Computational Coverage of TLG: The Montague Test

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Abstract

This paper reports on the empirical coverage of Type Logical Grammar (TLG) and on how it has been computer implemented. We analyse the Montague fragment computationally and we proffer this task as a challenge to computational grammar: the *Montague Test*.

Keywords: logical syntax and semantics; parsing as deduction; Montague grammar; computational grammar; Montague Test

1. Introduction

The Type Logical Grammar of (Morrill, 1994) and (Moortgat, 1997) is a powerful formalism with a transparent syntax-semantics interface operating through the Curry-Howard isomorphism. The version of the formalism used comprises 50 connectives shown in Figure 1.

The heart of the logic is the displacement calculus of (Morrill et al., 2011) which comprises twin continuous and discontinuous residuated families of connectives having a pure sequent calculus, the tree-based hypersequent calculus, and enjoying Cut-elimination (Valentín, 2012). Other primary connectives are additives, 1st order quantifiers, normal (i.e. distributive) modalities, bracket (i.e. nondistributive) modalities, and the non-linear exponentials, and contraction for anaphora.

We can draw a clear distinction between these primary connectives and the semantically inactive connectives and synthetic connectives which are abbreviatory and there merely for convenience. There are semantically inactive variants of the continuous and discontinuous multiplicatives, including the words as types predicate W, and semantically inactive variants of the additives, 1st order quantifiers, and normal modalities. Defined connectives divide into the continuous deterministic synthetic connectives of projection and injection, and the discontinuous, split and bridge, and the continuous nondeterministic synthetic connectives of nondirectional division and unordered product, and the discontinuous, nondeterministic extract, infix, and discontinuous product.

Finally there is the negation as failure of 'except' (formerly difference), a powerful device for expressing linguistic exceptions (Morrill and Valentín, 2014).

2. Rules and linguistic applications for primary connectives

In this section we present semantically labelled sequent rules for, and exemplify linguistic applications of, the primary connectives.

The continuous multiplicatives of Figure 2, the Lambek connectives, are the basic means of categorial categorization and subcategorization. The directional divisions over, $/$, and under, $\backslash$, are exemplified by assignments such as *the*: N/CN for *the man*: N and *sings*: $N\backslash S$ for *John sings*: S, and *loves*: $(N\backslash S)/N$ for *John loves Mary*: S. The continuous product $\bullet$ is exemplified by a 'small clause' assignment such as *considers*: $(N\backslash S)/(N\bullet(CN/CN))$ for *John considers Mary socialist*: S.[1] The continuous unit can be used together with additive disjunction to express the optionality of a complement as in *eats*: $(N\backslash S)/(N\oplus I)$ for

[1] But this makes no different empirical predictions from the more standard type of analysis in CG and G/HPSG which simply treats verbs like *consider* as taking a noun phrase and an infinitive.

	cont. mult.	disc. mult.	add.	qu.	norm. mod.	brack. mod.	exp.	contr. for anaph.
primary	/ \ • I	↑ ↓ ⊙ J	& ⊕	⋀ ⋁	□ ◇	$[\,]^{-1}$ ⟨⟩	! ?	|
semantically inactive variants	•– –∘ ∘– –• ◐ ◑ W	(semantically inactive ↑ ↓ variants) ◓ ◒	⊓ ⊔	∀ ∃	■ ♦			
det. synth.	$\lhd^{-1}$ $\rhd^{-1}$ $\lhd$ $\rhd$	ˇ ˆ						except
nondet. synth.	÷ ⊗	⇑ ⇓ ⊚						−

Figure 1: Table of categorial connectives

1. $$\frac{\Gamma \Rightarrow B\colon \psi \qquad \Delta\langle \overrightarrow{C}\colon z\rangle \Rightarrow D\colon \omega}{\Delta\langle \overrightarrow{C/B}\colon x, \Gamma\rangle \Rightarrow D\colon \omega\{(x\ \psi)/z\}}\ /L \qquad \frac{\Gamma, \overrightarrow{B}\colon y \Rightarrow C\colon \chi}{\Gamma \Rightarrow C/B\colon \lambda y\chi}\ /R$$

