

Computational Coverage of TLG: Displacement*

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Abstract

This paper reports on the coverage of TLG of Morrill (1994) and Moortgat (1997), and on how it has been computer implemented. We computer-analyse examples of displacement: discontinuous idioms, quantification, (medial) relativisation, VP ellipsis, (medial) pied piping, appositive relativisation, parentheticals, gapping, comparative subdeletion, and reflexivisation, and, in the appendix, Dutch verb raising and cross-serial dependency.

Keywords: logical syntax and semantics; parsing as deduction; displacement

1. Introduction

The version of the formalism used is essentially the categorial type logic of Morrill (2014) plus Morrill and Valentín (2014b), and comprises 50 connectives as shown:

	cont. mult.	disc. mult.	add.	qu.	norm. mod.	brack. mod.	exp.	contr. for anaph.
primary	/ • I	↑ ⊙ J	& ⊕	∧ ∨	□ ◇	[] ⁻¹ ⟨ ⟩	! ?	
semantically inactive variants	•- ∞ ●	◦- → ●	↑ ↓ ●	□	∨	■		
		W		∩	∃	◆		
det. synth.	◁ ⁻¹ ◁	▷ ⁻¹ ▷						except
nondet. synth.	÷ ⊗	↑↑ ⊙						-

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The heart of the logic is the displacement calculus of Morrill and Valentín (2010) and Morrill et al. (2011) which comprises twin continuous and discontinuous residuated families of connectives having a pure sequent calculus, the tree-based hypersequent calculus, and enjoying Cut-elimination (Valentín, 2012). Other primary connectives are additives, 1st order quantifiers, normal (i.e. distributive) modalities, bracket (i.e. nondistributive) modalities, and the non-linear exponentials, and contraction for anaphora.

We can draw a clear distinction between these primary connectives and the semantically inactive connectives and defined connectives which are abbreviatory and therefore merely for convenience.

There are semantically inactive variants of the continuous and discontinuous multiplicatives, including the words as types predicate W , and semantically inactive variants of the additives, 1st order quantifiers, and normal modalities.

Defined connectives divide into the continuous deterministic (unary) synthetic connectives of projection and injection, and the discontinuous, split and bridge, and the continuous nondeterministic (binary) synthetic connectives of nondirectional division and unordered product, and the discontinuous, nondeterministic extract, infix, and discontinuous product.

Finally there is the negation as failure of ‘except’ (formerly difference), a powerful device for expressing linguistic exceptions (Morrill and Valentín, 2014a).

2. Rules and linguistic applications for primary connectives

$$\begin{array}{l}
1. \quad \frac{\Gamma \Rightarrow B:\psi \quad \Delta\langle\vec{C}:z\rangle \Rightarrow D:\omega}{\Delta\langle\vec{C}/\vec{B}:x,\Gamma\rangle \Rightarrow D:\omega\{(x\ \psi)/z\}} /L \quad \frac{\Gamma,\vec{B}:y \Rightarrow C:\chi}{\Gamma \Rightarrow C/B:\lambda y\chi} /R \\
2. \quad \frac{\Gamma \Rightarrow A:\phi \quad \Delta\langle\vec{C}:z\rangle \Rightarrow D:\omega}{\Delta\langle\Gamma,A\vec{C}:y\rangle \Rightarrow D:\omega\{(y\ \phi)/z\}} \setminus L \quad \frac{\vec{A}:x,\Gamma \Rightarrow C:\chi}{\Gamma \Rightarrow A\setminus C:\lambda x\chi} \setminus R \\
3. \quad \frac{\Delta\langle\vec{A}:x,\vec{B}:y\rangle \Rightarrow D:\omega}{\Delta\langle\vec{A}\bullet\vec{B}:z\rangle \Rightarrow D:\omega\{\pi_1 z/x,\pi_2 z/y\}} \bullet L \quad \frac{\Gamma_1 \Rightarrow A:\phi \quad \Gamma_2 \Rightarrow B:\psi}{\Gamma_1,\Gamma_2 \Rightarrow A\bullet B:(\phi,\psi)} \bullet R \\
4. \quad \frac{\Delta\langle\Lambda\rangle \Rightarrow A:\phi}{\Delta\langle\vec{I}:x\rangle \Rightarrow A:\phi} IL \quad \frac{}{\Lambda \Rightarrow I:0} IR
\end{array}$$

Figure 1: Continuous multiplicatives

The continuous multiplicatives of Figure 1, the Lambek connectives, are the basic means of categorial categorization and subcategorization. The directional divisions over, /, and under, \, are exemplified by assignments such as **the**: N/CN for **the man**: N and **sings**: $N\setminus S$ for **John sings**: S , and **loves**: $(N\setminus S)/N$ for **John loves Mary**: S . The continuous product \bullet is exemplified by a ‘small clause’ assignment such as **considers**: $(N\setminus S)/(N\bullet(CN/CN))$ for **John considers Mary socialist**: S .¹ The continuous unit can be used together with additive disjunction to express the optionality of a complement as in **eats**: $(N\setminus S)/(N\oplus I)$

¹But this makes no different empirical predictions from the more standard type of analysis in CG and G/HPSG which simply treats verbs like consider as taking a noun phrase and an infinitive.

for **John eats fish**: S and **John eats**: S .² It can also be used in conjunction with the connective ‘except’ to prevent the null string being supplied as argument to an intensifier as in **very**: $(CN/CN)/((CN/CN) - I)$ for **very tall man**: CN but ***very man**: CN .

$$\begin{array}{l}
5. \quad \frac{\Gamma \Rightarrow B: \psi \quad \Delta \langle \vec{C}: z \rangle \Rightarrow D: \omega}{\Delta \langle \vec{C} \uparrow_k \vec{B}: x |_k \Gamma \rangle \Rightarrow D: \omega \{ (x \ \psi) / z \}} \uparrow_k L \quad \frac{\Gamma |_k \vec{B}: y \Rightarrow C: \chi}{\Gamma \Rightarrow C \uparrow_k B: \lambda y \chi} \uparrow_k R \\
6. \quad \frac{\Gamma \Rightarrow A: \phi \quad \Delta \langle \vec{C}: z \rangle \Rightarrow D: \omega}{\Delta \langle \Gamma |_k \vec{A} \downarrow_k \vec{C}: y \rangle \Rightarrow D: \omega \{ (y \ \phi) / z \}} \downarrow_k L \quad \frac{\vec{A}: x |_k \Gamma \Rightarrow C: \chi}{\Gamma \Rightarrow A \downarrow_k C: \lambda x \chi} \downarrow_k R \\
7. \quad \frac{\Delta \langle \vec{A}: x |_k \vec{B}: y \rangle \Rightarrow D: \omega}{\Delta \langle \vec{A} \odot_k \vec{B}: z \rangle \Rightarrow D: \omega \{ \pi_1 z / x, \pi_2 z / y \}} \odot_k L \quad \frac{\Gamma_1 \Rightarrow A: \phi \quad \Gamma_2 \Rightarrow B: \psi}{\Gamma_1 |_k \Gamma_2 \Rightarrow A \odot_k B} \odot_k R \\
8. \quad \frac{\Delta \langle 1 \rangle \Rightarrow A: \phi}{\Delta \langle \vec{J}: x \rangle \Rightarrow A: \phi} JL \quad \frac{}{1 \Rightarrow J: 0} JR
\end{array}$$

Figure 2: Discontinuous multiplicatives

The discontinuous multiplicatives of Figure 2, the displacement connectives, are defined in relation to intercalation. When the value of the k subscript is 1 it may be omitted. Circumfixation, \uparrow , is exemplified by a discontinuous idiom assignment **gives+1+the+cold+shoulder**: $(N \setminus S) \uparrow N$ for **Mary gives John the cold shoulder**: S , and infixation, \downarrow , and circumfixation together are exemplified by a quantifier phrase assignment **everyone**: $(S \uparrow N) \downarrow S$ simulating Montague’s S14 treatment of quantifying in. Circumfixation and discontinuous product, \odot , are illustrated in an assignment to a relative pronoun **that**: $(CN \setminus CN) / ((S \uparrow N) \odot I)$ allowing both peripheral and medial extraction: **that John likes**: $CN \setminus CN$ and **that John saw today**: $CN \setminus CN$. Use of the discontinuous product unit, J , in conjunction with except is illustrated in a pronoun assignment **him**: $((S \uparrow N) \uparrow_2 N) - (J \bullet ((N \setminus S) \uparrow N)) \downarrow_2 (S \uparrow N)$ preventing a subject antecedent (Principle B effect).

The additives of Figure 3 have application to polymorphism. For example the additive conjunction $\&$ can be used for **rice**: $N \& CN$ as in **rice grows**: S and **the rice grows**: S ,³ and the additive disjunction \oplus can be used for **is**: $(N \setminus S) / (N \oplus (CN / CN))$ as in **Bond is 007**: S and **Bond is teetotal**: S .

The quantifiers of Figure 4 have application to features. For example, singular and plural number in **sheep**: $\wedge nCNn$ for **the sheep grazes**: S and **the sheep graze**: S . And for a past, present or future tense finite sentence complement: **said**: $(N \setminus S) / \vee tSf(t)$ in **John said Mary walked**: S , **John said Mary walks**: S and **John said Mary will walk**: S .

With respect to the normal modalities of Figure 5, the universal has application to intensionality. For example, for a propositional attitude verb **believes**: $\square((N \setminus S) / \square S)$ with a modality outermost since the word has a sense, and its sentential complement is an intensional domain, but its subject is not.

The bracket modalities of Figure 6 have application to syntactical domains such as

²Note the advantage of this over simply listing intransitive and transitive lexical entries: empirically this latter does not capture the generalisation that in both cases the verb eats combines with a subject to the left, and computationally every lexical ambiguity doubles the lexical insertion search space. Appeal to lexical ambiguity is always available and never interesting, except where there is true ambiguity.

³Note the advantage of this approach over assuming an empty determiner: computationally it is not forbidden that there be any number of empty operators in any positions.

$$\begin{array}{c}
9. \quad \frac{\Gamma\langle\vec{A}:x\rangle \Rightarrow C:\chi}{\Gamma\langle\vec{A\&B}:z\rangle \Rightarrow C:\chi\{\pi_1z/x\}} \&L_1 \quad \frac{\Gamma\langle\vec{B}:y\rangle \Rightarrow C:\chi}{\Gamma\langle\vec{A\&B}:z\rangle \Rightarrow C:\chi\{\pi_2z/y\}} \&L_2 \\
\frac{\Gamma \Rightarrow A:\phi \quad \Gamma \Rightarrow B:\psi}{\Gamma \Rightarrow A\&B:(\phi,\psi)} \&R \\
10. \quad \frac{\Gamma\langle\vec{A}:x\rangle \Rightarrow C:\chi_1 \quad \Gamma\langle\vec{B}:y\rangle \Rightarrow C:\chi_2}{\Gamma\langle\vec{A\oplus B}:z\rangle \Rightarrow C:z \rightarrow x.\chi_1;y.\chi_2} \oplus L \\
\frac{\Gamma \Rightarrow A:\phi}{\Gamma \Rightarrow A\oplus B:\iota_1\phi} \oplus R_1 \quad \frac{\Gamma \Rightarrow B:\psi}{\Gamma \Rightarrow A\oplus B:\iota_2\psi} \oplus R_2
\end{array}$$

Figure 3: Additives

$$\begin{array}{c}
11. \quad \frac{\Gamma\langle\vec{A[t/v]}:x\rangle \Rightarrow B:\psi}{\Gamma\langle\vec{\bigwedge vA}:z\rangle \Rightarrow B:\psi\{(z\ t)/x\}} \wedge L \quad \frac{\Gamma \Rightarrow A[a/v]:\phi}{\Gamma \Rightarrow \bigwedge vA:\lambda v\phi} \wedge R^\dagger \\
12. \quad \frac{\Gamma\langle\vec{A[a/v]}:x\rangle \Rightarrow B:\psi}{\Gamma\langle\vec{\bigvee vA}:z\rangle \Rightarrow B:\psi\{\pi_2z/x\}} \vee L^\dagger \quad \frac{\Gamma \Rightarrow A[t/v]:\phi}{\Gamma \Rightarrow \bigvee vA:(t,\phi)} \vee R
\end{array}$$

Figure 4: Quantifiers, where \dagger indicates that there is no a in the conclusion

prosodic phrases and extraction islands. For example, **walks**: $\langle\rangle N \setminus S$ for the subject condition, and **before**: $[\]^{-1}(VP \setminus VP)/VP$ for the adverbial island constraint.

Finally, there are non-linear connectives. The exponentials of Figure 7 have application to sharing. Using the universal exponential, $!$, for which contraction induces island brackets, we can assign a relative pronoun type **that**: $(CN \setminus CN)/(S/!N)$ allowing parasitic extraction such as **paper that John filed without reading**: CN , where parasitic gaps can appear only in (weak) islands, but can be iterated in subislands, for example, **man who the fact that the friends of admire without praising surprises**.⁴

Using the existential exponential, $?$, we can assign a coordinator type **and**: $(?N \setminus N)/N$ allowing iterated coordination as in **John, Bill, Mary and Suzy**: N , or **and**: $(?(S/N) \setminus (S/N))/(S/N)$ for **John likes, Mary dislikes, and Bill hates, London** (iterated right node raising), and so on.

The limited contraction for anaphora, $|$, of Figure 8 also has application to sharing; it can be used for anaphora in an assignment like **it**: $(S \uparrow N) \downarrow (S|N)$ for, e.g., the company_{*i*} said it_{*i*} **flourished**: S , and it can be used for **such that** relativisation in an assignment **such that**: $(CN \setminus CN)/(S|N)$ for, say, **man such that_{*i*} he_{*i*} thinks Mary loves him_{*i*}**: CN .

