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GENERALISING DISCONTINUITY

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Résumé - Abstract

Cet article présente la notion de logique catégorielle multimodale avec *sortes*, et fait deux généralisations du calcul catégoriel pour les discontinuités. Dans la première, nous introduisons des modalités unaires qui font la transition entre les chaines continues et les discontinues. Dans la deuxième, les modes d'adjonction du calcul des discontinuités classique, concaténation, juxtaposition et interpolation, sont enrichis de variantes. Des illustrations et des motivations linguistiques sont données, concernant l'ellipse verbale, les phénomènes de "gapping" et les comparatives.

This paper expounds the notion of *sorted* multimodal categorial logic, and makes two generalisations of categorial calculus of discontinuity. In the first, we introduce unary modalities which mediate between continuous and discontinuous strings. In the second, the modes of adjunction of the classical discontinuity calculus, concatenation, juxtaposition and interpolation, are augmented with variants. Linguistic illustration and motivation is provided with reference to VP ellipsis, gapping as like-category coordination, and comparative subdeletion.

Mots Clefs - Keywords

Grammaire catégorielle, grammaire de logique des types, discontinuité, chaines a deux composantes, ellipse verbale, gapping, comparatives.

Categorial grammar, type logical grammar, discontinuity, split strings, VP ellipsis, gapping, comparative subdeletion.

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Introduction

The concern of the present paper, within the broad field of computational grammar, is the treatment of discontinuity in type logical grammar.[1] By *computational grammar* we mean description of language with the precision indigenous to computer science: such that the question of whether some algorithmic process realises the specification made is not a matter of impression, or of what the description might be supposed to mean, but rather of objective, i.e. mathematical, fact. By *logical grammar* Gamut (1991, Vol. 2, p.139) refer to grammar which undertakes to describe not only well-formedness, but also the logical semantic relations between signs of the object-language. We think of a language as a set of signs each comprising a *signifier*, represented by a *prosodic form*, and a *signified*, represented by a *semantic form*; these fall into various parts of speech or categories. The framework of *type logical grammar* (Morrill 1994a, Moortgat 1996a, Carpenter in press) is a theory of categorial grammar as a refinement of logical grammar in which the signs are classified by types such that a language model is determined entirely by lexical type assignment. The approach embodies a pure form of lexicalism which, rather than being monostratal, is no-stratal: there is no essential level of syntactic representation at all, but simply projection from the lexicon according to the mathematicised meaning of type-constructors.

It is natural to conceive of such a pure minimal architecture as an idealisation of the lexicalist trend over the past two decades. However, it is also natural to question whether "significant" linguistic phenomena can be rendered within the bounds of such a stringent purism. Indeed, these are two facets of the same scientific issue: we establish a methodological ideal, and devise increasingly severe tests on its adequacy. In the present context, the question comes down to determining whether there exists type logic suitable for the constructs of linguistic analysis.

The associative Lambek calculus (Lambek 1958) provides a logic of concatenation. Its types are specifications of concatenative comportment and by classifying words with respect to types, properties of strings of words are defined as deductive consequences. The non-associative Lambek calculus (Lambek 1961) is similarly a logic of juxtaposition, by which we mean putting side-by-side in a way which imposes grouping (concatenation, being associative, forgets grouping). But the existence of discontinuous phenomena in natural grammar guarantees that such logic of itself cannot be adequate. To take just one example, addressed in the course of this paper, consider the comparative subdeletion exhibited in the following.

(1) a. John ate more bagels than Mary ate donuts.
    b. John ate more bagels than Mary donuts.
    c. John ate more bagels than Mary.
    d. John ate more bagels than donuts.

In (1a) we see the basic pattern whereby a sentence containing 'more' in determiner position is conjoined by 'than' to one from which a determiner is lacking.

[1] We are grateful to Bob Carpenter and the reviewers for their comments on an earlier version of this article; any shortcomings, however, are fully our responsibility.