2. $$\frac{\Gamma \Rightarrow A\colon \phi \qquad \Delta\langle \overrightarrow{C}\colon z\rangle \Rightarrow D\colon \omega}{\Delta\langle \Gamma, \overrightarrow{A\backslash C}\colon y\rangle \Rightarrow D\colon \omega\{(y\ \phi)/z\}}\ \backslash L \qquad \frac{\overrightarrow{A}\colon x, \Gamma \Rightarrow C\colon \chi}{\Gamma \Rightarrow A\backslash C\colon \lambda x\chi}\ \backslash R$$

3. $$\frac{\Delta\langle \overrightarrow{A}\colon x, \overrightarrow{B}\colon y\rangle \Rightarrow D\colon \omega}{\Delta\langle \overrightarrow{A{\bullet}B}\colon z\rangle \Rightarrow D\colon \omega\{\pi_1 z/x, \pi_2 z/y\}}\ {\bullet}L \qquad \frac{\Gamma_1 \Rightarrow A\colon \phi \qquad \Gamma_2 \Rightarrow B\colon \psi}{\Gamma_1, \Gamma_2 \Rightarrow A{\bullet}B\colon (\phi, \psi)}\ {\bullet}R$$

4. $$\frac{\Delta\langle\Lambda\rangle \Rightarrow A\colon \phi}{\Delta\langle \overrightarrow{I}\colon x\rangle \Rightarrow A\colon \phi}\ IL \qquad \frac{}{\Lambda \Rightarrow I\colon 0}\ IR$$

Figure 2: Continuous multiplicatives

John eats fish: S and *John eats*: S.[2] It can also be used in conjunction with the connective 'except' to prevent the null string being supplied as argument to an intensifier as in *very*: $(CN/CN)/((CN/CN) - I)$ for *very tall man*: CN but $*$*very man*: CN.

The discontinuous multiplicatives of Figure 3, the displacement connectives, are defined in relation to intercalation. When the value of the k subscript is 1 it may be omitted. Circumfixation, ↑, is exemplified by a discontinuous idiom assignment *gives*+1+*the*+*cold*+*shoulder*: $(N\backslash S)\uparrow N$ for *Mary gives John the cold shoulder*: S, and infixation, ↓, and circumfixation together are exemplified by a quantifier phrase assignment *everyone*: $(S\uparrow N)\downarrow S$ simulating Montague's S14 treatment of quantifying in. Circumfixation and discontinuous product, ⊙, are illustrated in an assignment to a relative pronoun *that*: $(CN\backslash CN)/((S\uparrow N)\odot I)$ allowing both peripheral and medial extraction, *that John likes*: $CN\backslash CN$ and *that John saw today*: $CN\backslash CN$. Use of the discontinuous product unit, J, in conjunction with except is illustrated in a pronoun assignment *him*: $(((S\uparrow N)\uparrow_2 N) - (J\bullet((N\backslash S)\uparrow N)))\downarrow_2 (S\uparrow N)$ preventing a subject antecedent (Principle B effect).

[2]Note the advantage of this over simply listing intranstive and transitive lexical entries: empirically this latter does not capture the generalisation that in both cases *eats* combines with a subject to the left, and computationally every lexical ambiguity doubles the lexical insertion search space.

