Position Paper

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What is it that holds categorial grammar together, and what is it that holds it apart? What the various trends have in common seems to be a footing in mathematical philosophy. Where they differ is in their reach into linguistics and computation. Contemporary categorial grammar in computational linguistics can be perhaps fairly classified into three main schools: unificational, combinatory, and logical. The unificational school is represented by such names as Klein, Pollard, Uszkoreit; the combinatory by Steedman, Szabolcsi; the logical by van Benthem, Moortgat. This paper will offer a construal of the field which sees the logical school embracing the unificational and combinatory, which are reactions of the core to computational and linguistic demands respectively. This will be done by showing how the logical view explains the unificational and combinatory approaches.

The thesis of the combinatory school is that the description of (discontinuous) dependencies in certain natural language constructions requires a variety of functional operations: combinators. The logical perspective explains these combinatory primitives as consequences of type theory. In addition however, the lure of linguistic significance has lead the combinatory school to overreach itself in pursuit of both observational and descriptive goals.

In relation to observational adequacy, such operations as mixed composition violate the interpretaton of directional division formulated by Lambek; the logical correction required here is that rather than treat discontinuity by adding improper rules of formation, the requisite resource transformations should be controlled from within the logic by means of structural modalities or other type-constructors. In relation to descriptive adequacy, the claim has been offered that the cognitive (and presumably universal) possibilities of natural grammar are explained as being the computational properties of a handful of ‘useful’ primitive combinators (e.g. Szabolcsi 1987 seeks a ‘mystery’ combinator *X*, guided by concern with paradoxical combinators and

Gödel incompleteness, ...). The technical reality however is that the valid combination schemes cannot be finitely axiomatised, and the author has also claimed that the demands on combination made by empirical criteria must be declaratively described in their generality by recursive rules (Morrill 1988).

The unificational school is likewise transparently named: its tenet is that computational implementation of featural information flow in grammar processing should consist of unification. Again, the logical perspective expains unification as the implementation of type inference with universally quantified types. A computational outlook which regards rules as creating search space has lead to a tendency to try to limit modes of combination to those of the basic AB calculus, which it is generally necessary to augment with some kind of feature percolation devices in order to create the combinatorics for discontinuity. The type-theoretic basis however insists upon the inclusion of all valid combination schemes, and these themselves perform the role otherwise advanced more procedurally by percolation mechanisms. Observe for example that if an element is of a type (category) *A* then it *is* an element which combines with elements taking *A*s into *B*s, to form a *B*. Hence lifting is valid, whatever the temptation to believe that actual grammars may be completely implemented without it

THE RULE-TO-RULE FRAMEWORK

We attribute the architecture to be presented to Montague, though there are considerable terminological differences with his UG. We begin by assuming that the aspect of language we wish to model is an assocation between symbols and meanings in various parts of speech. (we could have targeted more or other linguistic dimensions). This sets a different context than the usual point of departure which takes as its goal the assignment of structural descriptions (syntactic structures) to the sentences of a language. Unless structural descriptions are interpreted in terms of empirically realised prosodic or semantic properties, they are literally meaningless and discussion of their nature is idle. Below, structural descriptions or syntactic structures are understood as derivational structures which do indeed predict a relation between prosodics and semantics, but they are just a side-effect of defining a relation between prosodics and semantics, being a summary of the way in which a linking is generated.

We assume *prosodic* and *semantic algrebras*. An algebra consists of a set of elements, and a set of operations over that domain; an algebraic element is identified with its properties in relation to the operations. What we call the prosodic algebra, Montague would have called the syntactic algebra. We understand syntax instead as a theoretical relation between prosodics and semantics; the term prosody is chosen as a dilution of the term phonology, often used to mean nothing more that word order.

A *sign* is a pair consisting of a prosodic object and a semantic object. A *type* is a set of signs. The product of the prosodic and semantic algebras is the set of all possible signs; its powerset is the set of possible types. A *language model* is just a set of types. Prosodic and semantic objects are designated structurally by *prosodic* and *semantic forms* which are interpreted in the prosodic and semantic algebras. Prosodic forms could be lists, trees, headed trees, labelled trees, etc.; their interpretation as prosodic objects is not necessarily one-to-one: e.g. trees with the same yield might be interpreted as having the same prosodic sequence object value. Semantic forms are typically terms of higher-order type logic; again the interpretation as semantic objects need not be one-to-one: distinct lambda terms may designate the same semantic object. An i*ndexed language model* is an indexed set of types. An indexed language model defines a language model which is the set of types indexed.