Semantically, the cardinality of one set (the bagels eaten by John) is asserted to be greater than that of another (the donuts eaten by Mary). There is discontinuous dependency between the determiner positions of the two sentences. Furthermore, in the paraphrase (1b) there is, in addition, ellipsis of the second transitive verb. In (1c) and (1d) other components are elided, with consequent semantic effects.

The present work continues in the line of others seeking to develop categorial type calculus of discontinuity (Moortgat 1988, 1990, 1996b, 1996c; Solias 1992; Morrill and Solias 1993; Morrill 1994a, chs. 4–5, 1995a, Calcagno 1995, Moortgat and Oehrle 1995). In particular, it generalises the sorted discontinuity calculus outlined in the appendix of Morrill (1995a). We begin by defining a general framework for *sorted multimodality* and we present and exemplify the sorted discontinuity calculus. We then introduce two generalisations: unary "split" and "bridge" operators mediating between strings and split strings, and binary operators for generalised concatenation, juxtaposition and interpolation adjunctions.

1 MULTIMODALITY AND DISCONTINUITY

The first attempts to formulate categorial logic for discontinuity appear in Moortgat (1988), although as documented in that work there are a number allusions to nonconcatenative operators in earlier, non-type logical, literature. Moortgat (1988) defines four binary discontinuous type-constructors: infixors and extractors, each with universal and existential varieties. But as he notes, none of these can be provided with both left and right rules in the Gentzen sequent format. Moortgat (1990) makes another important contribution with the introduction of a binary in situ binder type-constructor,[2] later generalised to a ternary one (Moortgat 1996b) suited, for example, to a Montagovian analysis of quantifying-in. However, it is again not possible to formulate a sufficiently general sequent calculus. Interestingly, the in situ binder is almost (but only almost) definable in terms of an extractor and an infixor.

In discontinuity calculus as presented in Morrill (1994a, chs. 4–5, 1995a) it is sought to combine and extend logic of concatenation and juxtaposition with logic of interpolation within a *multimodal* categorial framework (Moortgat and Morrill 1991). On the multimodal design, a family of type-constructors is defined for each of a set of modes of adjunction in a prosodic algebra. For each mode i we obtain as categorial type-constructors a kind of conjunction $\bullet_i$ (*product*) and its two residual implications (*divisions*) $\backslash_i$ and $/_i$. The expressivity of the multimodal scheme stems from the possibility of establishing interaction postulates between the various modes. Thus, in the discontinuity calculus the three modes $+$ (concatenation), $(\cdot,\cdot)$ (juxtaposition) and W (wrapping) are related by the interaction postulate $s_1{+}s_2{+}s_3 = (s_1, s_3)Ws_2$. This system has a sequent calculus, with left and right inference rules for all of the connectives, and it allows

[2] By "in situ binder" we mean an element which takes semantic scope in a wider constituent, binding the position in which it occurs, e.g. the quantifier phrase in 'John sent every book to Mary' which occurs in the middle of the verb phrase, but which takes sentential scope semantically.

the definition of in situ binding in terms of infixation and extraction in the way anticipated, though not realised, in Moortgat's proposals. It has been applied to a range of phenomena including discontinuous functors and idioms, subject- and object-oriented reflexivisation, quantification, gapping and pied-piping.

In this *unsorted* approach, concatenation, juxtaposition and interpolation are each assumed to be total functions in a single abstract total algebra, and the categorial types are formed from unsorted type-constructors without restriction.

The *sorted* discontinuity calculus is briefly introduced in the appendix of (Morrill 1995a). It is distinguished from the unsorted version in that instead of assuming all adjunctions to be total functions in an unsorted algebra, two sorts of object (string and split string) are assumed so that the adjunctions are sorted operations in a sorted algebra, and the categorial types come in a restricted form according to the sorted type-constructors. This formulation has particularly good computational properties; while the unsorted version has a logic programming implementation depending on matching under associativity and partial commutativity (Morrill 1995b), the sorted version has one depending on just unification of unstructured string positions (Morrill 1995), or difference lists (Lloré and Morrill 1995).