5. $$\frac{\Gamma \Rightarrow B\colon\psi \qquad \Delta\langle\overrightarrow{C}\colon z\rangle \Rightarrow D\colon\omega}{\Delta\langle\overrightarrow{C{\uparrow_k}B}\colon x\,|_k\,\Gamma\rangle \Rightarrow D\colon\omega\{(x\ \psi)/z\}}\uparrow_k L \qquad \frac{\Gamma\,|_k\,\overrightarrow{B}\colon y\Rightarrow C\colon\chi}{\Gamma\Rightarrow C{\uparrow_k}B\colon\lambda y\chi}\uparrow_k R$$

6. $$\frac{\Gamma \Rightarrow A\colon\phi \qquad \Delta\langle\overrightarrow{C}\colon z\rangle \Rightarrow D\colon\omega}{\Delta\langle\Gamma\,|_k\,\overrightarrow{A{\downarrow_k}C}\colon y\rangle \Rightarrow D\colon\omega\{(y\ \phi)/z\}}\downarrow_k L \qquad \frac{\overrightarrow{A}\colon x\,|_k\,\Gamma\Rightarrow C\colon\chi}{\Gamma\Rightarrow A{\downarrow_k}C\colon\lambda x\chi}\downarrow_k R$$

7. $$\frac{\Delta\langle\overrightarrow{A}\colon x\,|_k\,\overrightarrow{B}\colon y\rangle \Rightarrow D\colon\omega}{\Delta\langle\overrightarrow{A{\odot_k}B}\colon z\rangle \Rightarrow D\colon\omega\{\pi_1 z/x, \pi_2 z/y\}}\odot_k L \qquad \frac{\Gamma_1\Rightarrow A\colon\phi \qquad \Gamma_2\Rightarrow B\colon\psi}{\Gamma_1\,|_k\,\Gamma_2\Rightarrow A{\odot_k}B}\odot_k R$$

8. $$\frac{\Delta\langle 1\rangle \Rightarrow A\colon\phi}{\Delta\langle\overrightarrow{J}\colon x\rangle \Rightarrow A\colon\phi}JL \qquad \frac{}{1\Rightarrow J\colon 0}JR$$

Figure 3: Discontinuous multiplicatives

9. $$\frac{\Gamma\langle\overrightarrow{A}\colon x\rangle \Rightarrow C\colon\chi}{\Gamma\langle\overrightarrow{A\&B}\colon z\rangle \Rightarrow C\colon\chi\{\pi_1 z/x\}}\&L_1 \qquad \frac{\Gamma\langle\overrightarrow{B}\colon y\rangle \Rightarrow C\colon\chi}{\Gamma\langle\overrightarrow{A\&B}\colon z\rangle \Rightarrow C\colon\chi\{\pi_2 z/y\}}\&L_2$$

$$\frac{\Gamma\Rightarrow A\colon\phi \qquad \Gamma\Rightarrow B\colon\psi}{\Gamma\Rightarrow A\&B\colon(\phi,\psi)}\&R$$

10. $$\frac{\Gamma\langle\overrightarrow{A}\colon x\rangle \Rightarrow C\colon\chi_1 \qquad \Gamma\langle\overrightarrow{B}\colon y\rangle \Rightarrow C\colon\chi_2}{\Gamma\langle\overrightarrow{A\oplus B}\colon z\rangle \Rightarrow C\colon z\to x.\chi_1; y.\chi_2}\oplus L$$

$$\frac{\Gamma\Rightarrow A\colon\phi}{\Gamma\Rightarrow A\oplus B\colon\iota_1\phi}\oplus R_1 \qquad \frac{\Gamma\Rightarrow B\colon\psi}{\Gamma\Rightarrow A\oplus B\colon\iota_2\psi}\oplus R_2$$

Figure 4: Additives

The additives of Figure 4 have application to polymorphism. For example the additive conjunction & can be used for *rice*: $N\&CN$ as in *rice grows*: S and *the rice grows*: S,[3] and the additive disjunction $\oplus$ can be used for *is*: $(N\backslash S)/(N\oplus(CN/CN))$ as in *Bond is 007*: S and *Bond is teetotal*: S.

The quantifiers of Figure 5 have application to features. For example, singular and plural number in *sheep*: $\bigwedge nCNn$ for *the sheep grazes*: S and *the sheep graze*: S. And for a past, present or future tense finite sentence complement: *said*: $(N\backslash S)/\bigvee tSf(t)$ in: *John said Mary walked*: S, *John said Mary walks*: S and *John said Mary will walk*: S.