An *assignment* is a pair consisting of a prosodic form and a semantic form. A *category* is a set of assignments. A *formal language model* or *inhabitation* is an indexed set of categories. A formal language model defines an indexed language model which is the indexed set of signs that its assignments designate, and this in turn defines a language model. An ordered triple consisting of a prosodic form, a semantic form, and a type index is called a *type assignment*.

Operations over prosodic and semantic forms are called *prosodic* and *semantic constructors*. A *cohabitation condition* or *arrow* is a pairing of equal adicity prosodic and semantic constructors the inputs and outputs of which receive the same type indices. A *theory of habitation* is a set of arrows.

An inhabitation *satisfies an arrow* if and only if the result of applying the prosodic and semantic constructions to any assignments to its input type indices yields an assignment to its output type index. A inhabitation *satisfies a theory of habitation* if and only if it satisfies each arrow in that theory. A theory of habitation and an initial assignment define a formal language model which it the smallest extension of the initial assignment satisfying the theory of habitation.

In order to reference prosodic and semantic constructions, more or less formal meta-languages are used. These may talk loosely of how to construct one prosodic form from others (‘replace the first occurrence of a verb by its third person singular present form’ ...; the English semantic evaluation procedure in EFL), or more formal prosodic and semantic meta-languages may be employed (transformational grammar; IL of PTQ). The prosodic and semantic representation languages are composed of variables over and constuctors on prosodic and semantic forms. Constructions on prosodic and semantic forms are represented by prosodic and semantic terms.

A *statement of formation* consists of a prosodic term and a semantic term labelled by a *succedent type index*, with their free variables labelled by the same *antecedent type indices*. We write the following:

(1) $a_1 - x_1 : A_1, ..., a_n - x_n : A_n \Rightarrow \alpha(a_1, ..., a_n) - \phi(x_1, ..., x_n) : A$

A statement of formation designates a cohabitation condition. A *theory of formation* is a set of statements of formation including identity statements for each type formula, and which is closed under Cut:

(2) $a - x : A \Rightarrow a - x : A$ [id]

$$\frac{\Gamma \Rightarrow \beta - \psi : B \qquad \Delta(b - y : B) \Rightarrow \alpha - \phi : A}{\Delta(\Gamma) \Rightarrow \alpha[b \text{ €€} \leftarrow \beta] - \phi[y \leftarrow \psi] : A}\text{[Cut]}$$

A theory of formation designates a theory of habitation; id and Cut correspond to the fact that any inhabitation satisfying a theory of habitation respects the reflexivity and transitivity of set containment.

A *presentation of a theory of formation* is an inductive definition of a theory of formation consisting of *axiomatic* and *proper rules of formation*, where a *rule of formation* consists of a mapping from zero (axiomatic) or more (proper) *premise* statements of formation to *conclusion* statements of formation. A *derivation* is a sequence of statements of formation each of which is either axiomatic, or is the conclusion of a proper rule of formation the premises of which appear earlier in the sequence. A *derivational structure* is a graph summarising a derivation, in a way such as that in which a tree summarises a CF-PSG derivation.

With respect to prosodic and semantic formal algrebras, a *grammar formalism* contains a space of possible lexical assignments and theories of habitation. A *grammar presentation* consists of an initial assignment and a presentation of a theory of formation. Computational implementation of formal grammars might address procedures for solving a number of problems. One processing task which arises is recognition of signs. There are two versions. For a given language model, the *fixed language recognition problem* is the task of recognising signs in the language model. More generallly, for a given grammar formalism the *universal language recognition problem* is the task of determining for a given theory of habitation and sign whether that sign is in the language model defined.

Processing tasks more interesting than recognition are parsing and generation. The parsing problem is traditionally regarded as the task of recovering the ‘structural description’ assigned to a prosodic object by a grammar presentation. If the structural descriptions determine meaning, this is tantamount to computing the mapping from prosodic objects to semantic objects; it is this task which we wish to regard as parsing; generation is the reverse mapping.

Now we may talk about e.g. the universal generation problem for GPSG. We can address the decidability or computational complexity of such problems. In some formalisms, such as DCG and PATR-II, these problems are necessarily undecidable, a fact following from their simulation of Turing Machines. For others it is consistently found that the complexity of universal parsing problems is exponential in time, i.e. in a technical sense intractable/unfeasable.

