

# CatLog: A Categorical Parser/Theorem-Prover\*

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**Abstract.** We present CatLog, a parser/theorem-prover for logical categorical grammar. The logical fragment implemented is a displacement logic the multiplicative basis of which is the displacement calculus of Morrill, Valentín & Fadda (2011)[8].

(Logical) categorical grammar (Morrill 1994[9], 2011[10]; Moortgat 1997[6]; Carpenter 1998[1]; Jäger 2005[4]) originated with Lambek's (1958[5]) insight that a calculus of **grammatical types** (constituting a residuated monoid) can be formulated using **Gentzen's method**. **The result is an algebraic rendering** of grammar as logic and parsing as deduction. Although the design is, really, architecturally perfect and, by now, well-understood, linguistically it is strictly limited to continuity **by the fact that it deals with a residuated family with parent (the canonical extension of) concatenation: after all, the whole challenge** of modern linguistics for 50 years has been the ubiquity in natural grammar of *discontinuity*. In this relation Morrill, Valentín & Fadda (2011)[8] provides for discontinuity the **displacement calculus D**, deductively a conservative extension of the Lambek calculus **L** with residuated families with respect to both concatenation and intercalation. Like **L**, **D** is free of structural rules and enjoys Cut-elimination and its corollaries the subformula property, decidability, and the finite reading property.

CatLog is a categorical parser/theorem prover implementing a categorical logic extending **D**. It employs Cut-free backward chaining sequent theorem-proving. For **L** deductive spurious ambiguity can be removed by normalization (Hendriks 1993[3]). Because **D** is based on the same design principles, the same techniques can be adopted (Morrill 2011[7]) and CatLog depends on this. In addition to normalization CatLog uses sequent search space pruning by the count invariance of van Benthem (1991[11]). The type-constructors of the displacement logic of CatLog are shown in Fig. 1.

Version f1.2 of CatLog is **provisional in a number of respects**. In particular, not all spurious ambiguity is eliminated for the categorical logic fragment, and non-duplication of results is achieved by filtering according to a brute force duplication check. Furthermore, bracketing structure must be specified in the input, rather than be induced. **And the count-invariance check for multiplicatives**

\* This research was partially supported by BASMATI MICINN project (TIN2011-27479-C04-03) and by SGR2009-1428 (LARCA).

$I, \backslash, \cdot, /$	Lambek connectives
$J, \{\downarrow_k, \odot_k, \uparrow_k\}_{k \in \{>, <\}}$	displacement connectives
$\otimes, -$	nondeterministic continuous connectives
$\Downarrow, \odot, \Uparrow$	nondeterministic discontinuous connectives
$\{\overset{\cdot}{\wedge}_k, \overset{\cdot}{\vee}_k\}_{k \in \{>, <\}}$	bridge and split
$\overset{\cdot}{\wedge}, \overset{\cdot}{\vee}$	nondeterministic bridge and split
$\triangleleft^{-1}, \triangleleft, \triangleright, \triangleright^{-1}$	left and right projection and injection
$\&, +$	semantically active additives
$\square, \sqcup$	semantically inactive additives
$\forall, \exists$	first-order quantifiers
$i, ^+$	structural modalities
$[\ ]^{-1}, \langle \rangle$	bracket modalities
$\square, \blacksquare$	normal modalities
$ $	limited contraction for anaphora

Fig. 1. Type-constructors of CatLog

is not adapted to additives and structural modalities. These issues remain topics for future improvement. Nevertheless CatLog f1.2 already provides fast and wide-coverage Montague-like parsing.

The program comprises 3000 lines of Prolog implementing some 80 inference rules for the categorial logic fragment,  $\LaTeX$  outputting, lexicon, and sample sentences. Among the examples four blocks are distinguished: Dutch examples (cross-serial dependencies), relativization including islands and parasitic gaps, the Montague example sentences of Dowty, Wall and Peters (1981)[2] Chapter 7, and the example sentences of Morrill, Valentín and Fadda (2011)[8].

The functionality is as follows. Once CatLog has been loaded into Prolog, the query `?- pplex.` will cause the lexicon to be pretty printed in the console window, the Dutch part of which is as shown in Fig. 2. The query `?- pplexlatex.` has no visible effect but will cause the lexicon to be output in  $\LaTeX$  to a file named “s.tex”. Querying `t(N)` will test the examples unifying with term  $N$ . For example `?- t(rel(6)).` tests the relativization example 6, `?- t(rel(_)).` tests all the relativization examples, and `?- t(_).` tests all the examples. The analyses — the examples, the derivational proofs, and the semantic readings — appear in the Prolog window, and this information but without duplicate equivalent analyses is written in  $\LaTeX$  to a file named “t.tex”.  $\LaTeX$ ing the file “out.tex” will include s.tex and t.tex and format the lexicon and last analyses made. For example, `?- t(d(2)).` produces the contents in Fig. 3 in Prolog. The  $\LaTeX$  output for the Dutch part of the lexicon and the same example is as shown in Figs. 4 and 5.

