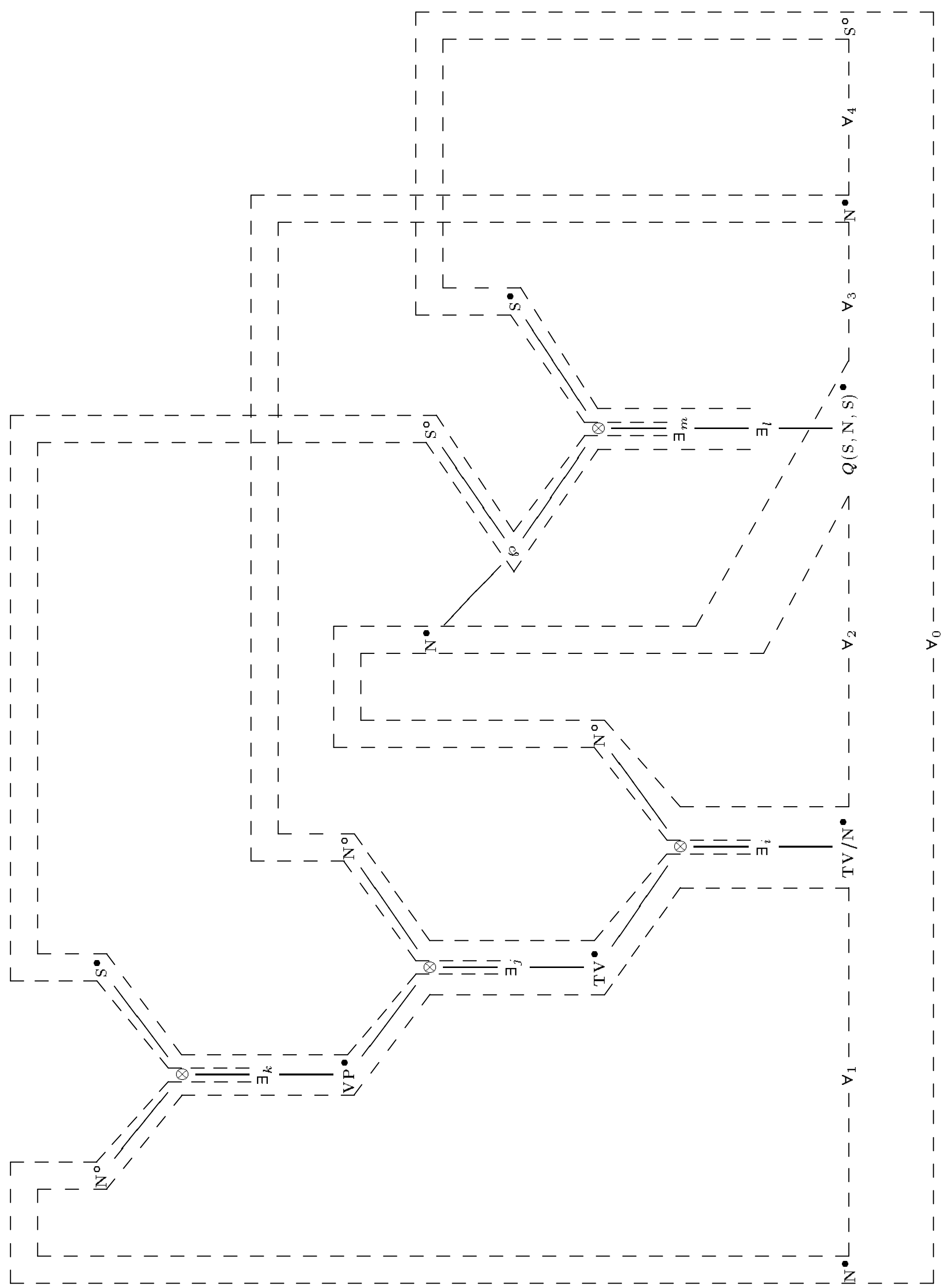

FIG. 19. Proof net for 'John bought someone Fido' via the in situ bindor

concatenation adjunction $+$ of functionality $L, L \to L$. In the combined models we have the following:

$$\begin{array}{lcl} [\![A{\downarrow}B]\!] & = & \{\langle s, v_2, v_3\rangle \mid \forall s_1, s_2, v_1, v_4, \langle s_1, s_2, v_1, v_2, v_3, v_4\rangle \in [\![A]\!] \to \\ & & \qquad\qquad\qquad \langle s_1{+}s{+}s_2, v_1, v_4\rangle \in [\![B]\!]\} \\ [\![B{\uparrow}A]\!] & = & \{\langle s_1, s_2, v_1, v_2, v_3, v_4\rangle \mid \langle s, v_2, v_3\rangle \in [\![A]\!] \to \\ & & \qquad\qquad\qquad \langle s_1{+}s{+}s_2, v_1, v_4\rangle \in [\![B]\!]\} \\ [\![A{\odot}B]\!] & = & \{\langle s_1{+}s{+}s_2, v_1, v_4\rangle \mid \exists v_2, v_3, \langle s_1, s_2, v_1, v_2, v_3, v_4\rangle \in [\![A]\!] \ \& \\ & & \qquad\qquad\qquad \langle s, v_2, v_3\rangle \in [\![B]\!]\} \end{array} \tag{3.2}$$

We have, then, $B{\uparrow}_e A = (B{\uparrow}A){\odot}I$[6] and $Q(B, A, C) = (B{\uparrow}A){\downarrow}C$.

We have already noted that giving sequent rules for categories of the variety $B{\uparrow}A$ is problematic: a category occurrence $B{\uparrow}A$ in an antecedent would fail to indicate where one is meant to interpolate. Our analysis is that in the sequent calculus of **L** a category occurrence signals two things: a resource, and the location of that resource with respect to others. This double service can be maintained in view of the continuity of concatenation, but discontinuity requires a distinction between signaling a resource, and its locations of action, which may be multiple. In particular, $B{\uparrow}A$ has two discontinuous components. Our solution is for a split string category formula to appear *twice* in a sequent, at its two loci of action. To mark that the two components are to be taken together as a resource, the occurrences are punctuated as roots, $\sqrt{\ }$.

Sequents come in two kinds, those Σ with sort string succedents, which have string antecedent configurations $\mathcal{O}$, and those Σ^2 with sort split string succedents, which have split string antecedent configurations $\mathcal{O}^2$:

$$\begin{array}{lcl} \Sigma & ::= & \mathcal{O} \Rightarrow \mathcal{F} \\ \Sigma^2 & ::= & \mathcal{O}^2 \Rightarrow \sqrt{\mathcal{F}^2} \\ \mathcal{O} & ::= & \Lambda \mid \mathcal{F}, \mathcal{O} \mid \sqrt[1]{\mathcal{F}^2}, \mathcal{O}, \sqrt[2]{\mathcal{F}^2} \\ \mathcal{O}^2 & ::= & \mathcal{O}, \sqrt{\mathcal{F}^2}, \mathcal{O} \mid \mathcal{O}, \sqrt[1]{\mathcal{F}^2}, \mathcal{O}^2, \sqrt[2]{\mathcal{F}^2}, \mathcal{O} \end{array} \tag{3.3}$$

Observe that configurations have balanced occurrences of parenthesising punctuation $\sqrt[1]{\ }$ and $\sqrt[2]{\ }$. These mark the two components of split antecedent categories. In a sequent with a split succedent category there is a $\sqrt{\ }$ in the antecedent marking the split point, and around which the parenthesising is balanced. The sequent rules are thus:

$$\text{a.}\quad \frac{\Gamma(\sqrt{A}) \Rightarrow \sqrt{A} \qquad \Delta(B) \Rightarrow C}{\Delta(\Gamma(A{\downarrow}B)) \Rightarrow C}{\downarrow}\mathrm{L} \qquad \frac{\sqrt[1]{A}, \Gamma, \sqrt[2]{A} \Rightarrow B}{\Gamma \Rightarrow A{\downarrow}B}{\downarrow}\mathrm{R} \tag{3.4}$$

$$\text{b.}\quad \frac{\Gamma \Rightarrow A \qquad \Delta(B) \Rightarrow C}{\Delta(\sqrt[1]{B{\uparrow}A}, \Gamma, \sqrt[2]{B{\uparrow}A}) \Rightarrow C}{\uparrow}\mathrm{L} \qquad \frac{\Gamma(A) \Rightarrow B}{\Gamma(\sqrt{B{\uparrow}A}) \Rightarrow \sqrt{B{\uparrow}A}}{\uparrow}\mathrm{R}$$

$$\text{c.}\quad \frac{\Gamma(\sqrt[1]{A}, B, \sqrt[2]{A}) \Rightarrow C}{\Gamma(A{\odot}B) \Rightarrow C}{\odot}\mathrm{L} \qquad \frac{\Gamma(\sqrt{A}) \Rightarrow \sqrt{A} \qquad \Delta \Rightarrow B}{\Gamma(\Delta) \Rightarrow A{\odot}B}{\odot}\mathrm{R}$$