With respect to the normal modalities of Figure 6, the universal has application to intensionality. For example, for a propositional attitude verb *believes*: $\Box((N\backslash S)/\Box S)$ with a modality outermost since the word has a sense, and its sentential complement is an intensional domain, but its subject is not.

The bracket modalities of Figure 7 have application to syntactical domains such as prosodic phrases and extraction islands. For example, *walks*: $\langle\rangle N\backslash S$ for the sentential subject condition, and *before*: $[\,]^{-1}(VP\backslash VP)/VP$ for the adverbial island constraint.

Finally, there are non-linear connectives. The exponentials of Figure 8 have application to sharing. Using the universal exponential, !, for which contraction induces island brackets, we can assign a relative pronoun type *that*: $(CN\backslash CN)/(S/!N)$ allowing parasitic extraction such as *paper that John filed without reading*: CN, where parasitic gaps can appear only in islands, but can be iterated in subis-

[3] Note the advantage of this approach over assuming an empty determiner: computationally it is not forbidden that there be any number of empty operators in any positions.

$$11.\quad \frac{\Gamma\langle\overrightarrow{A[t/v]}: x\rangle \Rightarrow B: \psi}{\Gamma\langle\overrightarrow{\bigwedge vA}: z\rangle \Rightarrow B: \psi\{(z\ t)/x\}} \bigwedge L \qquad \frac{\Gamma \Rightarrow A[a/v]: \phi}{\Gamma \Rightarrow \bigwedge vA: \lambda v\phi} \bigwedge R^{\dagger}$$

$$12.\quad \frac{\Gamma\langle\overrightarrow{A[a/v]}: x\rangle \Rightarrow B: \psi}{\Gamma\langle\overrightarrow{\bigvee vA}: z\rangle \Rightarrow B: \psi\{\pi_2 z/x\}} \bigvee L^{\dagger} \qquad \frac{\Gamma \Rightarrow A[t/v]: \phi}{\Gamma \Rightarrow \bigvee vA: (t, \phi)} \bigvee R$$

Figure 5: Quantifiers, where † indicates that there is no a in the conclusion

$$13.\quad \frac{\Gamma\langle\overrightarrow{A}: x\rangle \Rightarrow B: \psi}{\Gamma\langle\overrightarrow{\Box A}: z\rangle \Rightarrow B: \psi\{^{\vee}z/x\}} \Box L \qquad \frac{\Box/\blacksquare\Gamma \Rightarrow A: \phi}{\Box/\blacksquare\Gamma \Rightarrow \Box A: {}^{\wedge}\phi} \Box R$$

$$14.\quad \frac{\Box/\blacksquare\Gamma\langle\overrightarrow{A}: x\rangle \Rightarrow \Diamond/\blacklozenge B: \psi}{\Box\Gamma\langle\overrightarrow{\Diamond A}: z\rangle \Rightarrow \Diamond/\blacklozenge B: \psi\{^{\cup}z/x\}} \Diamond L \qquad \frac{\Gamma \Rightarrow A: \phi}{\Gamma \Rightarrow \Diamond A: {}^{\cap}\phi} \Diamond R$$

Figure 6: Normal modalities, where $\Box/\blacksquare\Gamma$ signifies a structure all the types of which have main connective $\Box$ or $\blacksquare$

$$15.\quad \frac{\Delta\langle\overrightarrow{A}: x\rangle \Rightarrow B: \psi}{\Delta\langle[\overrightarrow{[\,]^{-1}A}: x]\rangle \Rightarrow B: \psi} [\,]^{-1}L \qquad \frac{[\Gamma] \Rightarrow A: \phi}{\Gamma \Rightarrow [\,]^{-1}A: \phi} [\,]^{-1}R$$

$$16.\quad \frac{\Delta\langle[\overrightarrow{A}: x]\rangle \Rightarrow B: \psi}{\Delta\langle\overrightarrow{\langle\rangle A}: x\rangle \Rightarrow B: \psi} \langle\rangle L \qquad \frac{\Gamma \Rightarrow A: \phi}{[\Gamma] \Rightarrow \langle\rangle A: \phi} \langle\rangle R$$