⁴Morrill (2011b), Chapter 5. In the case that island violations are grammatical, as they are under certain conditions, we assume that the relative pronoun type is not $(CN \setminus CN)/(S/!N)$ but $(CN \setminus CN)/(S/\circ N)$ where \circ is an association and commutation structural modality (Morrill, 1994), Chapter 7. This explains how island violation is possible combinatorially but we leave unanswered the question of how the choice of the relative pronoun type is conditioned by processing factors.

$$\begin{array}{l}
13. \quad \frac{\Gamma \langle \vec{A}: x \rangle \Rightarrow B: \psi}{\Gamma \langle \overline{\square A}: z \rangle \Rightarrow B: \psi \{^{\vee} z/x\}} \square L \quad \frac{\square / \blacksquare \Gamma \Rightarrow A: \phi}{\square / \blacksquare \Gamma \Rightarrow \square A: \wedge \phi} \square R \\
14. \quad \frac{\square / \blacksquare \Gamma \langle \vec{A}: x \rangle \Rightarrow \diamond / \blacklozenge B: \psi}{\square \Gamma \langle \overline{\diamond A}: z \rangle \Rightarrow \diamond / \blacklozenge B: \psi \{^{\cup} z/x\}} \diamond L \quad \frac{\Gamma \Rightarrow A: \phi}{\Gamma \Rightarrow \diamond A: \cap \phi} \diamond R
\end{array}$$

Figure 5: Normal modalities, where $\square / \blacksquare \Gamma$ signifies a structure all the types of which have main connective \square or \blacksquare

$$\begin{array}{l}
15. \quad \frac{\Delta \langle \vec{A}: x \rangle \Rightarrow B: \psi}{\Delta \langle \overline{[\]^{-1} A}: x \rangle \Rightarrow B: \psi} [\]^{-1} L \quad \frac{[\] \Rightarrow A: \phi}{\Gamma \Rightarrow [\]^{-1} A: \phi} [\]^{-1} R \\
16. \quad \frac{\Delta \langle \overline{A}: x \rangle \Rightarrow B: \psi}{\Delta \langle \overline{\langle A \rangle}: x \rangle \Rightarrow B: \psi} \langle \rangle L \quad \frac{\Gamma \Rightarrow A: \phi}{[\] \Rightarrow \langle \rangle A: \phi} \langle \rangle R
\end{array}$$

Figure 6: Bracket modalities

3. Implementation

A computational lexicon and parser integrates the grammatical features of the previous section, and of the remaining connectives, which defines a fragment including:

- the PTQ examples of Dowty et al. (1981), Chapter 7;
- the discontinuity examples of Morrill et al. (2011);
- relativisation, including islands and parasitic gaps;
- constituent coordination, non-constituent coordination, coordination of 'unlike' types, ATBE, and a unitary lexical type analyses of simplex and complex gapping.

The implementation is CatLog2, a categorial parser/theorem-prover comprising 6000 lines of Prolog using backward chaining proof-search in the tree-based hypersequent calculus (Morrill, 2011a), and the focusing of Andreoli (Andreoli, 1992) rather than normalisation as in CatLog (Morrill, 2012). In addition to focusing, the implementation exploits the count-invariance of van Benthem (1991) and Valentín et al. (2013). In this paper we present the second item in the list above.

4. Displacement examples

In this section we analyse the displacement examples of the article Morrill et al. (2011) presenting the displacement calculus.⁵ The first example, however, is modified in view of Morrill and Valentín (2014b). It is a discontinuous idiom (we include the indexation of CatLog, which contains the numeration of the source, within the example displays):

$$(1) \text{ (tdc(43)) } [\mathbf{mary}] + \mathbf{gave} + \mathbf{the} + \mathbf{man} + \mathbf{the} + \mathbf{cold} + \mathbf{shoulder} : S f$$

⁵Note how in the input to CatLog brackets mark islands: single brackets for weak islands such as subjects and double brackets for strong islands such as relative clauses and coordinate structures (Morrill, 2011b), Chapter 5.

$$\begin{array}{c}
17. \quad \frac{\Gamma \langle A: x \rangle \Rightarrow B: \psi}{\Gamma \langle !A: x \rangle \Rightarrow B: \psi} !L \quad \frac{!A_1: x_1, \dots, !A_n: x_n \Rightarrow A: \phi}{!A_1: x_1, \dots, !A_n: x_n \Rightarrow !A: \phi} !R \\
\\
\frac{\Delta \langle !A: x, \Gamma \rangle \Rightarrow B: \psi}{\Delta \langle \Gamma, !A: x \rangle \Rightarrow B: \psi} !P \quad \frac{\Delta \langle \Gamma, !A: x \rangle \Rightarrow B: \psi}{\Delta \langle !A: x, \Gamma \rangle \Rightarrow B: \psi} !P \\
\\
\frac{\Delta \langle !A_0: x_0, \dots, !A_n: x_n, [!A_0: y_0, \dots, !A_n: y_0, \Gamma] \rangle \Rightarrow B: \psi}{\Delta \langle !A_0: x_0, \dots, !A_n: x_n, \Gamma \rangle \Rightarrow B: \psi \{x_0/y_0, \dots, x_n/y_n\}} !C \\
\\
18. \quad \frac{\Gamma \Rightarrow A: \phi}{\Gamma \Rightarrow ?A: [\phi]} ?R \quad \frac{\Gamma \Rightarrow A: \phi \quad \Delta \Rightarrow ?A: \psi}{\Gamma, \Delta \Rightarrow ?A: [\phi|\psi]} ?E
\end{array}$$

Figure 7: Exponentials

$$\begin{array}{c}
19. \quad \frac{\Gamma \Rightarrow A: \phi \quad \Delta \langle \vec{A}: x; \vec{B}: y \rangle \Rightarrow D: \omega}{\Delta \langle \Gamma; \vec{B} | \vec{A}: z \rangle \Rightarrow D: \omega \{ \phi/x, (z \phi)/y \}} !L \\
\\
\frac{\Gamma \langle \vec{B}_0: y_0; \dots; \vec{B}_n: y_n \rangle \Rightarrow D: \omega}{\Gamma \langle \vec{B}_0 | \vec{A}: z_0; \dots; \vec{B}_n | \vec{A}: z_n \rangle \Rightarrow D | A: \lambda x \omega \{ (z_0 \ x)/y_0, \dots, (z_n \ x)/y_n \}} |R
\end{array}$$

Figure 8: Limited contraction for anaphora

Lexical lookup yields:

$$\begin{array}{l}
(2) \ [\blacksquare \text{Nt}(s(f)) : m], \square((\langle \rangle \exists g \text{Nt}(s(g)) \setminus S f) / (\exists a \text{Na} \blacklozenge W[\text{the, cold, shoulder}])) : \\
\quad \hat{\lambda} A \lambda B (\text{Past}(\langle \sim \text{shun } A \rangle B)), \blacksquare \forall n (\text{Nt}(n) / \text{CNn}) : \iota, \square \text{CNs}(m) : \text{man}, \\
\quad \blacksquare W[\text{the, cold, shoulder}] : 0 \Rightarrow S f
\end{array}$$

There is the derivation:

$$\begin{array}{c}
\frac{\frac{\text{CNs}(m) \Rightarrow \text{CNs}(m)}{\square \text{CNs}(m) \Rightarrow \text{CNs}(m)} \square L \quad \frac{\text{Nt}(s(m)) \Rightarrow \text{Nt}(s(m))}{\square \text{Nt}(s(m)) \Rightarrow \text{Nt}(s(m))} \square L}{\text{Nt}(s(m)) / \text{CNs}(m), \square \text{CNs}(m) \Rightarrow \text{Nt}(s(m))} /L \\
\\
\frac{\frac{\text{Nt}(s(m)) / \text{CNs}(m), \square \text{CNs}(m) \Rightarrow \text{Nt}(s(m))}{\forall n (\text{Nt}(n) / \text{CNn}), \square \text{CNs}(m) \Rightarrow \text{Nt}(s(m))} \forall L \quad \frac{\text{Nt}(s(f)) \Rightarrow \text{Nt}(s(f))}{\blacksquare \text{Nt}(s(f)) \Rightarrow \text{Nt}(s(f))} \blacksquare L}{\blacksquare \forall n (\text{Nt}(n) / \text{CNn}), \square \text{CNs}(m) \Rightarrow \text{Nt}(s(m))} \blacksquare L \quad \frac{\text{W}[\text{the, cold, shoulder}] \Rightarrow \text{W}[\text{the, cold, shoulder}]}{\blacksquare \text{W}[\text{the, cold, shoulder}] \Rightarrow \text{W}[\text{the, cold, shoulder}]} \blacksquare L \\
\\
\frac{\blacksquare \forall n (\text{Nt}(n) / \text{CNn}), \square \text{CNs}(m) \Rightarrow \text{Nt}(s(m))}{\blacksquare \forall n (\text{Nt}(n) / \text{CNn}), \square \text{CNs}(m) \Rightarrow \exists a \text{Na}} \exists R \quad \frac{\text{W}[\text{the, cold, shoulder}] \Rightarrow \text{W}[\text{the, cold, shoulder}]}{\blacksquare \text{W}[\text{the, cold, shoulder}] \Rightarrow \text{W}[\text{the, cold, shoulder}]} \blacksquare L \quad \frac{\blacksquare \text{Nt}(s(f)) \Rightarrow \text{Nt}(s(f))}{\blacksquare \text{Nt}(s(f)) \Rightarrow \langle \rangle \exists g \text{Nt}(s(g))} \exists R \\
\\
\frac{\blacksquare \forall n (\text{Nt}(n) / \text{CNn}), \square \text{CNs}(m) \Rightarrow \exists a \text{Na} \quad \blacksquare \text{W}[\text{the, cold, shoulder}] \Rightarrow \text{W}[\text{the, cold, shoulder}]}{\blacksquare \forall n (\text{Nt}(n) / \text{CNn}), \square \text{CNs}(m), \blacksquare \text{W}[\text{the, cold, shoulder}]} \blacklozenge R \quad \frac{\blacksquare \text{Nt}(s(f)) \Rightarrow \langle \rangle \exists g \text{Nt}(s(g)) \quad S f \Rightarrow S f}{\blacksquare \text{Nt}(s(f)), \langle \rangle \exists g \text{Nt}(s(g)) \setminus S f \Rightarrow S f} \langle \rangle R \\
\\
\frac{\blacksquare \forall n (\text{Nt}(n) / \text{CNn}), \square \text{CNs}(m), \blacksquare \text{W}[\text{the, cold, shoulder}]}{\blacksquare \forall n (\text{Nt}(n) / \text{CNn}), \square(\langle \rangle \exists g \text{Nt}(s(g)) \setminus S f) / (\exists a \text{Na} \blacklozenge W[\text{the, cold, shoulder}])}, \blacksquare \forall n (\text{Nt}(n) / \text{CNn}), \square \text{CNs}(m), \blacksquare \text{W}[\text{the, cold, shoulder}] \Rightarrow S f} /L \\
\\
\frac{\blacksquare \forall n (\text{Nt}(n) / \text{CNn}), \square(\langle \rangle \exists g \text{Nt}(s(g)) \setminus S f) / (\exists a \text{Na} \blacklozenge W[\text{the, cold, shoulder}])}{\blacksquare \forall n (\text{Nt}(n) / \text{CNn}), \square(\langle \rangle \exists g \text{Nt}(s(g)) \setminus S f) / (\exists a \text{Na} \blacklozenge W[\text{the, cold, shoulder}])}, \blacksquare \forall n (\text{Nt}(n) / \text{CNn}), \square \text{CNs}(m), \blacksquare \text{W}[\text{the, cold, shoulder}] \Rightarrow S f} \square L
\end{array}$$

This delivers semantics:

$$(3) \ (\text{Past}(\langle \sim \text{shun } (\iota \sim \text{man}) \rangle m))$$

Similarly:

(4) (tdc(4343)) **[mary]+gave+john+the+cold+shoulder** : Sf

Lexical lookup yields:

(5) $[\blacksquare Nt(s(f)) : m], \square((\langle \rangle \exists g Nt(s(g)) \setminus S f) / (\exists a Na \bullet W[the, cold, shoulder])) : \wedge \lambda A \lambda B (Past ((\sim shun A) B)), \blacksquare Nt(s(m)) : j, \blacksquare W[the, cold, shoulder] : 0 \Rightarrow Sf$

There is the derivation:

$$\begin{array}{c}
 \frac{\frac{\frac{\frac{\frac{\blacksquare Nt(s(m)) \Rightarrow Nt(s(m))}{\blacksquare L}}{\blacksquare Nt(s(m)) \Rightarrow Nt(s(m))}{\exists R}}{\blacksquare Nt(s(m)) \Rightarrow \exists a Na}}{\blacksquare Nt(s(m)), \blacksquare W[the, cold, shoulder] \Rightarrow \exists a Na \bullet W[the, cold, shoulder]} \bullet R}{\frac{\frac{\frac{\frac{\frac{\frac{\frac{\blacksquare Nt(s(f)) \Rightarrow Nt(s(f))}{\blacksquare L}}{\blacksquare Nt(s(f)) \Rightarrow Nt(s(f))}{\exists R}}{\blacksquare Nt(s(f)) \Rightarrow \exists g Nt(s(g))}{\langle \rangle R}}{\blacksquare Nt(s(f)) \Rightarrow \langle \rangle \exists g Nt(s(g))} \setminus L}{\blacksquare Nt(s(f)), \langle \rangle \exists g Nt(s(g)) \setminus S f \Rightarrow S f} / L}{\frac{\blacksquare Nt(s(f)), \langle \rangle \exists g Nt(s(g)) \setminus S f}{\blacksquare Nt(s(f)), \langle \rangle \exists g Nt(s(g)) \setminus S f / (\exists a Na \bullet W[the, cold, shoulder])} \bullet L}}{\frac{\blacksquare Nt(s(m)), \blacksquare W[the, cold, shoulder] \Rightarrow S f}{\blacksquare Nt(s(m)), \blacksquare W[the, cold, shoulder] \Rightarrow S f} \square L}}{\blacksquare Nt(s(f)), \square((\langle \rangle \exists g Nt(s(g)) \setminus S f) / (\exists a Na \bullet W[the, cold, shoulder])), \blacksquare Nt(s(m)), \blacksquare W[the, cold, shoulder] \Rightarrow S f} / L
 \end{array}$$