[6] The semantic types are not quite identical, but there is a 1–1 correspondence between elements of D and elements of $D \times \{1\}$ (and $\{1\} \to D$).

By way of example, we assume the wrapping functor assignment (3.5) for the discontinuous idiom 'gave ... the cold shoulder'.

$$(\textbf{gave}, \textbf{the+cold+shoulder}) \quad - \quad \mathit{shun} \quad : \quad (\mathrm{N}\backslash \mathrm{S})\uparrow \mathrm{N} \tag{3.5}$$

Then 'John gave Mary the cold shoulder' is derived as follows:

$$\dfrac{\mathrm{N} \Rightarrow \mathrm{N} \qquad \dfrac{\mathrm{N} \Rightarrow \mathrm{N} \qquad \mathrm{S} \Rightarrow \mathrm{S}}{\mathrm{N}, \mathrm{N}\backslash \mathrm{S} \Rightarrow \mathrm{S}}\backslash L}{\mathrm{N}, \sqrt[1]{(\mathrm{N}\backslash \mathrm{S})\uparrow \mathrm{N}}, \mathrm{N}, \sqrt[2]{(\mathrm{N}\backslash \mathrm{S})\uparrow \mathrm{N}} \Rightarrow \mathrm{S}}\uparrow L \tag{3.6}$$

The medial relativisation (1.7) is treated as follows:

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathrm{N} \Rightarrow \mathrm{N} \qquad \dfrac{\mathrm{PP} \Rightarrow \mathrm{PP} \qquad \dfrac{\mathrm{N} \Rightarrow \mathrm{N} \qquad \mathrm{S} \Rightarrow \mathrm{S}}{\mathrm{N}, \mathrm{N}\backslash \mathrm{S} \Rightarrow \mathrm{S}}\backslash L}{\mathrm{N}, (\mathrm{N}\backslash \mathrm{S})/\mathrm{PP}, \mathrm{PP} \Rightarrow \mathrm{S}}/L}{\mathrm{N}, ((\mathrm{N}\backslash \mathrm{S})/\mathrm{PP})/\mathrm{N}, \mathrm{N}, \mathrm{PP} \Rightarrow \mathrm{S}}/L}{\mathrm{N}, ((\mathrm{N}\backslash \mathrm{S})/\mathrm{PP})/\mathrm{N}, \sqrt{\mathrm{S}\uparrow \mathrm{N}}, \mathrm{PP} \Rightarrow \sqrt{\mathrm{S}\uparrow \mathrm{N}}}\uparrow R \qquad \Rightarrow I}{\mathrm{N}, ((\mathrm{N}\backslash \mathrm{S})/\mathrm{PP})/\mathrm{N}, \mathrm{PP} \Rightarrow (\mathrm{S}\uparrow \mathrm{N})\odot I}\odot R \qquad \mathrm{R} \Rightarrow \mathrm{R}}{\mathrm{R}/((\mathrm{S}\uparrow \mathrm{N})\odot I), \mathrm{N}, ((\mathrm{N}\backslash \mathrm{S})/\mathrm{PP})/\mathrm{N}, \mathrm{PP} \Rightarrow \mathrm{R}}/R \tag{3.7}$$

The obligatory extraction (1.10a) receives the sequent derivation (3.8).