WHAT *IS* THE LOGICAL SCHOOL?

The central tenet of the logical school is that type indices are formulas freely generated by type-constructors and compositionally interpreted according to the meaning of the type-constructors as operations on sets of signs. Thus the theory of habitation is fixed once and for all by the meaning of the type-constructors. This position is summed up by the slogan: ‘All and only logical rules of formation’. The theory of habitation is invarient, and the only ‘parameter’ of grammar is the lexical assignment. Then the entire theory of habitation dimension disappears from the universal language tasks: we consider only variation in lexical assignment. In more detail, the logical school has the following closely related characteristics: logic without structural rules as a formalism for grammar of prosodic forms; classification of signs by means of type formulas with a universal logic – i.e. full lexicalism; ‘formulas-as-types’ semantics.

Interpretation of type formulas as sets of signs is, for example, thus:

() $[[A/B]] = \{\langle a, x\rangle \mid \text{for all } \langle b, y\rangle \text{ in } [[B]], \langle a+b, (x\ y)\rangle \text{ in } [[\mathrm{A}]]\}$

A consequence of the compositional denotational interpretation of complex type formulas is that since basic type inhabitation determines all type inhabitation, the ‘data’ consisting of signs inhabiting basic types, determines the correct grammars: those inducing the same inhabitation as the one following by the interpretation of type-constructors from the inhabitation of basic types.

STATE OF THE ART AND FUTURE PROSPECTS

The state of the art as regards categorial processing is as follows. Moortgat (1988) explains the basis of cut-free ‘parsing-as-deduction’. The principle problem arising is derivational equivalence. For the division fragment, a normalisation approach is developed in König (1989) and Hepple (1990a). This is adapted, insofar as that is straightforward, to a more general logic fragment in Morrill (1990c). However, proper treatment of derivational equivalence in parsing for more general logic fragments, by means of normalisation or otherwise (e.g. the 'proof nets' of linear logic), is a large research topic. Generation, even for just Lambek's division, is a major research topic on which work has barely begun.

In relation to complexity, note that conjunctive type-construction means that any finite number of lexical types A_1, A_2, ..., A_n can be trivially compressed into a single lexical entry of type $A_1 \wedge A_2 \wedge \ldots \wedge A_n$. Of course the processing complexity arising from lexical ambiguity does not thereby disappear, but becomes transferred to the work load of theorem proving. Complexity is reduced by building the polymorphism deeper into types, capturing whatever generalisations are possible. A potential additional parameter is that of main clause type(s) (Morrill and Gavarró 1991): rather than assuming a necessarily atomic distinguished type, we allow a complex one Disjunctive type-construction means that any finite number of target types A_1, A_2, ..., A_n can be trivially compressed into a single distinguished type $A_1 \vee A_2 \vee \ldots \vee A_n$.

Our survey of the logical field will by structured by families of type-constructor: multiplicatives, intensional modalities, quantified types, Boolean types, and structural modalities.

The product and dual directional division basis is given in Lambek (1958, 1961). Moortgat (1988, 1990a, 1990b) offers extraction and infixation operators $\uparrow$ and $\Uparrow$. A matter arising is the inablity of Lambek's Gentzen sequent format to express complete logics for these type-constructors, a problem Moortgat has addressed by moving from non-commutative statements of formation with implicit left-to-right prosodic constructions, to multiset statements of formation with explicit prosodic terms. A methodological question is whether it is advisable to use such high-level type-constructors as $\uparrow$ and $\Uparrow$, or whether we should seek to 'decompose' these into structural modalities.

A second class of type-constructor is given by the semantic modalities (Morrill 1989, 90). A matter arising here is formulation of polymodal logic for polyindexical intensional semantics. Assume for instance modalities $\mathrm{L_w}$ and $\mathrm{L_t}$ for worlds and times; a simple juxtaposition of independent inference rules would not derive commuted statements of formation such as $\mathrm{L_w L_t} A \Rightarrow \mathrm{L_t L_w} A$. The proposals for semantic modality were originally made with close attention to possible applications in the description of locality. A development in Hepple (1990b) pursues modality without semantic commitment; such syntactic polymodality raises similar technical questions to those arising semantically; methodologically however, the employment of apparatus geared soley towards the capture of boundedness and islandhood can appear *ad hoc*.