Figure 7: Bracket modalites

lands.[4, 5]

Using the existential exponential, ?, we can assign a coordinator type *and*: $(?N\backslash N)/N$ allowing iterated coordination as in *John, Bill, Mary and Suzy*: N, or *and*: $(?(S/N)\backslash(S/N))/(S/N)$ for *John likes*, *Mary dislikes*, *and Bill hates*, *London* (iterated right node raising), and so on.

The limited contraction for anaphora, $|$, of Figure 9 also has application to sharing; it can be used for anaphora in an assignment like *it*: $(S{\uparrow}N){\downarrow}(S|N)$ for, e.g., *the company$_i$ said it$_i$ flourished*: S, and it can be used for *such that* relativisation in an as-

$$17.\quad \frac{\Gamma\langle A: x\rangle \Rightarrow B: \psi}{\Gamma\langle !A: x\rangle \Rightarrow B: \psi} !L \qquad \frac{!A_1: x_1, \ldots, !A_n: x_n \Rightarrow A: \phi}{!A_1: x_1, \ldots, !A_n: x_n \Rightarrow !A: \phi} !R$$

$$\frac{\Delta\langle !A: x, \Gamma\rangle \Rightarrow B: \psi}{\Delta\langle \Gamma, !A: x\rangle \Rightarrow B: \psi} !P \qquad \frac{\Delta\langle \Gamma, !A: x\rangle \Rightarrow B: \psi}{\Delta\langle !A: x, \Gamma\rangle \Rightarrow B: \psi} !P$$

$$\frac{\Delta\langle !A_0: x_0, \ldots, !A_n: x_n, [!A_0: y_0, \ldots, !A_n: y_0, \Gamma]\rangle \Rightarrow B: \psi}{\Delta\langle !A_0: x_0, \ldots, !A_n: x_n, \Gamma\rangle \Rightarrow B: \psi\{x_0/y_0, \ldots, x_n/y_n\}} !C$$

$$18.\quad \frac{\Gamma \Rightarrow A: \phi}{\Gamma \Rightarrow ?A: [\phi]} ?R \qquad \frac{\Gamma \Rightarrow A: \phi \qquad \Delta \Rightarrow ?A: \psi}{\Gamma, \Delta \Rightarrow ?A: [\phi|\psi]} ?E$$

Figure 8: Exponentials

signment *such that*: $(CN\backslash CN)/(S|N)$ for, say, *man such that$_i$ he$_i$ thinks Mary loves him$_i$*: CN.

3. Implementation

A computational lexicon and parser integrates the grammatical features of the previous section, and of the remaining connectives, which defines a fragment including:

- the PTQ examples of (Dowty et al., 1981), Chapter 7;
- the discontinuity examples of (Morrill et al., 2011);
- relativisation, including islands and parasitic gaps;
- constituent coordination, non-constituent coordination, coordination of 'unlike'

[4] For example, *man who the fact that the friends of admire without praising surprises*.

[5] In the case that island violations are grammatical, as they are under certain conditions, we assume that the relative pronoun type is not $(CN\backslash CN)/(S/!N)$ but $(CN\backslash CN)/(S/\bigcirc N)$ where $\bigcirc$ is an association and commutation structural modality.This explains how island violation is possible combinatorially but we leave unanswered the question of how the choice of the relative pronoun type is conditioned by processing factors.