$$\dfrac{\dfrac{\mathrm{N} \Rightarrow \mathrm{N} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\mathrm{N} \Rightarrow \mathrm{N} \qquad \mathrm{N}, (\mathrm{N}\backslash \mathrm{S})/\mathrm{VP}, \mathrm{VP} \Rightarrow \mathrm{S}}{\mathrm{N}, \sqrt[1]{((\mathrm{N}\backslash \mathrm{S})/\mathrm{VP})\uparrow \mathrm{N}}, \mathrm{N}, \sqrt[2]{((\mathrm{N}\backslash \mathrm{S})/\mathrm{VP})\uparrow \mathrm{N}}, \mathrm{VP} \Rightarrow \mathrm{S}}\uparrow L}{\mathrm{N}, \sqrt[1]{((\mathrm{N}\backslash \mathrm{S})/\mathrm{VP})\uparrow \mathrm{N}}, \sqrt{\mathrm{S}\uparrow \mathrm{N}}, \sqrt[2]{((\mathrm{N}\backslash \mathrm{S})/\mathrm{VP})\uparrow \mathrm{N}}, \mathrm{VP} \Rightarrow \sqrt{\mathrm{S}\uparrow \mathrm{N}}}\uparrow R \qquad I \Rightarrow I}{\mathrm{N}, \sqrt[1]{((\mathrm{N}\backslash \mathrm{S})/\mathrm{VP})\uparrow \mathrm{N}}, I, \sqrt[2]{((\mathrm{N}\backslash \mathrm{S})/\mathrm{VP})\uparrow \mathrm{N}}, \mathrm{VP} \Rightarrow (\mathrm{S}\uparrow \mathrm{N})\odot I}\odot R}{\mathrm{N}, (((\mathrm{N}\backslash \mathrm{S})/\mathrm{VP})\uparrow \mathrm{N})\odot I, \mathrm{VP} \Rightarrow (\mathrm{S}\uparrow \mathrm{N})\odot I}\odot L}{\mathrm{N}, ((((\mathrm{N}\backslash \mathrm{S})/\mathrm{VP})\uparrow \mathrm{N})\odot I)/\mathrm{N}, \mathrm{N}, \mathrm{VP} \Rightarrow (\mathrm{S}\uparrow \mathrm{N})\odot I}/L \qquad \mathrm{R} \Rightarrow \mathrm{R}}{\mathrm{R}/((\mathrm{S}\uparrow \mathrm{N})\odot I), \mathrm{N}, ((((\mathrm{N}\backslash \mathrm{S})/\mathrm{VP})\uparrow \mathrm{N})\odot I)/\mathrm{N}, \mathrm{N}, \mathrm{VP} \Rightarrow \mathrm{R}}/L \tag{3.8}$$

The ungrammaticality of (1.10b) corresponds to the invalidity of the following:

$$\dfrac{\mathrm{N} \Rightarrow \mathrm{N} \qquad \dfrac{\dfrac{I \Rightarrow \mathrm{N} \qquad \cdots}{\mathrm{N}, \sqrt[1]{((\mathrm{N}\backslash \mathrm{S})/\mathrm{VP})\uparrow \mathrm{N}}, I, \sqrt[2]{((\mathrm{N}\backslash \mathrm{S})/\mathrm{VP})\uparrow \mathrm{N}}, \mathrm{N}, \mathrm{VP} \Rightarrow \mathrm{S}}\uparrow L}{\mathrm{N}, (((\mathrm{N}\backslash \mathrm{S})/\mathrm{VP})\uparrow \mathrm{N})\odot I, \mathrm{N}, \mathrm{VP} \Rightarrow \mathrm{S}}\odot L}{\mathrm{N}, ((((\mathrm{N}\backslash \mathrm{S})/\mathrm{VP})\uparrow \mathrm{N})\odot I)/\mathrm{N}, \mathrm{N}, \mathrm{N}, \mathrm{VP} \Rightarrow \mathrm{S}}/L \tag{3.9}$$

In situ binding such as the quantification in (1.11a) is derived thus:

$$\dfrac{\dfrac{\mathrm{N}, ((\mathrm{N}\backslash \mathrm{S})/\mathrm{N})/\mathrm{N}, \mathrm{N}, \mathrm{N} \Rightarrow \mathrm{S}}{\mathrm{N}, ((\mathrm{N}\backslash \mathrm{S})/\mathrm{N})/\mathrm{N}, \sqrt{\mathrm{S}\uparrow \mathrm{N}}, \mathrm{N} \Rightarrow \sqrt{\mathrm{S}\uparrow \mathrm{N}}}\uparrow R \qquad \mathrm{S} \Rightarrow \mathrm{S}}{\mathrm{N}, ((\mathrm{N}\backslash \mathrm{S})/\mathrm{N})/\mathrm{N}, (\mathrm{S}\uparrow \mathrm{N})\downarrow \mathrm{S}, \mathrm{N} \Rightarrow \mathrm{S}}\downarrow L \tag{3.10}$$