This delivers semantics:

(6) $(Past ((\sim shun j) m))$

The next example has medial quantification:

(7) (tdc(47)) **[john]+gave+every+book+to+mary** : Sf

Lexical lookup yields:

(8) $[\blacksquare Nt(s(m)) : j], \square((\langle \rangle \exists a Na \setminus S f) / (\exists b Nb \bullet P P t o)) : \wedge \lambda A \lambda B (Past (((\sim give \pi_2 A) \pi_1 A) B)), \blacksquare \forall g (\forall f ((S f \uparrow Nt(s(g))) \downarrow S f) / C N s(g)) : \lambda C \lambda D \forall E [(C E) \rightarrow (D E)], \square C N s(n) : book, \blacksquare ((P P t o / \exists a Na) \square \forall n ((\langle \rangle N n \setminus S i) / (\langle \rangle N n \setminus S b))) : \lambda F F, \blacksquare Nt(s(f)) : m \Rightarrow S f$

There is the derivation:



This delivers semantics:

$$(9) \forall C[(\sim \text{book } C) \rightarrow (\text{Past } (((\sim \text{give } m) C) j))]$$

The following example has subordinate clause existential quantification, exhibiting de re/de dicto ambiguity:

$$(10) (\text{tdc}(50)) [\text{mary}] + \text{thinks} + [\text{someone}] + \text{left} : S f$$

Lexical lookup yields:

$$(11) [\blacksquare Nt(s(f)) : m], \Box((\langle \rangle \exists g Nt(s(g)) \setminus S f) / (C P that \sqcup \Box S f)) : \hat{\lambda} A \lambda B (Pres ((\sim think A) B)), \\ [\Box \forall f ((S f \uparrow \blacksquare \forall g Nt(g)) \downarrow S f) : \hat{\lambda} C \exists D [(\sim person D) \wedge (C D)], \Box(\langle \rangle \exists g Nt(s(g)) \setminus S f) : \\ \hat{\lambda} E (Pres (\sim leave E)) \Rightarrow S f$$

There is the de re derivation:

$$\begin{array}{c} \frac{\frac{\frac{\frac{\Box Nt(s(A)) \Rightarrow Nt(s(A))}{\Box L}}{\forall g Nt(g) \Rightarrow Nt(s(A))}{\blacksquare L}}{\blacksquare \forall g Nt(g) \Rightarrow Nt(s(A))}{\exists R}}{\blacksquare \forall g Nt(g) \Rightarrow \langle \rangle \exists g Nt(s(g))}{\langle R}} \\ \frac{\frac{\frac{\frac{\blacksquare \forall g Nt(g), \langle \rangle \exists g Nt(s(g)) \setminus S f \Rightarrow S f}{\Box L}}{\blacksquare \forall g Nt(g), \Box(\langle \rangle \exists g Nt(s(g)) \setminus S f) \Rightarrow S f}{\Box R}}{\blacksquare \forall g Nt(g), \Box(\langle \rangle \exists g Nt(s(g)) \setminus S f) \Rightarrow \Box S f}{\Box R}}}{\blacksquare \forall g Nt(g), \Box(\langle \rangle \exists g Nt(s(g)) \setminus S f) \Rightarrow C P that \sqcup \Box S f}{\Box R}} \\ \frac{\frac{\frac{\frac{\frac{\frac{\Box Nt(s(f)) \Rightarrow Nt(s(f))}{\blacksquare L}}{\blacksquare Nt(s(f)) \Rightarrow \langle \rangle \exists g Nt(s(g))}{\exists R}}{\blacksquare Nt(s(f)) \Rightarrow \langle \rangle \exists g Nt(s(g)) \setminus S f}{\langle R}}}{\blacksquare Nt(s(f)), \langle \rangle \exists g Nt(s(g)) \setminus S f \Rightarrow S f}{\Box L}}}{\blacksquare Nt(s(f)), \langle \rangle \exists g Nt(s(g)) \setminus S f \Rightarrow S f}{\Box L}} \\ \frac{\frac{\frac{\blacksquare Nt(s(f)), \langle \rangle \exists g Nt(s(g)) \setminus S f \Rightarrow S f}{\Box L}}{\blacksquare Nt(s(f)), \Box(\langle \rangle \exists g Nt(s(g)) \setminus S f) \Rightarrow S f}{\Box L}}}{\blacksquare Nt(s(f)), \Box(\langle \rangle \exists g Nt(s(g)) \setminus S f) \Rightarrow S f \uparrow \blacksquare \forall g Nt(g)}{\uparrow R}} \\ \frac{\frac{\frac{\frac{\blacksquare Nt(s(f)), \Box(\langle \rangle \exists g Nt(s(g)) \setminus S f) \Rightarrow S f \uparrow \blacksquare \forall g Nt(g)}{\Box L}}{\blacksquare Nt(s(f)), \Box((\langle \rangle \exists g Nt(s(g)) \setminus S f) / (C P that \sqcup \Box S f))}{\Box L}}}{\blacksquare Nt(s(f)), \Box((\langle \rangle \exists g Nt(s(g)) \setminus S f) / (C P that \sqcup \Box S f)), [\blacksquare \forall g Nt(g), \Box(\langle \rangle \exists g Nt(s(g)) \setminus S f) \Rightarrow S f]}{\Box L}} \\ \frac{\frac{\frac{\blacksquare Nt(s(f)), \Box((\langle \rangle \exists g Nt(s(g)) \setminus S f) / (C P that \sqcup \Box S f)), [\blacksquare \forall g Nt(g), \Box(\langle \rangle \exists g Nt(s(g)) \setminus S f) \Rightarrow S f]}{\Box L}}{\blacksquare Nt(s(f)), \Box((\langle \rangle \exists g Nt(s(g)) \setminus S f) / (C P that \sqcup \Box S f)), [\forall f((S f \uparrow \blacksquare \forall g Nt(g)) \downarrow S f)], \Box(\langle \rangle \exists g Nt(s(g)) \setminus S f) \Rightarrow S f}{\Box L}}}{\blacksquare Nt(s(f)), \Box((\langle \rangle \exists g Nt(s(g)) \setminus S f) / (C P that \sqcup \Box S f)), [\Box \forall f((S f \uparrow \blacksquare \forall g Nt(g)) \downarrow S f)], \Box(\langle \rangle \exists g Nt(s(g)) \setminus S f) \Rightarrow S f}{\Box L}} \end{array}$$

This delivers semantics:

$$(12) \exists B[(\sim person B) \wedge (Pres ((\sim think \hat{\lambda} (Pres (\sim leave B)) m)))]$$

And the de dicto derivation:

$$\begin{array}{c}
\frac{}{\boxed{Nt(s(A))} \Rightarrow Nt(s(A))} \\
\frac{}{\boxed{\forall g Nt(g)} \Rightarrow Nt(s(A))} \quad \forall L \\
\frac{}{\boxed{\blacksquare \forall g Nt(g)} \Rightarrow Nt(s(A))} \quad \blacksquare L \\
\frac{}{\boxed{\blacksquare \forall g Nt(g)} \Rightarrow \boxed{\exists g Nt(s(g))}} \quad \exists R \\
\frac{}{\boxed{\blacksquare \forall g Nt(g)} \Rightarrow \boxed{\langle \exists g Nt(s(g)) \rangle}} \quad \langle \rangle R \\
\frac{}{\boxed{\blacksquare \forall g Nt(g)} \Rightarrow \boxed{\langle \exists g Nt(s(g)) \rangle} \quad \boxed{Sf} \Rightarrow Sf} \quad \backslash L \\
\frac{}{\boxed{\blacksquare \forall g Nt(g)}, \boxed{\langle \exists g Nt(s(g)) \rangle Sf} \Rightarrow Sf} \quad \square L \\
\frac{}{\boxed{\blacksquare \forall g Nt(g)}, \boxed{\square(\langle \exists g Nt(s(g)) \rangle Sf)} \Rightarrow Sf} \quad \square L \\
\frac{}{\boxed{1}, \boxed{\square(\langle \exists g Nt(s(g)) \rangle Sf)} \Rightarrow Sf \uparrow \boxed{\blacksquare \forall g Nt(g)} \quad \boxed{Sf} \Rightarrow Sf} \quad \uparrow R \\
\frac{}{\boxed{\blacksquare \forall g Nt(g)}, \boxed{\langle \exists g Nt(s(g)) \rangle Sf} \Rightarrow Sf} \quad \downarrow L \\
\frac{}{\boxed{\blacksquare \forall g Nt(g)}, \boxed{\square(\langle \exists g Nt(s(g)) \rangle Sf)} \Rightarrow Sf} \quad \square L \\
\frac{}{\boxed{\square \forall f((Sf \uparrow \blacksquare \forall g Nt(g)) \downarrow Sf)}, \boxed{\square(\langle \exists g Nt(s(g)) \rangle Sf)} \Rightarrow Sf} \quad \square L \\
\frac{}{\boxed{\square \forall f((Sf \uparrow \blacksquare \forall g Nt(g)) \downarrow Sf)}, \boxed{\square(\langle \exists g Nt(s(g)) \rangle Sf)} \Rightarrow \square Sf} \quad \square R \\
\frac{}{\boxed{\square \forall f((Sf \uparrow \blacksquare \forall g Nt(g)) \downarrow Sf)}, \boxed{\square(\langle \exists g Nt(s(g)) \rangle Sf)} \Rightarrow \boxed{CThat \sqcup \square Sf}} \quad \sqcup R \\
\frac{}{\boxed{\blacksquare Nt(s(f))}, \boxed{\langle \exists g Nt(s(g)) \rangle Sf} \Rightarrow Sf} \quad \blacksquare L \\
\frac{}{\boxed{\blacksquare Nt(s(f))} \Rightarrow Nt(s(f))} \quad \blacksquare L \\
\frac{}{\boxed{\blacksquare Nt(s(f))} \Rightarrow \boxed{\exists g Nt(s(g))}} \quad \exists R \\
\frac{}{\boxed{\blacksquare Nt(s(f))} \Rightarrow \boxed{\langle \exists g Nt(s(g)) \rangle} \quad \boxed{Sf} \Rightarrow Sf} \quad \langle \rangle R \\
\frac{}{\boxed{\blacksquare Nt(s(f))}, \boxed{\langle \exists g Nt(s(g)) \rangle Sf} \Rightarrow Sf} \quad \backslash L \\
\frac{}{\boxed{\blacksquare Nt(s(f))}, \boxed{\langle \exists g Nt(s(g)) \rangle Sf} \Rightarrow Sf} \quad /L \\
\frac{}{\boxed{\blacksquare Nt(s(f))}, \boxed{\langle \exists g Nt(s(g)) \rangle Sf} \Rightarrow Sf} \quad \square L \\
\frac{}{\boxed{\blacksquare Nt(s(f))}, \boxed{\square(\langle \exists g Nt(s(g)) \rangle Sf) / (CThat \sqcup \square Sf)}, \boxed{\square \forall f((Sf \uparrow \blacksquare \forall g Nt(g)) \downarrow Sf)}, \boxed{\square(\langle \exists g Nt(s(g)) \rangle Sf)} \Rightarrow Sf} \quad \square L \\
\frac{}{\boxed{\blacksquare Nt(s(f))}, \boxed{\square(\langle \exists g Nt(s(g)) \rangle Sf) / (CThat \sqcup \square Sf)}, \boxed{\square \forall f((Sf \uparrow \blacksquare \forall g Nt(g)) \downarrow Sf)}, \boxed{\square(\langle \exists g Nt(s(g)) \rangle Sf)} \Rightarrow Sf} \quad \square L
\end{array}$$

This delivers semantics:

$$(13) (Pres ((\sim think \hat{\exists} D[(\sim person D) \wedge (Pres (\sim leave D))]) m))$$

The next example exhibits classical quantifier scope ambiguity:

$$(14) (tdc(53)) [\mathbf{everyone}] + \mathbf{loves} + \mathbf{someone} : Sf$$

There is the subject wide scope reading (cf. everyone loves their (respective) mother) and the object wide scope reading (cf. everyone loves (one and) the (same) queen). Lexical lookup yields:

$$(15) [\square \forall f((Sf \uparrow \forall g Nt(g)) \downarrow Sf) : \hat{\lambda} A \forall B[(\sim person B) \rightarrow (A B)], \square(\langle \exists g Nt(s(g)) \rangle Sf) / \exists a Na) : \hat{\lambda} C \lambda D (Pres ((\sim love C) D)), \square \forall f((Sf \uparrow \blacksquare \forall g Nt(g)) \downarrow Sf) : \hat{\lambda} E \exists F[(\sim person F) \wedge (E F)] \Rightarrow Sf$$

There is the subject wide scope derivation as follows in which the subject quantifier is processed closest to the root:

This delivers the semantics:

$$(17) \exists B[(\sim person B) \wedge \forall E[(\sim person E) \rightarrow (Pres ((\sim love B) E))]]$$