A third class consists of quantified types. First-order and second-order cases, and their relation to unification, are discussed in Morrill

(1990a) and van Benthem (1989) respectively. For application of second-order quantification to type-assignment to natural language quantifiers see Emms (1990). The principle technical issue raised is decidability and completeness of type inference.

A fourth class is made up what are in linear logic terminology called additives. These have a natural set-theoretic interpretation as intersection and union on types (see Morrill 1990a); however the existing logic is incomplete with respect to this interpretation. This matter is addressed in a proposed reconstrual in Roorda (forthcoming).

Finally there are structural modalities (Morrill *et al.* and Barry *et al*.), or what have been called in linear logic exponentials. These type constructors control structural operations such as contraction, weakening, and permutation. Dropping these structural rules lead to type systems with increasing structure on resources. Associative categorial logic deals with sequences of formulas; non-associative with bracketed sequences, i.e. trees. A recent development is to attribute even more structure, in the form of headed trees (Moortgat and Morrill 1991); traditional transformational grammar dealt with (category-) labelled trees. In these various cases, structural modalities may be used to license flexibility where this is required. In general, there is good space for both application of existing apparatus in empirical work, and effort towards an increased theoretical understanding of it in terms of interpretation and completeness.

The logical school has been closely aligned to Montague Semantics; so has the combinatory school; the same is not true of the unificational, which has tended towards discourse semantics.

The purpose of a grammar regarding syntax is to define amongst possible sentences those which are well-formed. The purpose regarding semantics is to define amongst the meanings of sentences such notions as synonomy, antynomy, tautology, contradiction, and entailment. In truth-conditional model-theoretic semantics the semantic relations are derived as follows: a class of mathematical structures called 'models' is defined that represent situations with respect to which meanings may be considered. In each model a meaning maps to a value, in particular, sentence meanings map to truth values. Thus a meaning is identified with its truth conditions on models representing world situations. Sentence meanings are synonyms iff they have the same truth values in every model, antonyms iff they have opposite truth values in every model; a sentence meaning is a tautology iff it is true in every model, a contradiction iff it is false in every model; and one sentence meaning entails another iff there is no model in which the one is true and the other false.

The two principle achievements of Montague Semantics are its analysis of intensionality, and quantification. Categorial logic has recently ‘caught up’ with the type discipline that Montague surely wanted to observe. In respect of intensionality see Morrill (1990b); in respect of quantification there are currently two proposals: Moortgat (1990a) and Emms (1990).

In Montague Grammar a sentence is syntactically well-formed iff it has at least one reading/construel/meaning, thus well-formedness may be seen as a derivative of meaningfulness. This position paper began with a rejection of syntax as a level of description, in favour of syntax as a theory of description. A possibility left open there was for 'more or other' linguistic dimensions than prosodics and semantics, but surely not 'syntax'. A matter now is whether we should regard finer-grained levels such as phonetics and pragmatics as additional, or as alternative, more informative, refinements. In respect of discourse semantics there is reason to believe that pragmatics need not be additional to semantics, but could subsume it, e.g. with 'static' semantics being a derivative of 'dynamic semantics'. Categorial accomodation of such a perspective is a subject for research. Likewise for prosodics, it remains to be seen if descent to detail supercedes or supplements word order description. The 'multi-dimensional' outlook is presented in Oehrle (1988). However this is resolved, type-theory must extend beyond its current rudiments as regards its applicability to description of natural language grammar.

The position presented is summarised in the following points:

1. The grammar architecture is of the rule-to-rule design used by Montague.
2. Type (category) indices are formulas freely generated by type-constructors and compositionally interpreted according to the meaning of the type-constructors.
3. From 2 it follows that the type logic is fixed, being the embodiment of the interpretation of type-constructors; the laws of formation cannot be varied for different languages.
4. From 3 it follows that the Montague lexicon plus rules design loses enirely the rule dimension, i.e. there is total lexicalism; we allow however language-particular specification of a main clause type.
5. Parsing is the task of computing the set of semantic objects associated with a given prosodic object in a given type.
6. Generation is the task of computing the set of prosodic objects associated with a given semantic object in a given type.
7. From 4 it follows that rules are not a variable in the universal parsing and generation tasks, hence their solution in terms of the pure logic provides universal parsing and generation procedures.

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