Similarly, for the reflexivisation (1.11b):

$$\dfrac{\dfrac{((N\backslash S)/N)/N,\ N,\ N \Rightarrow N\backslash S}{((N\backslash S)/N)/N,\ \sqrt{(N\backslash S){\uparrow}N},\ N \Rightarrow \sqrt{(N\backslash S){\uparrow}N}}{\uparrow}R \qquad N,\ N\backslash S \Rightarrow S}{N,\ ((N\backslash S)/N)/N,\ ((N\backslash S){\uparrow}N){\downarrow}(N\backslash S),\ N \Rightarrow S}{\downarrow}L \tag{3.11}$$

Finally, (1.14) is derived thus:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{N \Rightarrow N \qquad N,\ N\backslash S \Rightarrow S}{N,\ \sqrt[1]{(N\backslash S){\uparrow}N},\ N,\ \sqrt[2]{(N\backslash S){\uparrow}N} \Rightarrow S}{\uparrow}L}{N,\ \sqrt[1]{(N\backslash S){\uparrow}N},\ \sqrt{S{\uparrow}N},\ \sqrt[2]{(N\backslash S){\uparrow}N} \Rightarrow \sqrt{S{\uparrow}N}}{\uparrow}R \qquad S \Rightarrow S}{N,\ \sqrt[1]{(N\backslash S){\uparrow}N},\ (S{\uparrow}N){\downarrow}S,\ \sqrt[2]{(N\backslash S){\uparrow}N} \Rightarrow S}{\downarrow}L}{\sqrt[1]{(N\backslash S){\uparrow}N},\ (S{\uparrow}N){\downarrow}S,\ \sqrt[2]{(N\backslash S){\uparrow}N} \Rightarrow N\backslash S}\backslash R}{(S{\uparrow}N){\downarrow}S \Rightarrow ((N\backslash S){\uparrow}N){\downarrow}(N\backslash S)}{\downarrow}R \tag{3.12}$$

This sequent methodology can be extended straightforwardly to include generalisations of discontinuity such as those in Morrill and Merenciano (1996).

4 Proof nets for sorted discontinuity calculus

The two incident parameter edges of the binary relational predication of formulas of sort string are notated in expanded proof nets according to (2.12); the four incident parameter edges of the quaternary relational predication of formulas of sort split string are notated in expanded proof nets according to (4.1):

$$\text{start}_1\ \text{end}_2\ A^{\bullet}\ \text{start}_2\ \text{end}_1 \qquad \text{end}_1\ \text{start}_2\ A^{\circ}\ \text{end}_2\ \text{start}_1 \tag{4.1}$$

The subscripts refer to the first (left) and second (right) string components of a split string; note that, again, the input and output orderings are mirror-images, which promotes visual symmetry. The expanded proof links for the discontinuity connectives are given in figure 20.

Prosodic traversal visits split string conclusions *twice*. On the first occasion the parameter start_1 of the first component of a split string input conclusion is visited, and travel continues up at the parameter end_1; on the second occasion the parameter start_2 of the second component is visited, and travel continues up at end_2; the material visited meanwhile is interpolated between the two components. Thus the result of prosodic extraction for figure 21 is **John+gave+Mary+the+cold+shoulder**. The result of semantic extraction is (*shun m j*).

Again, the proof net methodology we have illustrated proffers prospects for extension to partially commutative categorial logics in general.