The next example is of medial relativisation:

$$(18) (tdc(54)) \mathbf{dog}+[[\mathbf{that}+[\mathbf{mary}]+\mathbf{saw}+\mathbf{today}]] : CNs(n)$$

But it is not analysed as in Morrill et al. (2011) but as in Morrill (2011b), Chapter 5. Note the double brackets for the strong island relative clause. The lexical lookup yields:

$$(19) \square CNs(n) : \mathbf{dog}, [[\blacksquare \forall n([\]^{-1} [\]^{-1} (CNn \setminus CNn) / \blacksquare ((\langle \rangle Nt(n) \square! \blacksquare Nt(n)) \setminus S f)) : \lambda A \lambda B \lambda C [(B C) \wedge (A C)], [\blacksquare Nt(s(f)) : m], \square ((\langle \rangle \exists a Na \setminus S f) / (\exists a Na \oplus CPthat)) : \wedge \lambda D \lambda E (Past ((D \rightarrow F. (\sim see F); G. (\sim see G)) E)), \square \forall a \forall f ((\langle \rangle Na \setminus S f) \setminus (\langle \rangle Na \setminus S f)) : \wedge \lambda H \lambda I ((\sim today (H I)))] \Rightarrow CNs(n)$$

There is the derivation in Figure 9. This delivers semantics:

$$(20) \lambda C[(\sim dog C) \wedge (\sim today (Past ((\sim see C) m)))]$$

The next example is of VP ellipsis:

$$(21) (tdc(58a)) [\mathbf{john}]+\mathbf{slept}+\mathbf{before}+[\mathbf{mary}]+\mathbf{did} : S f$$

$$(22) [\blacksquare Nt(s(m)) : j], \square (\langle \rangle \exists g Nt(s(g)) \setminus S f) : \wedge \lambda A (Past (\sim sleep A)), \blacksquare (\forall a \forall f ((\langle \rangle Na \setminus S f) \setminus (\langle \rangle Na \setminus S f)) / S f) : \lambda B \lambda C \lambda D ((before B) (C D)), [\blacksquare Nt(s(f)) : m], \blacksquare \forall a \forall g \forall b \forall h (((\langle \rangle Na \setminus S g) \uparrow (\langle \rangle Nb \setminus S h)) / (\exists c \langle \rangle Nc \setminus S f)) \setminus ((\langle \rangle Na \setminus S g) \uparrow (\langle \rangle Nb \setminus S h))) : \lambda E \lambda F ((E F) F) \Rightarrow S f$$

There is the derivation in Figure 10. This delivers the semantics:

$$(23) ((before (Past (\sim sleep m))) (Past (\sim sleep j)))$$

In tdc(64) there is medial pied-piping:

$$(24) (tdc(64)) \mathbf{mountain}+[[\mathbf{the}+\mathbf{painting}+\mathbf{of}+\mathbf{which}+\mathbf{by}+\mathbf{cezanne}+[\mathbf{john}]+\mathbf{sold}+\mathbf{for}+\mathbf{tenmilliondollars}]] : CNs(n)$$

Lexical lookup yields:

$$(25) \square CNs(n) : \mathbf{mountain}, [[\blacksquare \forall n(Nt(n) / CNn) : \iota, \square (CNs(n) / PPof) : \wedge \lambda A ((\sim of A) \sim painting), \square ((\forall n (CNn \setminus CNn) / \blacksquare \exists b Nb) \& (PPof / \exists a Na)) : \wedge (\sim of, \lambda BB), \blacksquare \forall n \forall m ((Nt(n) \uparrow Nt(m)) \downarrow ([]^{-1} []^{-1} (CNm \setminus CNm) / \blacksquare ((\langle \rangle Nt(n) \square! \blacksquare Nt(n)) \setminus S f))) : \lambda C \lambda D \lambda E \lambda F [(E F) \wedge (D (C F))], \square (\forall n (CNn \setminus CNn) / \exists a Na) : \wedge \lambda G \lambda H ((\sim by G) H), \blacksquare Nt(s(m)) : c, [\blacksquare Nt(s(m)) : j], \square ((\langle \rangle \exists a Na \setminus S f) / (\exists b Nb \bullet PPfor)) : \wedge \lambda I \lambda J (Past (((\sim sell \pi_2 I) \pi_1 I) J)), \blacksquare (PPfor / \exists a Na) : \lambda KK, \square Nt(s(n)) : \mathbf{tenmilliondollars}]] \Rightarrow CNs(n)$$

There is the derivation:



Figure 10: Derivation of **John slept before Mary did**

This delivers semantics:

$$(26) \lambda D[(\sim mountain D) \wedge (Past (((\sim sell \sim tenmilliondollars) (\iota ((\sim by c) ((\sim of D) \sim painting)))))) j))]$$

In appositive relativisation a relative clause, marked off by a prosodic phrase, modifies a full noun phrase:

$$(27) (tdc(67)) \mathbf{[john+[who+jogs]]+sneezed} : Sf$$

The lexical lookup yields:

$$(28) [\blacksquare Nt(s(m)) : j, \mathbf{[\blacksquare \forall h \forall n ([\blacksquare]^{-1} [\blacksquare]^{-1} (Nt(n) \backslash ((Sh \uparrow Nt(n)) \downarrow Sh)) / \blacksquare ((\langle \rangle Nt(n) \uparrow \blacksquare Nt(n)) \backslash Sf))} : \lambda A \lambda B \lambda C [(A B) \wedge (C B)], \square (\langle \rangle \exists g Nt(s(g)) \backslash Sf) : \hat{\lambda} D (Pres (\sim jog D))], \square (\langle \rangle \exists g Nt(s(g)) \backslash Sf) : \hat{\lambda} E (Past (\sim sneeze E)) \Rightarrow Sf]$$

There is the derivation:

$$\begin{array}{c}
\frac{}{Nt(s(m)) \Rightarrow Nt(s(m))} \\
\frac{}{Nt(s(m)) \Rightarrow \exists g Nt(s(g))} \exists R \\
\frac{[Nt(s(m))] \Rightarrow \langle \rangle \exists g Nt(s(g)) \quad [Sf] \Rightarrow Sf}{[Nt(s(m))] \Rightarrow \langle \rangle \exists g Nt(s(g)) \backslash Sf} \langle R \\
\frac{[Nt(s(m))] \Rightarrow \langle \rangle \exists g Nt(s(g)) \backslash Sf}{[Nt(s(m))] \Rightarrow \square (\langle \rangle \exists g Nt(s(g)) \backslash Sf)} \square L \\
\frac{[Nt(s(m))] \Rightarrow \square (\langle \rangle \exists g Nt(s(g)) \backslash Sf)}{\langle \rangle Nt(s(m)), \square (\langle \rangle \exists g Nt(s(g)) \backslash Sf) \Rightarrow Sf} \langle L \\
\frac{\langle \rangle Nt(s(m)), \square (\langle \rangle \exists g Nt(s(g)) \backslash Sf) \Rightarrow Sf}{\langle \rangle Nt(s(m)) \uparrow \blacksquare Nt(s(m))} \uparrow R \\
\frac{\langle \rangle Nt(s(m)) \uparrow \blacksquare Nt(s(m)) \quad [Sf] \Rightarrow Sf}{\square (\langle \rangle \exists g Nt(s(g)) \backslash Sf) \Rightarrow (\langle \rangle Nt(s(m)) \uparrow \blacksquare Nt(s(m))) \backslash Sf} \square R \\
\frac{\square (\langle \rangle \exists g Nt(s(g)) \backslash Sf) \Rightarrow (\langle \rangle Nt(s(m)) \uparrow \blacksquare Nt(s(m))) \backslash Sf}{\square (\langle \rangle \exists g Nt(s(g)) \backslash Sf) \Rightarrow \blacksquare (\langle \rangle Nt(s(m)) \uparrow \blacksquare Nt(s(m))) \backslash Sf} \blacksquare R \\
\frac{\square (\langle \rangle \exists g Nt(s(g)) \backslash Sf) \Rightarrow \blacksquare (\langle \rangle Nt(s(m)) \uparrow \blacksquare Nt(s(m))) \backslash Sf}{[\blacksquare Nt(s(m)), \square (\langle \rangle \exists g Nt(s(g)) \backslash Sf) \backslash Sf] \Rightarrow Sf} \square L \\
\frac{[\blacksquare Nt(s(m)), \square (\langle \rangle \exists g Nt(s(g)) \backslash Sf) \backslash Sf] \Rightarrow Sf}{[\blacksquare Nt(s(m)), \blacksquare Nt(s(m)) \backslash Sf] \Rightarrow Sf} \blacksquare L \\
\frac{[\blacksquare Nt(s(m)), \blacksquare Nt(s(m)) \backslash Sf] \Rightarrow Sf}{[\blacksquare Nt(s(m)), \blacksquare Nt(s(m)) \backslash (\langle \rangle Nt(s(m)) \uparrow \blacksquare Nt(s(m))) \backslash Sf] \Rightarrow Sf} \square L \\
\frac{[\blacksquare Nt(s(m)), \blacksquare Nt(s(m)) \backslash (\langle \rangle Nt(s(m)) \uparrow \blacksquare Nt(s(m))) \backslash Sf] \Rightarrow Sf}{[\blacksquare Nt(s(m)), \blacksquare Nt(s(m)) \backslash (\blacksquare Nt(s(m)) \uparrow \blacksquare Nt(s(m))) \backslash Sf] \Rightarrow Sf} \square L \\
\frac{[\blacksquare Nt(s(m)), \blacksquare Nt(s(m)) \backslash (\blacksquare Nt(s(m)) \uparrow \blacksquare Nt(s(m))) \backslash Sf] \Rightarrow Sf}{[\blacksquare Nt(s(m)), \blacksquare Nt(s(m)) \backslash ([\blacksquare]^{-1} [\blacksquare]^{-1} (Nt(s(m)) \backslash (\langle \rangle Nt(s(m)) \uparrow \blacksquare Nt(s(m))) \backslash Sf)) \backslash Sf] \Rightarrow Sf} \square L \\
\frac{[\blacksquare Nt(s(m)), \blacksquare Nt(s(m)) \backslash ([\blacksquare]^{-1} [\blacksquare]^{-1} (Nt(s(m)) \backslash (\langle \rangle Nt(s(m)) \uparrow \blacksquare Nt(s(m))) \backslash Sf)) \backslash Sf] \Rightarrow Sf}{[\blacksquare Nt(s(m)), \blacksquare Nt(s(m)) \backslash ([\blacksquare]^{-1} [\blacksquare]^{-1} (Nt(s(m)) \backslash (\langle \rangle Nt(s(m)) \uparrow \blacksquare Nt(s(m))) \backslash Sf)) \backslash (\langle \rangle Nt(s(m)) \uparrow \blacksquare Nt(s(m))) \backslash Sf] \Rightarrow Sf} \square L \\
\frac{[\blacksquare Nt(s(m)), \blacksquare Nt(s(m)) \backslash ([\blacksquare]^{-1} [\blacksquare]^{-1} (Nt(s(m)) \backslash (\langle \rangle Nt(s(m)) \uparrow \blacksquare Nt(s(m))) \backslash Sf)) \backslash (\langle \rangle Nt(s(m)) \uparrow \blacksquare Nt(s(m))) \backslash Sf] \Rightarrow Sf}{[\blacksquare Nt(s(m)), \blacksquare Nt(s(m)) \backslash (\forall n ([\blacksquare]^{-1} [\blacksquare]^{-1} (Nt(n) \backslash ((Sf \uparrow Nt(n)) \downarrow Sf)) / \blacksquare (\langle \rangle Nt(n) \uparrow \blacksquare Nt(n)) \backslash Sf)) \backslash (\langle \rangle \exists g Nt(s(g)) \backslash Sf)] \Rightarrow Sf} \forall L \\
\frac{[\blacksquare Nt(s(m)), \blacksquare Nt(s(m)) \backslash (\forall n ([\blacksquare]^{-1} [\blacksquare]^{-1} (Nt(n) \backslash ((Sf \uparrow Nt(n)) \downarrow Sf)) / \blacksquare (\langle \rangle Nt(n) \uparrow \blacksquare Nt(n)) \backslash Sf)) \backslash (\langle \rangle \exists g Nt(s(g)) \backslash Sf)] \Rightarrow Sf}{[\blacksquare Nt(s(m)), \blacksquare Nt(s(m)) \backslash (\forall h \forall n ([\blacksquare]^{-1} [\blacksquare]^{-1} (Nt(n) \backslash ((Sh \uparrow Nt(n)) \downarrow Sh)) / \blacksquare (\langle \rangle Nt(n) \uparrow \blacksquare Nt(n)) \backslash Sf)) \backslash (\langle \rangle \exists g Nt(s(g)) \backslash Sf)] \Rightarrow Sf} \forall L \\
\frac{[\blacksquare Nt(s(m)), \blacksquare Nt(s(m)) \backslash (\forall h \forall n ([\blacksquare]^{-1} [\blacksquare]^{-1} (Nt(n) \backslash ((Sh \uparrow Nt(n)) \downarrow Sh)) / \blacksquare (\langle \rangle Nt(n) \uparrow \blacksquare Nt(n)) \backslash Sf)) \backslash (\langle \rangle \exists g Nt(s(g)) \backslash Sf)] \Rightarrow Sf}{[\blacksquare Nt(s(m)), \blacksquare Nt(s(m)) \backslash (\forall h \forall n ([\blacksquare]^{-1} [\blacksquare]^{-1} (Nt(n) \backslash ((Sh \uparrow Nt(n)) \downarrow Sh)) / \blacksquare (\langle \rangle Nt(n) \uparrow \blacksquare Nt(n)) \backslash Sf)) \backslash (\langle \rangle \exists g Nt(s(g)) \backslash Sf)] \Rightarrow Sf} \blacksquare L \\
\frac{[\blacksquare Nt(s(m)), \blacksquare Nt(s(m)) \backslash (\forall h \forall n ([\blacksquare]^{-1} [\blacksquare]^{-1} (Nt(n) \backslash ((Sh \uparrow Nt(n)) \downarrow Sh)) / \blacksquare (\langle \rangle Nt(n) \uparrow \blacksquare Nt(n)) \backslash Sf)) \backslash (\langle \rangle \exists g Nt(s(g)) \backslash Sf)] \Rightarrow Sf}{[\blacksquare Nt(s(m)), \blacksquare Nt(s(m)) \backslash (\blacksquare \forall h \forall n ([\blacksquare]^{-1} [\blacksquare]^{-1} (Nt(n) \backslash ((Sh \uparrow Nt(n)) \downarrow Sh)) / \blacksquare (\langle \rangle Nt(n) \uparrow \blacksquare Nt(n)) \backslash Sf)) \backslash (\langle \rangle \exists g Nt(s(g)) \backslash Sf)] \Rightarrow Sf} \blacksquare L
\end{array}$$