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wil: (NA\S*i*)in(NA\S*f*): LBLC((want (B C)) C)  
 wil: Q/~(S*f*ex((NA\S*i*)in(NA\S*f*)): LB(B LCLD((want (C D)) D))  
 alles: (SAexNt(s(n)))inSA: LBAC[(thing C) -> (B C)]  
 boeken: Np(n): books  
 cecilia: Nt(s(f)): c  
 de: Nt(s(A))/CNA: the  
 helpen: |>-1((NA\S*i*)in(NB\NA\S*i*)): LCLD((help (C D)) D)  
 henk: Nt(s(m)): h  
 jan: Nt(s(m)): j  
 kan: (NA\S*i*)in(NA\S*f*): LBLC((isable (B C)) C)  
 kunnen: |>-1((NA\S*i*)in(NA\S*f*)): LBLC((isable (B C)) C)  
 las: NA\Nt(s(B))\S*f*: read  
 lezen: |>-1(NA\NB\S*i*): read  
 nijlpaarden: CNp(n): hippos  
 voeren: |>-1(NA\NB\S*i*): feed  
 zag: (Nt(s(A))\S*i*)in(NB\Nt(s(A))\S*f*): LCLD((saw (C D)) D)

Fig. 2. Dutch part of lexicon

(d(2)) jan+boeken+kan+lezen S\_647

Nt(s(m)): j, Np(n): books, (NA\S*i*)in(NA\S*f*): LBLC((isable (B C)) C), |>-1(NB\NE\S*i*): read => SF

Nt(s(m)), Np(n), (Nt(s(m))\S*i*)in(Nt(s(m))\S*f*), |>-1(Np(n)\Nt(s(m))\S*i*) => Sf [inL]  
 Np(n), 1, |>-1(Np(n)\Nt(s(m))\S*i*) => Nt(s(m))\S*i* [\R]  
 Nt(s(m)), Np(n), 1, |>-1(Np(n)\Nt(s(m))\S*i*) => Si [|>-1L]  
 Nt(s(m)), Np(n), Np(n)\Nt(s(m))\S*i*{1} => Si [\L]  
 Np(n) => Np(n)  
 Nt(s(m)), Nt(s(m))\S*i*{1} => Si [\L]  
 Nt(s(m)) => Nt(s(m))  
 S*i*{1} => S*i*  
 Nt(s(m)), Nt(s(m))\S*f* => Sf [\L]  
 Nt(s(m)) => Nt(s(m))  
 S*f* => S*f*

((isable ((read books) j)) j)

Fig. 3. Dutch verb raising

wil: (NA\S*i*)\NA\S*f*:  $\lambda B \lambda C ((want (B C)) C)$   
 wil: Q/~(S*f*↑((NA\S*i*)\NA\S*f*)):  $\lambda B (B \lambda C \lambda D ((want (C D)) D))$   
 alles: (SA↑Nt(s(n)))\SA:  $\lambda B \forall C [(thing C) \rightarrow (B C)]$   
 boeken: Np(n): books  
 cecilia: Nt(s(f)): c  
 de: Nt(s(A))/CNA: the  
 helpen:  $\triangleright^{-1}((NA\S*i*)\NB\NA\S*i*): \lambda C \lambda D ((help (C D)) D)$   
 henk: Nt(s(m)): h  
 jan: Nt(s(m)): j  
 kan: (NA\S*i*)\NA\S*f*:  $\lambda B \lambda C ((isable (B C)) C)$   
 kunnen:  $\triangleright^{-1}((NA\S*i*)\NA\S*f*): \lambda B \lambda C ((isable (B C)) C)$   
 las: NA\Nt(s(B))\S*f*: read  
 lezen:  $\triangleright^{-1}(NA\NB\S*i*): read$   
 nijlpaarden: CNp(n): hippos  
 voeren:  $\triangleright^{-1}(NA\NB\S*i*): feed$   
 zag: (Nt(s(A))\S*i*)\NB\Nt(s(A))\S*f*:  $\lambda C \lambda D ((saw (C D)) D)$

Fig. 4. Dutch part of lexicon

(d(2)) **jan+boeken+kan+lezen** :  $S_647$

$Nt(s(m)) : j, Np(n) : books, (NA \setminus Si) \downarrow (NA \setminus Sf) : \lambda B \lambda C ((isable (B C)) C), \triangleright^{-1}(ND \setminus (NE \setminus Si)) :$   
 $read \Rightarrow SF$

$$\begin{array}{c}
 \frac{\frac{\frac{Nt(s(m)) \Rightarrow Nt(s(m)) \quad Si\{1\} \Rightarrow Si}{Np(n) \Rightarrow Np(n)} \quad \frac{Nt(s(m)), \boxed{Nt(s(m)) \setminus Si\{1\}} \Rightarrow Si}{Nt(s(m)), Np(n), \boxed{Np(n) \setminus (Nt(s(m)) \setminus Si)\{1\}} \Rightarrow Si} \quad \frac{Nt(s(m)), Np(n), 1, \boxed{\triangleright^{-1}(Np(n) \setminus (Nt(s(m)) \setminus Si))} \Rightarrow Si}{Np(n), 1, \triangleright^{-1}(Np(n) \setminus (Nt(s(m)) \setminus Si)) \Rightarrow Nt(s(m)) \setminus Si} \quad \frac{Nt(s(m)) \Rightarrow Nt(s(m)) \quad Sf \Rightarrow Sf}{Nt(s(m)), \boxed{Nt(s(m)) \setminus Sf} \Rightarrow Sf}}{\frac{Nt(s(m)), Np(n), \boxed{(Nt(s(m)) \setminus Si) \downarrow (Nt(s(m)) \setminus Sf)} \quad \triangleright^{-1}(Np(n) \setminus (Nt(s(m)) \setminus Si)) \Rightarrow Sf}}{\downarrow L}
 \end{array}$$

$((isable ((read books) j)) j)$

**Fig. 5.** Dutch verb raising

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