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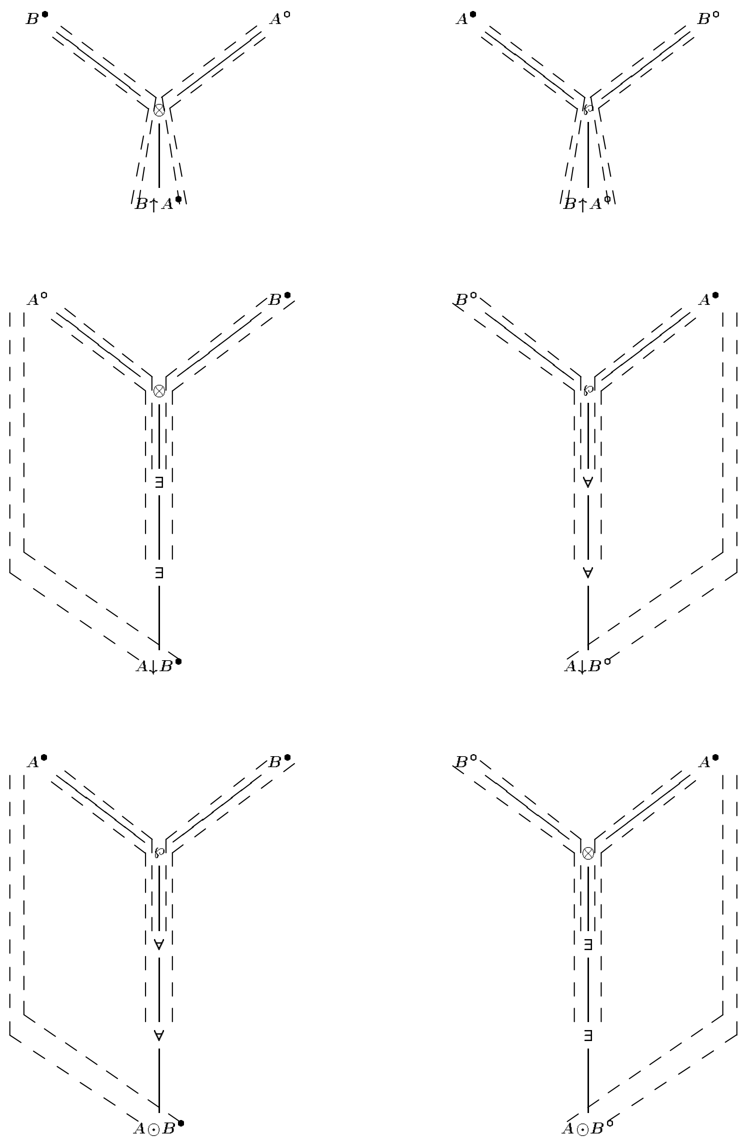

FIG. 20. Expanded proof links for the discontinuity connectives

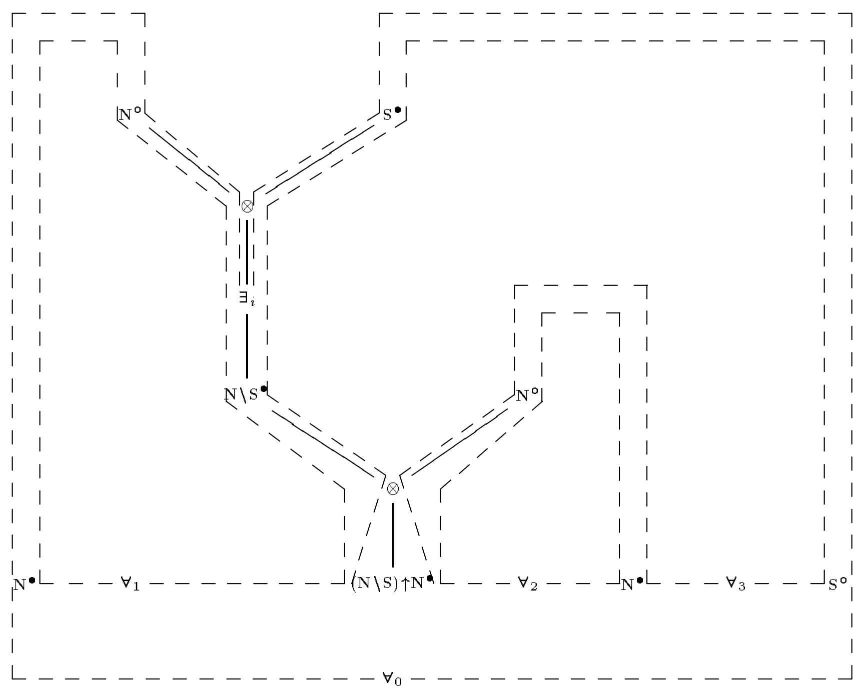

FIG. 21. Proof net for 'John gave Mary the cold shoulder' via a wrapping functor

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