This delivers semantics:

$$(29) [(Pres (\sim jog j)) \wedge (Past (\sim sneeze j))]$$

There follow four placements of a parenthetical adverbial within a sentence all derived from the same types and all assigning the same semantics. First:

$$(30) (tdc(70a)) \mathbf{fortunately+[john]+has+perseverance} : Sf$$

$$(31) \square \forall f (\sim Sf \downarrow Sf) : fortunately, [\blacksquare Nt(s(m)) : j], \square (\langle \rangle \exists g Nt(s(g)) \backslash Sf) / \exists a N a : \hat{\lambda} A \lambda B (Pres ((\sim have A) B)), \square (Nt(s(n)) \& CNs(n)) : \hat{\lambda} ((gen \sim perseverance), \sim perseverance) \Rightarrow Sf$$

$$\begin{array}{c}
\frac{\frac{\frac{Nt(s(n)) \Rightarrow Nt(s(n))}{\square(Nt(s(n)) \& CNs(n)) \Rightarrow Nt(s(n))} \&L}{\square(Nt(s(n)) \& CNs(n)) \Rightarrow Nt(s(n))} \square L}{\square(Nt(s(n)) \& CNs(n)) \Rightarrow \exists a Na} \exists R \\
\frac{\frac{\frac{Nt(s(m)) \Rightarrow Nt(s(m))}{\blacksquare Nt(s(m)) \Rightarrow Nt(s(m))} \blacksquare L}{\blacksquare Nt(s(m)) \Rightarrow \exists g Nt(s(g))} \exists R}{\blacksquare Nt(s(m)) \Rightarrow (\langle \rangle \exists g Nt(s(g)))} \langle \rangle R \\
\frac{\frac{\frac{\frac{\frac{\frac{\frac{Sf \Rightarrow Sf}{\square(Nt(s(n)) \& CNs(n)) \Rightarrow Nt(s(n))} \&L}{\square(Nt(s(n)) \& CNs(n)) \Rightarrow Nt(s(n))} \square L}{\square(Nt(s(n)) \& CNs(n)) \Rightarrow \exists a Na} \exists R}{\blacksquare Nt(s(m)), (\langle \rangle \exists g Nt(s(g)) \setminus Sf} \Rightarrow Sf} \setminus L}{\blacksquare Nt(s(m)), (\langle \rangle \exists g Nt(s(g)) \setminus Sf / \exists a Na, \square(Nt(s(n)) \& CNs(n)) \Rightarrow Sf} /L}{\blacksquare Nt(s(m)), \square((\langle \rangle \exists g Nt(s(g)) \setminus Sf) / \exists a Na), \square(Nt(s(n)) \& CNs(n)) \Rightarrow Sf} \square L}{\blacksquare Nt(s(m)), \square((\langle \rangle \exists g Nt(s(g)) \setminus Sf) / \exists a Na), \square(Nt(s(n)) \& CNs(n)) \Rightarrow Sf} \square L}{\blacksquare Nt(s(m)), 1, \square((\langle \rangle \exists g Nt(s(g)) \setminus Sf) / \exists a Na), \square(Nt(s(n)) \& CNs(n)) \Rightarrow \sim Sf} \sim R}{1, \blacksquare Nt(s(m)), \square((\langle \rangle \exists g Nt(s(g)) \setminus Sf) / \exists a Na), \square(Nt(s(n)) \& CNs(n)) \Rightarrow \sim Sf} \downarrow L}{\sim Sf \setminus Sf, [\blacksquare Nt(s(m)), \square((\langle \rangle \exists g Nt(s(g)) \setminus Sf) / \exists a Na), \square(Nt(s(n)) \& CNs(n)) \Rightarrow Sf} \forall L}{\forall f(\sim Sf \setminus Sf), [\blacksquare Nt(s(m)), \square((\langle \rangle \exists g Nt(s(g)) \setminus Sf) / \exists a Na), \square(Nt(s(n)) \& CNs(n)) \Rightarrow Sf} \square L}{\square \forall f(\sim Sf \setminus Sf), [\blacksquare Nt(s(m)), \square((\langle \rangle \exists g Nt(s(g)) \setminus Sf) / \exists a Na), \square(Nt(s(n)) \& CNs(n)) \Rightarrow Sf} \square L}
\end{array}$$

(32) (\sim fortunately (*Pres* (\sim have (*gen* \sim perseverance))) *j*))

Second:

(33) (tdc(70b)) [**john**]+**fortunately**+**has**+**perseverance** : *Sf*

(34) [$\blacksquare Nt(s(m)) : j$], $\square \forall f(\sim Sf \setminus Sf) :$ fortunately, $\square((\langle \rangle \exists g Nt(s(g)) \setminus Sf) / \exists a Na) :$
 $\hat{\lambda}A\lambda B(Pres((\sim$ have A) $B)), \square(Nt(s(n)) \& CNs(n)) : \hat{\sim}((gen \sim perseverance), \sim perseverance) \Rightarrow Sf$

$$\begin{array}{c}
\frac{\frac{\frac{Nt(s(n)) \Rightarrow Nt(s(n))}{\square(Nt(s(n)) \& CNs(n)) \Rightarrow Nt(s(n))} \&L}{\square(Nt(s(n)) \& CNs(n)) \Rightarrow Nt(s(n))} \square L}{\square(Nt(s(n)) \& CNs(n)) \Rightarrow \exists a Na} \exists R \\
\frac{\frac{\frac{Nt(s(m)) \Rightarrow Nt(s(m))}{\blacksquare Nt(s(m)) \Rightarrow Nt(s(m))} \blacksquare L}{\blacksquare Nt(s(m)) \Rightarrow \exists g Nt(s(g))} \exists R}{\blacksquare Nt(s(m)) \Rightarrow (\langle \rangle \exists g Nt(s(g)))} \langle \rangle R \\
\frac{\frac{\frac{\frac{\frac{\frac{Sf \Rightarrow Sf}{\square(Nt(s(n)) \& CNs(n)) \Rightarrow Nt(s(n))} \&L}{\square(Nt(s(n)) \& CNs(n)) \Rightarrow Nt(s(n))} \square L}{\square(Nt(s(n)) \& CNs(n)) \Rightarrow \exists a Na} \exists R}{\blacksquare Nt(s(m)), (\langle \rangle \exists g Nt(s(g)) \setminus Sf} \Rightarrow Sf} \setminus L}{\blacksquare Nt(s(m)), (\langle \rangle \exists g Nt(s(g)) \setminus Sf / \exists a Na, \square(Nt(s(n)) \& CNs(n)) \Rightarrow Sf} /L}{\blacksquare Nt(s(m)), \square((\langle \rangle \exists g Nt(s(g)) \setminus Sf) / \exists a Na), \square(Nt(s(n)) \& CNs(n)) \Rightarrow Sf} \square L}{\blacksquare Nt(s(m)), \square((\langle \rangle \exists g Nt(s(g)) \setminus Sf) / \exists a Na), \square(Nt(s(n)) \& CNs(n)) \Rightarrow Sf} \square L}{\blacksquare Nt(s(m)), 1, \square((\langle \rangle \exists g Nt(s(g)) \setminus Sf) / \exists a Na), \square(Nt(s(n)) \& CNs(n)) \Rightarrow \sim Sf} \sim R}{\blacksquare Nt(s(m)), \square((\langle \rangle \exists g Nt(s(g)) \setminus Sf) / \exists a Na), \square(Nt(s(n)) \& CNs(n)) \Rightarrow \sim Sf} \downarrow L}{\blacksquare Nt(s(m)), \sim Sf \setminus Sf, \square((\langle \rangle \exists g Nt(s(g)) \setminus Sf) / \exists a Na), \square(Nt(s(n)) \& CNs(n)) \Rightarrow Sf} \forall L}{\blacksquare Nt(s(m)), \forall f(\sim Sf \setminus Sf), \square((\langle \rangle \exists g Nt(s(g)) \setminus Sf) / \exists a Na), \square(Nt(s(n)) \& CNs(n)) \Rightarrow Sf} \square L}{\blacksquare Nt(s(m)), \square \forall f(\sim Sf \setminus Sf), \square((\langle \rangle \exists g Nt(s(g)) \setminus Sf) / \exists a Na), \square(Nt(s(n)) \& CNs(n)) \Rightarrow Sf} \square L}
\end{array}$$

(35) (\sim fortunately (*Pres* (\sim have (*gen* \sim perseverance))) *j*))

Third:

(36) (tdc(70c)) [**john**]+**has**+**fortunately**+**perseverance** : *Sf*

(37) $[\blacksquare Nt(s(m)) : j], \square((\langle \rangle \exists g Nt(s(g)) \setminus S f) / \exists a Na) : \hat{\lambda} A \lambda B (Pres (\tilde{\sim} have A) B), \square \forall f (\tilde{\sim} S f \downarrow S f) :$
fortunately, $\square(Nt(s(n)) \& CNs(n)) : \hat{\sim}((gen \tilde{\sim} perseverance), \tilde{\sim} perseverance) \Rightarrow S f$

$$\begin{array}{c}
\frac{}{Nt(s(n)) \Rightarrow Nt(s(n))} \quad \frac{}{Nt(s(m)) \Rightarrow Nt(s(m))} \blacksquare L \\
\frac{}{Nt(s(n)) \& CNs(n) \Rightarrow Nt(s(n))} \& L \quad \frac{}{\blacksquare Nt(s(m)) \Rightarrow Nt(s(m))} \blacksquare L \\
\frac{}{\square(Nt(s(n)) \& CNs(n)) \Rightarrow Nt(s(n))} \square L \quad \frac{}{\blacksquare Nt(s(m)) \Rightarrow \exists g Nt(s(g))} \exists R \\
\frac{}{\square(Nt(s(n)) \& CNs(n)) \Rightarrow Nt(s(n))} \square L \quad \frac{}{[\blacksquare Nt(s(m))] \Rightarrow \langle \rangle \exists g Nt(s(g))} \langle \rangle R \quad \frac{}{S f \Rightarrow S f} \\
\frac{}{\square(Nt(s(n)) \& CNs(n)) \Rightarrow \exists a Na} \exists R \quad \frac{}{[\blacksquare Nt(s(m))], \langle \rangle \exists g Nt(s(g)) \setminus S f \Rightarrow S f} \setminus L \\
\frac{}{\square(Nt(s(n)) \& CNs(n)) \Rightarrow \exists a Na} \exists R \quad \frac{}{[\blacksquare Nt(s(m))], \langle \rangle \exists g Nt(s(g)) \setminus S f \Rightarrow S f} \setminus L \\
\frac{}{[\blacksquare Nt(s(m))], \langle \rangle \exists g Nt(s(g)) \setminus S f / \exists a Na, \square(Nt(s(n)) \& CNs(n)) \Rightarrow S f} \setminus L \\
\frac{}{[\blacksquare Nt(s(m))], \square(\langle \rangle \exists g Nt(s(g)) \setminus S f / \exists a Na), \square(Nt(s(n)) \& CNs(n)) \Rightarrow S f} \square L \\
\frac{}{[\blacksquare Nt(s(m))], \square(\langle \rangle \exists g Nt(s(g)) \setminus S f / \exists a Na), \square(Nt(s(n)) \& CNs(n)) \Rightarrow S f} \square L \\
\frac{}{[\blacksquare Nt(s(m))], \square(\langle \rangle \exists g Nt(s(g)) \setminus S f / \exists a Na), 1, \square(Nt(s(n)) \& CNs(n)) \Rightarrow \tilde{\sim} S f} \tilde{\sim} R \quad \frac{}{S f \Rightarrow S f} \\
\frac{}{[\blacksquare Nt(s(m))], \square(\langle \rangle \exists g Nt(s(g)) \setminus S f / \exists a Na), \tilde{\sim} S f \downarrow S f, \square(Nt(s(n)) \& CNs(n)) \Rightarrow S f} \downarrow L \\
\frac{}{[\blacksquare Nt(s(m))], \square(\langle \rangle \exists g Nt(s(g)) \setminus S f / \exists a Na), \forall f (\tilde{\sim} S f \downarrow S f), \square(Nt(s(n)) \& CNs(n)) \Rightarrow S f} \forall L \\
\frac{}{[\blacksquare Nt(s(m))], \square(\langle \rangle \exists g Nt(s(g)) \setminus S f / \exists a Na), \square \forall f (\tilde{\sim} S f \downarrow S f), \square(Nt(s(n)) \& CNs(n)) \Rightarrow S f} \square L \\
\frac{}{[\blacksquare Nt(s(m))], \square(\langle \rangle \exists g Nt(s(g)) \setminus S f / \exists a Na), \square \forall f (\tilde{\sim} S f \downarrow S f), \square(Nt(s(n)) \& CNs(n)) \Rightarrow S f} \square L
\end{array}$$