It does not seem necessary to choose just one of the two approaches, sorted or unsorted: each may have its merits. We adopt here the sorted approach because it has until now received less attention, but it is straightforward to reconstrue the generalisations of discontinuity calculus presented here in terms of interaction postulates in an unsorted context. We shall first give a general definition of sorted multimodal calculi, and then define and exemplify the sorted version of the "classical" discontinuity calculus, before going on to consider generalisations.

1.1 Sorted multimodality

We let a sorted prosodic algebra be a structure $\langle\{L_\omega\}_{\omega\in\Omega}, \{+_i\}_{i\in I}\rangle$ with a set Ω of sorts, a set I of adjunctions, a domain L_ω for each sort $\omega\in\Omega$ and a binary operation $+_i$ for each adjunction $i\in I$. We have functionality mappings σ_l, σ_r and σ from I into Ω: each $+_i$ is of functionality $L_{\sigma_l(i)}, L_{\sigma_r(i)}\rightarrow L_{\sigma(i)}$. The well-formed category type formulas $\mathcal{F}_\omega$ for each sort $\omega\in\Omega$ are defined by mutual recursion. Given a set $\mathcal{A}_\omega$ of atomic category formulas of sort ω for each $\omega\in\Omega$, the well-formed category type formulas are given by all instances of (2).

(2)
$$\begin{array}{lcl}
\mathcal{F}_\omega & ::= & \mathcal{A}_\omega\\
\mathcal{F}_{\sigma(i)} & ::= & \mathcal{F}_{\sigma_l(i)}\bullet_i\mathcal{F}_{\sigma_r(i)}\\
\mathcal{F}_{\sigma_r(i)} & ::= & \mathcal{F}_{\sigma_l(i)}\backslash_i\mathcal{F}_{\sigma(i)}\\
\mathcal{F}_{\sigma_l(i)} & ::= & \mathcal{F}_{\sigma(i)}/_i\mathcal{F}_{\sigma_r(i)}
\end{array}$$

A formula of sort ω is to be interpreted as a subset of L_ω. Where $*_i$ is the set-wise generalisation of $+_i$,[3] a (prosodic) interpretation is a function $[\![\cdot]\!]$ mapping each formula of sort ω to a subset of L_ω satisfying the following interpretation clauses.

[3] I.e. for sets S, S': $S *_i S' = \{s{+_i}s' |\ s\in S\ \&\ s'\in S'\}$.

(We use $A, B, \ldots$ to denote category formulas.)

$$
(3) \quad \begin{array}{lcl}
[\![A\bullet_i B]\!] & = & [\![A]\!] *_i [\![B]\!] \\
[\![A\backslash_i B]\!] & = & \{s \in L_{\sigma_r(i)} | \ [\![A]\!] *_i \{s\} \subseteq [\![B]\!]\} \\
[\![B/_i A]\!] & = & \{s \in L_{\sigma_l(i)} | \ \{s\} *_i [\![A]\!] \subseteq [\![B]\!]\}
\end{array}
$$

We shall formulate reasoning using a Prawitz-style (i.e. tree-like) labelled linear natural deduction. Categorial logics of the kind we consider here are occurrence logics: as in linear logic (Girard 1987), the resources represented by formulas cannot be freely reused or left unused but must (except in certain specifically controlled circumstances) be used exactly once. In natural deduction, this resource-consciousness is reflected by tightening the standard conditionalisation procedure of closing any number of assumptions to require closure of a unique assumption.

However, the systems we use are *sub*linear in the space of logics arising from removal of standard structural rules. Thus while linear logic works with unordered resources, the present systems lack free commutativity also. This means that all theorems must be valid when reading divisions and products as the linear (multiplicative) implication and conjunction. Linear validity is a necessary condition for validity, though it is not sufficient because further sublinear structural conditions must be respected. We express these sublinear properties by labelling formulas and placing conditions on labels to further regulate inference.