19. $$\frac{\Gamma \Rightarrow A\colon \phi \qquad \Delta\langle \overrightarrow{A}\colon x; \overrightarrow{B}\colon y\rangle \Rightarrow D\colon \omega}{\Delta\langle \Gamma; \overrightarrow{B|A}\colon z\rangle \Rightarrow D\colon \omega\{\phi/x, (z\ \phi)/y\}}\,|L$$

$$\frac{\Gamma\langle \overrightarrow{B_0}\colon y_0; \ldots; \overrightarrow{B_n}\colon y_n\rangle \Rightarrow D\colon \omega}{\Gamma\langle \overrightarrow{B_0|A}\colon z_0; \ldots; \overrightarrow{B_n|A}\colon z_n\rangle \Rightarrow D|A\colon \lambda x\omega\{(z_0\ x)/y_0, \ldots, (z_n\ x)/y_n\}}\,|R$$

Figure 9: Limited contraction for anaphora

types, ATBE, and a unitary lexical type analyses of simplex and complex gapping.

The implementation is CatLog2, a categorial parser/theorem-prover comprising 6000 lines of Prolog using backward chaining proof-search in the tree-based hypersequent calculus and the focusing of Andreoli (Andreoli, 1992). In addition to focusing, the implementation exploits the count-invariance of (van Benthem, 1991) and (Valentín et al., 2013). This paper presents just the first item in the list above.

4. The Montague Test

In this section we give derivations of the Montague grammar fragment examples analysed in Chapter 7 of (Dowty et al., 1981), DWP.[6] (We include the indexation of CatLog, which contains the numeration of the source, within the example displays.)

(1) (dwp((7-7))) [**john**]+**walks** : Sf

Recall that in our syntactical forms the subjects are bracketed domains — implementing that subjects are weak islands. Lookup in our lexicon yields the following semantically labelled sequent:

(2) $[\blacksquare Nt(s(m)) : j], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) : {}^{\wedge}\lambda A(Pres\ ({}^{\vee}walk\ A)) \Rightarrow Sf$

As always the lexical types are semantically modalized outermost — implementing that word meanings are intensions/senses; the modality of the proper name subject is semantically inactive (proper names are rigid designators), while the modality of the tensed verb is semantically active (the interpretation of tensed verbs depends on the temporal reference points). The verb projects a finite sentence (feature f) when it combines with a third person singular (bracketed) subject of any gender; the actual subject is masculine (feature m).

The derivation is as follows:

$$\frac{\dfrac{\dfrac{\dfrac{\dfrac{}{\boxed{Nt(s(m))} \Rightarrow Nt(s(m))}}{\boxed{\blacksquare Nt(s(m))} \Rightarrow Nt(s(m))}\,\blacksquare L}{\blacksquare Nt(s(m)) \Rightarrow \boxed{\exists gNt(s(g))}}\,\exists R}{[\blacksquare Nt(s(m))] \Rightarrow \boxed{\langle\rangle\exists gNt(s(g))}}\,\langle\rangle R \qquad \dfrac{}{\boxed{Sf} \Rightarrow Sf}}{\dfrac{[\blacksquare Nt(s(m))], \boxed{\langle\rangle\exists gNt(s(g))\backslash Sf} \Rightarrow Sf}{[\blacksquare Nt(s(m))], \boxed{\Box(\langle\rangle\exists gNt(s(g))\backslash Sf)} \Rightarrow Sf}\,\Box L}\,\backslash L$$

The semantics delivered by the derivation of this example is:

(3) $(Pres\ ({}^{\vee}walk\ j))$

The full paper proceeds with illustration of the computational analysis of the examples of Chapter 7 of (Dowty et al., 1981).

4. Conclusion

This paper reports a formal, computational, logical and mathematical approach to syntax, semantics, and the syntax-semantics interface. In the paper we report a type logical computational cover grammar of the Montague PTQ fragment. In relation to comparison between theoretical frameworks, this so-called Montague Test could represent, in our opinion, a sort of challenge/baseline for any syntax-semantics framework.

We propose to call the task of covering the PTQ fragment computationally the *Montague Test*, and we issue the Montague Test as a challenge to all other grammar frameworks.

[6]Note how in the input to CatLog brackets mark islands: single brackets for weak islands such as subjects and double brackets for strong islands such as coordinate structures

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