(38) $(\tilde{\sim} \text{fortunately} (Pres (\tilde{\sim} have (gen \tilde{\sim} perseverance)) j)))$

And fourth:

(39) (tdc(70d)) **[john]+has+perseverance+fortunately : S f**

(40) $[\blacksquare Nt(s(m)) : j], \square((\langle \rangle \exists g Nt(s(g)) \setminus S f) / \exists a Na) : \hat{\lambda} A \lambda B (Pres (\tilde{\sim} have A) B), \square(Nt(s(n)) \& CNs(n)) :$
 $\hat{\sim}((gen \tilde{\sim} perseverance), \tilde{\sim} perseverance), \square \forall f (\tilde{\sim} S f \downarrow S f) : \text{fortunately} \Rightarrow S f$

$$\begin{array}{c}
\frac{}{Nt(s(n)) \Rightarrow Nt(s(n))} \quad \frac{}{Nt(s(m)) \Rightarrow Nt(s(m))} \blacksquare L \\
\frac{}{Nt(s(n)) \& CNs(n) \Rightarrow Nt(s(n))} \& L \quad \frac{}{\blacksquare Nt(s(m)) \Rightarrow Nt(s(m))} \blacksquare L \\
\frac{}{\square(Nt(s(n)) \& CNs(n)) \Rightarrow Nt(s(n))} \square L \quad \frac{}{\blacksquare Nt(s(m)) \Rightarrow \exists g Nt(s(g))} \exists R \\
\frac{}{\square(Nt(s(n)) \& CNs(n)) \Rightarrow Nt(s(n))} \square L \quad \frac{}{[\blacksquare Nt(s(m))] \Rightarrow \langle \rangle \exists g Nt(s(g))} \langle \rangle R \quad \frac{}{S f \Rightarrow S f} \\
\frac{}{\square(Nt(s(n)) \& CNs(n)) \Rightarrow \exists a Na} \exists R \quad \frac{}{[\blacksquare Nt(s(m))], \langle \rangle \exists g Nt(s(g)) \setminus S f \Rightarrow S f} \setminus L \\
\frac{}{\square(Nt(s(n)) \& CNs(n)) \Rightarrow \exists a Na} \exists R \quad \frac{}{[\blacksquare Nt(s(m))], \langle \rangle \exists g Nt(s(g)) \setminus S f \Rightarrow S f} \setminus L \\
\frac{}{[\blacksquare Nt(s(m))], \langle \rangle \exists g Nt(s(g)) \setminus S f / \exists a Na, \square(Nt(s(n)) \& CNs(n)) \Rightarrow S f} \setminus L \\
\frac{}{[\blacksquare Nt(s(m))], \square(\langle \rangle \exists g Nt(s(g)) \setminus S f / \exists a Na), \square(Nt(s(n)) \& CNs(n)) \Rightarrow S f} \square L \\
\frac{}{[\blacksquare Nt(s(m))], \square(\langle \rangle \exists g Nt(s(g)) \setminus S f / \exists a Na), \square(Nt(s(n)) \& CNs(n)) \Rightarrow S f} \square L \\
\frac{}{[\blacksquare Nt(s(m))], \square(\langle \rangle \exists g Nt(s(g)) \setminus S f / \exists a Na), \square(Nt(s(n)) \& CNs(n)), 1 \Rightarrow \tilde{\sim} S f} \tilde{\sim} R \quad \frac{}{S f \Rightarrow S f} \\
\frac{}{[\blacksquare Nt(s(m))], \square(\langle \rangle \exists g Nt(s(g)) \setminus S f / \exists a Na), \square(Nt(s(n)) \& CNs(n)), \tilde{\sim} S f \downarrow S f \Rightarrow S f} \downarrow L \\
\frac{}{[\blacksquare Nt(s(m))], \square(\langle \rangle \exists g Nt(s(g)) \setminus S f / \exists a Na), \square(Nt(s(n)) \& CNs(n)), \forall f (\tilde{\sim} S f \downarrow S f) \Rightarrow S f} \forall L \\
\frac{}{[\blacksquare Nt(s(m))], \square(\langle \rangle \exists g Nt(s(g)) \setminus S f / \exists a Na), \square(Nt(s(n)) \& CNs(n)), \square \forall f (\tilde{\sim} S f \downarrow S f) \Rightarrow S f} \square L \\
\frac{}{[\blacksquare Nt(s(m))], \square(\langle \rangle \exists g Nt(s(g)) \setminus S f / \exists a Na), \square(Nt(s(n)) \& CNs(n)), \square \forall f (\tilde{\sim} S f \downarrow S f) \Rightarrow S f} \square L
\end{array}$$

(41) $(\tilde{\sim} \text{fortunately} (Pres (\tilde{\sim} have (gen \tilde{\sim} perseverance)) j)))$

The next example is gapping:

This delivers semantics:

$$(44) [(Pres ((\sim study (gen \sim logic)) j)) \wedge (Pres ((\sim study (gen \sim phonetics)) c))]$$

Example tdc(75) contains comparative subdeletion:

$$(45) (tdc(75)) [\mathbf{john}] + \mathbf{ate} + \mathbf{more} + \mathbf{donuts} + \mathbf{than} + [\mathbf{mary}] + \mathbf{bought} + \mathbf{bagels} : Sf$$

Lexical lookup yields:

$$(46) [\blacksquare Nt(s(m)) : j], \square((\langle \exists aNa \setminus Sf \rangle / \exists aNa) : \hat{\lambda} A \lambda B (Past ((\sim eat A) B)), \\ \blacksquare \forall h \forall g \forall f ((Sf \uparrow ((Sh \uparrow Nt(p(g))) \downarrow Sh) / CNp(g))) \downarrow (Sf \wedge (Cp \text{than} \uparrow \blacksquare ((Sh \uparrow Nt(p(g))) \downarrow Sh) / CNp(g)))) : \\ \lambda C \lambda D [\lambda E (C \lambda F \lambda G ((F E) \wedge (G E))] > \lambda H \sim (D \lambda I \lambda J [(I H) \wedge (J H)]), \square(Nt(p(n)) \& CNp(n)) : \\ \hat{\sim} ((gen \sim donuts), \sim donuts), \blacksquare(Cp \text{than} / \square Sf) : \lambda K K, [\blacksquare Nt(s(f)) : m], \square((\langle \exists aNa \setminus Sf \rangle / \exists aNa) : \\ \hat{\sim} \lambda L \lambda M (Past ((\sim buy L) M)), \square(Nt(p(n)) \& CNp(n)) : \hat{\sim} ((gen \sim bagels), \sim bagels) \Rightarrow Sf$$

There is the derivation:

$$\begin{array}{c}
 \frac{\frac{\frac{Nt(s(m))}{\square Nt(s(m))} \Rightarrow Nt(s(m))}{\blacksquare Nt(s(m)) \Rightarrow Nt(s(m))} \blacksquare L \\
 \frac{\frac{\blacksquare Nt(s(m)) \Rightarrow Nt(s(m))}{\blacksquare Nt(s(m)) \Rightarrow \exists aNa} \exists R \\
 \frac{Nt(p(n)) \Rightarrow Nt(p(n)) \quad \frac{[\blacksquare Nt(s(m))] \Rightarrow \langle \exists aNa \rangle \quad \frac{Sf}{Sf \Rightarrow Sf} \langle R}}{\frac{Nt(p(n)) \Rightarrow \exists aNa \quad [\blacksquare Nt(s(m))] \Rightarrow \langle \exists aNa \setminus Sf \rangle \Rightarrow Sf}{\blacksquare Nt(s(m)), \langle \exists aNa \setminus Sf \rangle \Rightarrow Sf} \backslash L}}{\blacksquare Nt(s(m)), \langle \exists aNa \setminus Sf \rangle / \exists aNa, Nt(p(n)) \Rightarrow Sf} /L \\
 \frac{\frac{\frac{\frac{CNp(n)}{CNp(n)} \Rightarrow CNp(n)}{Nt(p(n)) \& CNp(n)} \Rightarrow CNp(n)}{\square(Nt(p(n)) \& CNp(n))} \Rightarrow CNp(n)}{\blacksquare Nt(s(m)), \square((\langle \exists aNa \setminus Sf \rangle / \exists aNa), Nt(p(n)) \Rightarrow Sf} \square L \\
 \frac{\frac{[\blacksquare Nt(s(m))] \Rightarrow \langle \exists aNa \setminus Sf \rangle}{\blacksquare Nt(s(m)), \square((\langle \exists aNa \setminus Sf \rangle / \exists aNa), 1 \Rightarrow Sf \uparrow Nt(p(n))} \uparrow R \\
 \frac{\frac{\frac{Nt(p(n)) \& CNp(n)}{CNp(n)} \Rightarrow CNp(n)}{\square(Nt(p(n)) \& CNp(n))} \Rightarrow CNp(n)}{\blacksquare Nt(s(m)), \square((\langle \exists aNa \setminus Sf \rangle / \exists aNa), (Sf \uparrow Nt(p(n))) \downarrow Sf} \downarrow L \\
 \frac{\blacksquare Nt(s(m)), \square((\langle \exists aNa \setminus Sf \rangle / \exists aNa), (Sf \uparrow Nt(p(n))) \downarrow Sf / CNp(n)}{\blacksquare Nt(s(m)), \square((\langle \exists aNa \setminus Sf \rangle / \exists aNa), 1, \square(Nt(p(n)) \& CNp(n)) \Rightarrow Sf \uparrow ((Sf \uparrow Nt(p(n))) \downarrow Sf) / CNp(n)} \uparrow R \\
 \frac{\blacksquare Nt(s(m)), \square((\langle \exists aNa \setminus Sf \rangle / \exists aNa), 1, \square(Nt(p(n)) \& CNp(n)) \Rightarrow Sf \uparrow ((Sf \uparrow Nt(p(n))) \downarrow Sf) / CNp(n)}{\blacksquare Nt(s(m)), \square((\langle \exists aNa \setminus Sf \rangle / \exists aNa), 1, \square(Nt(p(n)) \& CNp(n)) \Rightarrow Sf \uparrow ((Sf \uparrow Nt(p(n))) \downarrow Sf) / CNp(n)} \oplus
 \end{array}$$

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Appendix: CatLog output for Dutch verb raising and cross-serial dependency

kan : $(NA \setminus Si) \downarrow (NA \setminus Sf) : \lambda B \lambda C ((isable (B C)) C)$
las : $NA \setminus (Nt(s(B)) \setminus Sf) : read$
wil : $(NA \setminus Si) \downarrow (NA \setminus Sf) : \lambda B \lambda C ((want (B C)) C)$
kan : $Q / \wedge (Sf \uparrow ((NA \setminus Si) \downarrow (NA \setminus Sf))) : \lambda B (B \lambda C \lambda D ((isable (C D)) D))$
las : $Q / \wedge (Sf \uparrow (NA \setminus (Nt(s(B)) \setminus Sf))) : \lambda C (C read)$
wil : $Q / \wedge (Sf \uparrow ((NA \setminus Si) \downarrow (NA \setminus Sf))) : \lambda B (B \lambda C \lambda D ((want (C D)) D))$
alles : $(SA \uparrow Nt(s(n))) \downarrow SA : \lambda B \forall C [(thing C) \rightarrow (B C)]$
boeken : $Np(n) : books$
cecilia : $Nt(s(f)) : c$
de : $Nt(s(A)) / CNA : the$
helpen : $\triangleright^{-1} ((NA \setminus Si) \downarrow (NB \setminus (NA \setminus Si))) : \lambda C \lambda D ((help (C D)) D)$
henk : $Nt(s(m)) : h$
jan : $Nt(s(m)) : j$
kunnen : $\triangleright^{-1} ((NA \setminus Si) \downarrow (NA \setminus Si)) : \lambda B \lambda C ((isable (B C)) C)$
lezen : $\triangleright^{-1} (NA \setminus (NB \setminus Si)) : read$
nijlpaarden : $CNp(n) : hippos$
voeren : $\triangleright^{-1} (NA \setminus (NB \setminus Si)) : feed$
zag : $(Nt(s(A)) \setminus Si) \downarrow (NB \setminus (Nt(s(A)) \setminus Sf)) : \lambda C \lambda D ((saw (C D)) D)$

(d(1)) **jan+boeken+las** : Sf

$Nt(s(m)) : j, Np(n) : books, NA \setminus (Nt(s(B)) \setminus Sf) : read \Rightarrow Sf$

$$\begin{array}{c}
 \frac{}{Np(n) \Rightarrow Np(n)} \quad \frac{}{Nt(s(m)) \Rightarrow Nt(s(m))} \quad \frac{}{Sf \Rightarrow Sf} \\
 \hline
 \frac{Np(n) \Rightarrow Np(n) \quad Nt(s(m)), \boxed{Nt(s(m)) \setminus Sf} \Rightarrow Sf}{Nt(s(m)), Np(n), \boxed{Np(n) \setminus (Nt(s(m)) \setminus Sf)} \Rightarrow Sf} \backslash L
 \end{array}$$

((read books) j)

$Nt(s(m)) : j, Np(n) : books, Q / \wedge (Sf \uparrow (NA \setminus (Nt(s(B)) \setminus Sf))) : \lambda C (C read) \Rightarrow Sf$

(d(2)) **jan+boeken+kan+lezen** : Sf

$Nt(s(m)) : j, Np(n) : books, (NA \setminus Si) \downarrow (NA \setminus Sf) : \lambda B \lambda C ((isable (B C)) C), \triangleright^{-1} (ND \setminus (NE \setminus Si)) : read \Rightarrow Sf$

$$\begin{array}{c}
 \frac{}{Np(n) \Rightarrow Np(n)} \quad \frac{}{Nt(s(m)) \Rightarrow Nt(s(m))} \quad \frac{}{Si \{1\} \Rightarrow Si} \\
 \hline
 \frac{Np(n) \Rightarrow Np(n) \quad Nt(s(m)), \boxed{Nt(s(m)) \setminus Si \{1\}} \Rightarrow Si}{Nt(s(m)), Np(n), \boxed{Np(n) \setminus (Nt(s(m)) \setminus Si \{1\})} \Rightarrow Si} \backslash L \\
 \hline
 \frac{Nt(s(m)), Np(n), \boxed{\triangleright^{-1} (Np(n) \setminus (Nt(s(m)) \setminus Si))} \Rightarrow Si}{Np(n), \boxed{\triangleright^{-1} (Np(n) \setminus (Nt(s(m)) \setminus Si))} \Rightarrow Nt(s(m)) \setminus Si} \triangleright^{-1} L \quad \frac{}{Nt(s(m)) \Rightarrow Nt(s(m))} \quad \frac{}{Sf \Rightarrow Sf} \\
 \hline
 \frac{Np(n), \boxed{\triangleright^{-1} (Np(n) \setminus (Nt(s(m)) \setminus Si))} \Rightarrow Nt(s(m)) \setminus Si \quad Nt(s(m)), \boxed{Nt(s(m)) \setminus Sf} \Rightarrow Sf}{Nt(s(m)), Np(n), \boxed{(Nt(s(m)) \setminus Si) \downarrow (Nt(s(m)) \setminus Sf)} \triangleright^{-1} (Np(n) \setminus (Nt(s(m)) \setminus Si)) \Rightarrow Sf} \backslash R \quad \downarrow L
 \end{array}$$

((isable ((read books) j) j)

$Nt(s(m)) : j, Np(n) : books, Q / \wedge (Sf \uparrow ((NA \setminus Si) \downarrow (NA \setminus Sf))) : \lambda B (B \lambda C \lambda D ((isable (C D)) D)), \triangleright^{-1} (NE \setminus (NF \setminus Si)) : read \Rightarrow Sf$

(d(3)) **jan+boeken+wil+kunnen+lezen** : Sf

$Nt(s(m)) : j, Np(n) : books, (NA \setminus Si) \downarrow (NA \setminus Sf) : \lambda B \lambda C ((want (B C)) C), \triangleright^{-1} ((ND \setminus Si) \downarrow (ND \setminus Si)) : \lambda E \lambda F ((isable (E F)) F), \triangleright^{-1} (NG \setminus (NH \setminus Si)) : read \Rightarrow Sf$

$$\begin{array}{c}
\frac{}{Nt(s(m)) \Rightarrow Nt(s(m)) \quad \boxed{Si1} \Rightarrow Si} \\
\frac{}{Np(n) \Rightarrow Np(n) \quad Nt(s(m)), \boxed{Nt(s(m))\backslash Si1} \Rightarrow Si} \backslash L \\
\frac{}{Nt(s(m)), Np(n), \boxed{Np(n)\backslash(Nt(s(m))\backslash Si)1} \Rightarrow Si} \\
\frac{}{Nt(s(m)), Np(n), 1, \boxed{\triangleright^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si))} \Rightarrow Si} \triangleright^{-1} L \quad \frac{}{Nt(s(m)) \Rightarrow Nt(s(m)) \quad \boxed{Si1} \Rightarrow Si} \\
\frac{}{Np(n), 1, \triangleright^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si)) \Rightarrow Nt(s(m))\backslash Si} \backslash R \quad \frac{}{Nt(s(m)), \boxed{Nt(s(m))\backslash Si1} \Rightarrow Si} \backslash L \\
\frac{}{Nt(s(m)), Np(n), \boxed{(Nt(s(m))\backslash Si)\downarrow(Nt(s(m))\backslash Si)1} \Rightarrow Si} \triangleright^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si)) \Rightarrow Si} \triangleright^{-1} L \\
\frac{}{Nt(s(m)), Np(n), 1, \boxed{\triangleright^{-1}((Nt(s(m))\backslash Si)\downarrow(Nt(s(m))\backslash Si))} \Rightarrow Si} \triangleright^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si)) \Rightarrow Si} \triangleright^{-1} L \\
\frac{}{Np(n), 1, \triangleright^{-1}((Nt(s(m))\backslash Si)\downarrow(Nt(s(m))\backslash Si)), \triangleright^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si)) \Rightarrow Nt(s(m))\backslash Si} \backslash R \quad \frac{}{Nt(s(m)) \Rightarrow Nt(s(m)) \quad \boxed{Sf} \Rightarrow Sf} \\
\frac{}{Nt(s(m)), Np(n), \boxed{(Nt(s(m))\backslash Si)\downarrow(Nt(s(m))\backslash Sf)} \Rightarrow Sf} \triangleright^{-1}((Nt(s(m))\backslash Si)\downarrow(Nt(s(m))\backslash Si)), \triangleright^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si)) \Rightarrow Sf} \triangleright^{-1} L \\
\frac{}{Nt(s(m)), Np(n), \boxed{(Nt(s(m))\backslash Si)\downarrow(Nt(s(m))\backslash Sf)} \Rightarrow Sf} \triangleright^{-1}((Nt(s(m))\backslash Si)\downarrow(Nt(s(m))\backslash Si)), \triangleright^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si)) \Rightarrow Sf} \triangleright^{-1} L
\end{array}$$

((want ((isable ((read books) j)) j)) j)

$Nt(s(m)) : j, Np(n) : books, Q/\wedge(Sf\uparrow((NA)\backslash Si)\downarrow(NA)\backslash Sf)) : AB(B \ ACAD(want(C \ D)) \ D), \triangleright^{-1}((NE)\backslash Si)\downarrow(NE)\backslash Si) : \lambda F \lambda G((isable(F \ G)) \ G), \triangleright^{-1}(NH)\backslash(NI)\backslash Si) : read \Rightarrow Sf$

(d(4)) **jan+alles+las** : Sf

$Nt(s(m)) : j, (SA\uparrow Nt(s(n)))\downarrow SA : \lambda B\uparrow C((thing \ C) \rightarrow (B \ C)), ND)\backslash(Nt(s(E))\backslash Sf) : read \Rightarrow Sf$

$$\begin{array}{c}
\frac{}{Nt(s(m)) \Rightarrow Nt(s(m)) \quad \boxed{Sf} \Rightarrow Sf} \\
\frac{}{Nt(s(n)) \Rightarrow Nt(s(n)) \quad Nt(s(m)), \boxed{Nt(s(m))\backslash Sf} \Rightarrow Sf} \backslash L \\
\frac{}{Nt(s(m)), Nt(s(n)), \boxed{Nt(s(n))\backslash(Nt(s(m))\backslash Sf)} \Rightarrow Sf} \\
\frac{}{Nt(s(m)), 1, Nt(s(n))\backslash(Nt(s(m))\backslash Sf) \Rightarrow Sf\uparrow Nt(s(n))} \uparrow R \quad \frac{}{\boxed{Sf} \Rightarrow Sf} \\
\frac{}{Nt(s(m)), \boxed{(Sf\uparrow Nt(s(n)))\downarrow Sf} \Rightarrow Sf} \downarrow L \quad \frac{}{Nt(s(n))\backslash(Nt(s(m))\backslash Sf) \Rightarrow Sf} \downarrow L
\end{array}$$

$\forall B[(thing \ B) \rightarrow ((read \ B) \ j)]$

$Nt(s(m)) : j, (SA\uparrow Nt(s(n)))\downarrow SA : \lambda B\uparrow C((thing \ C) \rightarrow (B \ C)), Q/\wedge(Sf\uparrow(ND)\backslash(Nt(s(E))\backslash Sf)) : \lambda F(F \ read) \Rightarrow Sf$

(d(5)) **jan+alles+kan+lezen** : Sf

$Nt(s(m)) : j, (SA\uparrow Nt(s(n)))\downarrow SA : \lambda B\uparrow C((thing \ C) \rightarrow (B \ C)), (ND)\backslash Si)\downarrow(ND)\backslash Sf) : \lambda E \lambda F((isable(E \ F)) \ F), \triangleright^{-1}(NG)\backslash(NH)\backslash Si) : read \Rightarrow Sf$

$$\begin{array}{c}
\frac{}{Nt(s(m)) \Rightarrow Nt(s(m)) \quad \boxed{Si1} \Rightarrow Si} \\
\frac{}{Nt(s(n)) \Rightarrow Nt(s(n)) \quad Nt(s(m)), \boxed{Nt(s(m))\backslash Si1} \Rightarrow Si} \backslash L \\
\frac{}{Nt(s(m)), Nt(s(n)), \boxed{Nt(s(n))\backslash(Nt(s(m))\backslash Si)1} \Rightarrow Si} \\
\frac{}{Nt(s(m)), Nt(s(n)), 1, \boxed{\triangleright^{-1}(Nt(s(n))\backslash(Nt(s(m))\backslash Si))} \Rightarrow Si} \triangleright^{-1} L \quad \frac{}{Nt(s(m)) \Rightarrow Nt(s(m)) \quad \boxed{Sf} \Rightarrow Sf} \\
\frac{}{Nt(s(n)), 1, \triangleright^{-1}(Nt(s(n))\backslash(Nt(s(m))\backslash Si)) \Rightarrow Nt(s(m))\backslash Si} \backslash R \quad \frac{}{Nt(s(m)), \boxed{Nt(s(m))\backslash Sf} \Rightarrow Sf} \backslash L \\
\frac{}{Nt(s(m)), Nt(s(n)), \boxed{(Nt(s(m))\backslash Si)\downarrow(Nt(s(m))\backslash Sf)} \Rightarrow Sf} \triangleright^{-1}(Nt(s(n))\backslash(Nt(s(m))\backslash Si)) \Rightarrow Sf} \triangleright^{-1} L \\
\frac{}{Nt(s(m)), 1, (Nt(s(m))\backslash Si)\downarrow(Nt(s(m))\backslash Sf), \triangleright^{-1}(Nt(s(n))\backslash(Nt(s(m))\backslash Si)) \Rightarrow Sf\uparrow Nt(s(n))} \uparrow R \quad \frac{}{\boxed{Sf} \Rightarrow Sf} \\
\frac{}{Nt(s(m)), \boxed{(Sf\uparrow Nt(s(n)))\downarrow Sf} \Rightarrow Sf} \downarrow L \quad \frac{}{(Nt(s(m))\backslash Si)\downarrow(Nt(s(m))\backslash Sf), \triangleright^{-1}(Nt(s(n))\backslash(Nt(s(m))\backslash Si)) \Rightarrow Sf} \downarrow L
\end{array}$$

$\forall B[(thing \ B) \rightarrow ((isable((read \ B) \ j)) \ j)]$

$Nt(s(m)) : j, (SA\uparrow Nt(s(n)))\downarrow SA : \lambda B\uparrow C((thing \ C) \rightarrow (B \ C)), Q/\wedge(Sf\uparrow((ND)\backslash Si)\downarrow(ND)\backslash Sf)) : \lambda E(E \ \lambda F \lambda G((isable(F \ G)) \ G)), \triangleright^{-1}(NH)\backslash(NI)\backslash Si) : read \Rightarrow Sf$

(d(6)) **jan+cecilia+henk+de+nijlpaarden+zag+helpen+voeren** : Sf

$Nt(s(m)) : j, Nt(s(f)) : c, Nt(s(m)) : h, Nt(s(A))/CNA : the, CNp(n) : hippos, (Nt(s(B))\backslash Si)\downarrow(NC)\backslash(Nt(s(B))\backslash Sf) : \lambda D \lambda E((saw(D \ E)) \ E), \triangleright^{-1}(NF)\backslash Si)\downarrow(NF)\backslash Si) :$

$\lambda HAI((\text{help } (H \ I)) \ I), \triangleright^{-1}(NJ \setminus (NK \setminus Si)) : \text{feed} \Rightarrow Sf$

$$\begin{array}{c} \begin{array}{l} \frac{\frac{\frac{CNp(a) \Rightarrow CNp(a)}{\boxed{N(s(p(a)))}} \quad \frac{\frac{N(s(m)) \Rightarrow N(s(m))}{\boxed{Sf1}} \Rightarrow Sf}{\downarrow L} \\ \boxed{N(s(p(a)))/CNp(a)} \Rightarrow N(s(p(a))) \\ \frac{N(s(m)), \boxed{N(s(m)) \setminus Sf1}}{\downarrow L} \Rightarrow Sf \end{array} \\ \frac{\boxed{N(s(p(a)))/CNp(a)} \Rightarrow N(s(p(a)))}{\downarrow L} \\ \begin{array}{l} \frac{N(s(m)), N(s(p(a)))/CNp(a), CNp(a), \boxed{N(s(p(a))) \setminus Sf1}}{\downarrow L} \Rightarrow Sf \\ \frac{N(s(m)), N(s(p(a)))/CNp(a), CNp(a), \downarrow \triangleright^{-1}(N(s(p(a)) \setminus (N(s(m)) \setminus Sf))}{\downarrow L} \Rightarrow Sf \\ \frac{N(s(m)), N(s(p(a)))/CNp(a), CNp(a), \downarrow \triangleright^{-1}(N(s(p(a)) \setminus (N(s(m)) \setminus Sf))}{\downarrow L} \Rightarrow Sf \\ \frac{N(s(m)), N(s(p(a)))/CNp(a), \downarrow \triangleright^{-1}(N(s(p(a)) \setminus (N(s(m)) \setminus Sf))}{\downarrow L} \Rightarrow Sf \\ \frac{N(s(m)), N(s(p(a)))/CNp(a), \downarrow \triangleright^{-1}(N(s(p(a)) \setminus (N(s(m)) \setminus Sf))}{\downarrow L} \Rightarrow Sf \end{array} \\ \frac{\frac{\frac{N(s(m)), N(s(p(a)))/CNp(a), CNp(a), \boxed{N(s(p(a)) \setminus Sf1}}{\downarrow L} \Rightarrow Sf}{\downarrow L} \\ \frac{\frac{N(s(m)), N(s(p(a)))/CNp(a), CNp(a), \downarrow \triangleright^{-1}(N(s(p(a)) \setminus (N(s(m)) \setminus Sf))}{\downarrow L} \Rightarrow Sf}{\downarrow L} \\ \frac{\frac{N(s(m)), N(s(p(a)))/CNp(a), CNp(a), \downarrow \triangleright^{-1}(N(s(p(a)) \setminus (N(s(m)) \setminus Sf))}{\downarrow L} \Rightarrow Sf}{\downarrow L} \\ \frac{N(s(m)), N(s(p(a)))/CNp(a), CNp(a), \downarrow \triangleright^{-1}(N(s(p(a)) \setminus (N(s(m)) \setminus Sf))}{\downarrow L} \Rightarrow Sf \end{array}$$

((saw ((help ((feed (the hippos) h) h) c) c) j)

(d(7)) **wil+jan+boeken+lezen** : Q

$(NA \setminus Si) \downarrow (NA \setminus Sf) : \lambda B \lambda C ((want (B C) C), Nt(s(m)) : j, Np(n) : books, \triangleright^{-1}(ND \setminus (NE \setminus Si)) : read \Rightarrow Q$

$Q / \wedge (Sf \uparrow ((NA \setminus Si) \downarrow (NA \setminus Sf))) : \lambda B (\lambda C \lambda D ((want (C D) D)), Nt(s(m)) : j, Np(n) : books, \triangleright^{-1}(NE \setminus (NF \setminus Si)) : read \Rightarrow Q$

$$\begin{array}{c}
 \frac{\frac{Nt(s(m)) \Rightarrow Nt(s(m)) \quad \boxed{Si\{1\}} \Rightarrow Si}{\text{NL}}}{Np(n) \Rightarrow Np(n) \quad Nt(s(m)), \boxed{Nt(s(m)) \setminus Si\{1\}} \Rightarrow Si} \text{NL} \\
 \frac{\frac{Nt(s(m)), Np(n), \boxed{Np(n) \setminus (Nt(s(m)) \setminus Si)\{1\}} \Rightarrow Si}{\text{NL}}}{Nt(s(m)), Np(n), 1, \boxed{\triangleright^{-1}(Np(n) \setminus (Nt(s(m)) \setminus Si))} \Rightarrow Si} \text{NL} \\
 \frac{\frac{Nt(s(m)) \Rightarrow Nt(s(m)) \quad \boxed{Sf} \Rightarrow Sf}{\text{NL}}}{Np(n), 1, \triangleright^{-1}(Np(n) \setminus (Nt(s(m)) \setminus Si)) \Rightarrow Nt(s(m)) \setminus Si} \text{NL} \\
 \frac{\frac{Nt(s(m)), \boxed{Nt(s(m)) \setminus Sf} \Rightarrow Sf}{\text{NL}}}{Nt(s(m)), Np(n), \boxed{(Nt(s(m)) \setminus Si) \downarrow (Nt(s(m)) \setminus Sf)} \triangleright^{-1}(Np(n) \setminus (Nt(s(m)) \setminus Si)) \Rightarrow Sf} \text{NL} \\
 \frac{\frac{Nt(s(m)), Np(n), 1, \triangleright^{-1}(Np(n) \setminus (Nt(s(m)) \setminus Si)) \Rightarrow Sf \uparrow ((Nt(s(m)) \setminus Si) \downarrow (Nt(s(m)) \setminus Sf))}{\text{NR}}}{Nt(s(m)), Np(n), \triangleright^{-1}(Np(n) \setminus (Nt(s(m)) \setminus Si)) \Rightarrow \boxed{Sf \uparrow ((Nt(s(m)) \setminus Si) \downarrow (Nt(s(m)) \setminus Sf))}} \text{NR} \\
 \frac{\frac{\boxed{Q} \Rightarrow Q}{\text{NL}}}{\boxed{Q / \wedge (Sf \uparrow ((Nt(s(m)) \setminus Si) \downarrow (Nt(s(m)) \setminus Sf)))}, Nt(s(m)), Np(n), \triangleright^{-1}(Np(n) \setminus (Nt(s(m)) \setminus Si)) \Rightarrow Q} \text{NL}
 \end{array}$$

((want ((read books) j) j)

(d(8)) **jan+wil+boeken+lezen** : $Nt(s(m)) \bullet \wedge (Q \uparrow Nt(s(m)))$

$Nt(s(m)) : j, (NA \setminus Si) \downarrow (NA \setminus Sf) : \lambda B \lambda C ((want (B C) C), Np(n) : books, \triangleright^{-1}(ND \setminus (NE \setminus Si)) : read \Rightarrow Nt(s(m)) \bullet \wedge (Q \uparrow Nt(s(m)))$

$Nt(s(m)) : j, Q / \wedge (Sf \uparrow ((NA \setminus Si) \downarrow (NA \setminus Sf))) : \lambda B (\lambda C \lambda D ((want (C D) D)), Np(n) : books, \triangleright^{-1}(NE \setminus (NF \setminus Si)) : read \Rightarrow Nt(s(m)) \bullet \wedge (Q \uparrow Nt(s(m)))$

$$\begin{array}{c}
 \frac{\frac{Np(n) \Rightarrow Np(n) \quad \boxed{Si\{1\}} \Rightarrow Si}{\text{NL}}}{Nt(s(m)) \Rightarrow Nt(s(m)) \quad Np(n), \boxed{Np(n) \setminus Si\{1\}} \Rightarrow Si} \text{NL} \\
 \frac{\frac{Np(n), Nt(s(m)), \boxed{Nt(s(m)) \setminus (Np(n) \setminus Si)\{1\}} \Rightarrow Si}{\text{NL}}}{Np(n), Nt(s(m)), 1, \boxed{\triangleright^{-1}(Nt(s(m)) \setminus (Np(n) \setminus Si))} \Rightarrow Si} \text{NL} \\
 \frac{\frac{Np(n) \Rightarrow Np(n) \quad \boxed{Sf} \Rightarrow Sf}{\text{NL}}}{Nt(s(m)), 1, \triangleright^{-1}(Nt(s(m)) \setminus (Np(n) \setminus Si)) \Rightarrow Np(n) \setminus Si} \text{NL} \\
 \frac{\frac{Np(n), \boxed{Np(n) \setminus Sf} \Rightarrow Sf}{\text{NL}}}{Np(n), Nt(s(m)), \boxed{(Np(n) \setminus Si) \downarrow (Np(n) \setminus Sf)} \triangleright^{-1}(Nt(s(m)) \setminus (Np(n) \setminus Si)) \Rightarrow Sf} \text{NL} \\
 \frac{\frac{Np(n), Nt(s(m)), 1, \triangleright^{-1}(Nt(s(m)) \setminus (Np(n) \setminus Si)) \Rightarrow Sf \uparrow ((Np(n) \setminus Si) \downarrow (Np(n) \setminus Sf))}{\text{NR}}}{Np(n), Nt(s(m)), \triangleright^{-1}(Nt(s(m)) \setminus (Np(n) \setminus Si)) \Rightarrow \boxed{Sf \uparrow ((Np(n) \setminus Si) \downarrow (Np(n) \setminus Sf))}} \text{NR} \\
 \frac{\frac{\boxed{Q} \Rightarrow Q}{\text{NL}}}{\boxed{Q / \wedge (Sf \uparrow ((Np(n) \setminus Si) \downarrow (Np(n) \setminus Sf)))}, Np(n), Nt(s(m)), \triangleright^{-1}(Nt(s(m)) \setminus (Np(n) \setminus Si)) \Rightarrow Q} \text{NL} \\
 \frac{\frac{Q / \wedge (Sf \uparrow ((Np(n) \setminus Si) \downarrow (Np(n) \setminus Sf))), Np(n), 1, \triangleright^{-1}(Nt(s(m)) \setminus (Np(n) \setminus Si)) \Rightarrow Q \uparrow Nt(s(m))}{\text{NR}}}{Nt(s(m)) \Rightarrow Nt(s(m)) \quad Q / \wedge (Sf \uparrow ((Np(n) \setminus Si) \downarrow (Np(n) \setminus Sf))), Np(n), \triangleright^{-1}(Nt(s(m)) \setminus (Np(n) \setminus Si)) \Rightarrow \boxed{Q \uparrow Nt(s(m))}} \text{NR} \\
 \frac{\frac{\boxed{Nt(s(m)) \bullet \wedge (Q \uparrow Nt(s(m)))}}{\text{NR}}}{Nt(s(m)), Q / \wedge (Sf \uparrow ((Np(n) \setminus Si) \downarrow (Np(n) \setminus Sf))), Np(n), \triangleright^{-1}(Nt(s(m)) \setminus (Np(n) \setminus Si)) \Rightarrow \boxed{Nt(s(m)) \bullet \wedge (Q \uparrow Nt(s(m)))}} \text{NR}
 \end{array}$$

(j, $\lambda A ((want ((read A) books)) books)$)

$$\begin{array}{c}
\frac{\frac{Nt(s(m)) \Rightarrow Nt(s(m)) \quad \boxed{Si(1)} \Rightarrow Si}{\text{---}} \quad \downarrow L}{Np(n) \Rightarrow Np(n) \quad \frac{Nt(s(m)), \boxed{Nt(s(m)) \setminus Si(1)} \Rightarrow Si}{\text{---}} \quad \downarrow L} \\
\frac{Nt(s(m)), Np(n), \boxed{Np(n) \setminus (Nt(s(m)) \setminus Si(1))} \Rightarrow Si}{\text{---}} \quad \triangleright^{-1} L}{Nt(s(m)), Np(n), 1, \boxed{\triangleright^{-1}(Np(n) \setminus (Nt(s(m)) \setminus Si))} \Rightarrow Si} \quad \downarrow R \quad \frac{Nt(s(m)) \Rightarrow Nt(s(m)) \quad \boxed{Sf} \Rightarrow Sf}{\text{---}} \quad \downarrow L \\
\frac{Np(n), 1, \triangleright^{-1}(Np(n) \setminus (Nt(s(m)) \setminus Si)) \Rightarrow Nt(s(m)) \setminus Si}{\text{---}} \quad \downarrow L \quad \frac{Nt(s(m)), \boxed{Nt(s(m)) \setminus Sf} \Rightarrow Sf}{\text{---}} \\
\frac{Nt(s(m)), Np(n), \boxed{(Nt(s(m)) \setminus Si) \downarrow (Nt(s(m)) \setminus Sf)} \quad \triangleright^{-1}(Np(n) \setminus (Nt(s(m)) \setminus Si)) \Rightarrow Sf}{\text{---}} \quad \uparrow R \\
\frac{Nt(s(m)), Np(n), 1, \triangleright^{-1}(Np(n) \setminus (Nt(s(m)) \setminus Si)) \Rightarrow Sf \uparrow ((Nt(s(m)) \setminus Si) \downarrow (Nt(s(m)) \setminus Sf))}{\text{---}} \quad \uparrow R \\
\frac{Nt(s(m)), Np(n), \triangleright^{-1}(Np(n) \setminus (Nt(s(m)) \setminus Si)) \Rightarrow \boxed{(Sf \uparrow ((Nt(s(m)) \setminus Si) \downarrow (Nt(s(m)) \setminus Sf)))}}{\text{---}} \quad \uparrow R \quad \frac{\boxed{Q} \Rightarrow Q}{\text{---}} \quad \downarrow L \\
\frac{\boxed{Q / (Sf \uparrow ((Nt(s(m)) \setminus Si) \downarrow (Nt(s(m)) \setminus Sf)))} \quad Nt(s(m)), Np(n), \triangleright^{-1}(Np(n) \setminus (Nt(s(m)) \setminus Si)) \Rightarrow Q}{\text{---}} \quad \uparrow R \\
\frac{Q / (Sf \uparrow ((Nt(s(m)) \setminus Si) \downarrow (Nt(s(m)) \setminus Sf))), 1, Np(n), \triangleright^{-1}(Np(n) \setminus (Nt(s(m)) \setminus Si)) \Rightarrow Q \uparrow Nt(s(m))}{\text{---}} \quad \uparrow R \\
\frac{Nt(s(m)) \Rightarrow Nt(s(m)) \quad Q / (Sf \uparrow ((Nt(s(m)) \setminus Si) \downarrow (Nt(s(m)) \setminus Sf))), Np(n), \triangleright^{-1}(Np(n) \setminus (Nt(s(m)) \setminus Si)) \Rightarrow \boxed{(Q \uparrow Nt(s(m)))}}{\text{---}} \quad \uparrow R \\
\frac{Nt(s(m)), Q / (Sf \uparrow ((Nt(s(m)) \setminus Si) \downarrow (Nt(s(m)) \setminus Sf))), Np(n), \triangleright^{-1}(Np(n) \setminus (Nt(s(m)) \setminus Si)) \Rightarrow \boxed{Nt(s(m)) \bullet (Q \uparrow Nt(s(m)))}}{\text{---}} \quad \bullet R
\end{array}$$

(j, λA (want (read books A) A) A)