In labels we use boldface romans as constants, and we use $\alpha, \beta, \gamma, \ldots$ as variables over prosodic terms. A prosodically labelled formula has the form α: A. The labelled natural deduction rules use metavariables or Skolem constants according to quantifications implicit in the interpretation clauses and the polarity of their context. Thus for example, in $\backslash_i$E α and γ are variables since the meaning of $\backslash_i$ is that any element in $A\backslash B$ right-adjoins to any element in A to yield an element in B; and in $\backslash_i$I the assumption A is labelled with a Skolem constant **a** (i.e. a uniquely occurring constant denoting an arbitrary element in A) since the meaning of $A\backslash_i B$ requires us to show, in order to assert that γ is in $A\backslash_i B$, that γ forms a B element when right-adjoined to every A element we care to choose.[4]

[4] We give in the main text only the introduction rule for product since the elimination rule is slightly complex, and little motivated linguistically. However, a product elimination rule can be given as follows:

$$
\text{(i)} \quad \cfrac{\delta\colon A\bullet_i B \qquad \begin{array}{c} \overline{\mathbf{a}_1\colon A}^{\,n} \qquad \overline{\mathbf{a}_2\colon B}^{\,n} \\ \vdots \\ \gamma(\mathbf{a}_1{+}\mathbf{a}_2)\colon C \end{array}}{\gamma(\delta)\colon C}\ \bullet_i \mathrm{E}^n
$$

I.e. given δ in $A\bullet_i B$, we can derive $\gamma(\delta)$ in C (γ containing subterm δ at a distinguished position) if by assuming $\mathbf{a}_1$ in A and $\mathbf{a}_2$ in B we can derive $\gamma(\mathbf{a}_1{+}\mathbf{a}_2)$ in C (γ containing subterm $\mathbf{a}_1{+}\mathbf{a}_2$ in that position.)

(4)
$$\frac{\begin{array}{c}\vdots\\ \alpha\text{: } A\end{array}\qquad\begin{array}{c}\vdots\\ \gamma\text{: } A\backslash_i B\end{array}}{\alpha{+_i}\gamma\text{: } B}\backslash_i\mathrm{E} \qquad\qquad \frac{\begin{array}{c}\overline{\mathbf{a}\text{: } A}^{\,n}\\ \vdots\\ \mathbf{a}{+_i}\gamma\text{: } B\end{array}}{\gamma\text{: } A\backslash_i B}\backslash_i\mathrm{I}^n$$

(5)
$$\frac{\begin{array}{c}\vdots\\ \gamma\text{: } B/_i A\end{array}\qquad\begin{array}{c}\vdots\\ \alpha\text{: } A\end{array}}{\gamma{+_i}\alpha\text{: } B}/_i\mathrm{E} \qquad\qquad \frac{\begin{array}{c}\overline{\mathbf{a}\text{: } A}^{\,n}\\ \vdots\\ \gamma{+_i}\mathbf{a}\text{: } B\end{array}}{\gamma\text{: } B/_i A}/_i\mathrm{I}^n$$

(6)
$$\frac{\begin{array}{c}\vdots\\ \alpha\text{: } A\end{array}\qquad\begin{array}{c}\vdots\\ \beta\text{: } B\end{array}}{\alpha{+_i}\beta\text{: } A\bullet_i B}\bullet_i\mathrm{I}$$

This proof format is uniform for each mode of adjunction; it remains to add manipulation of labels according to the specific relations between modes in the prosodic algebra. This may be done by adding equations for label manipulation according to the algebraic laws, or by structuring labels in rules (we shall do the latter).

Also uniform for each mode is the extension of interpretation to include a semantic dimension. We assume as a semantic structure a family of sets $\{D_\tau\}_{\tau\in\mathcal{T}}$ where $\mathcal{T} ::= \mathcal{B} \mid \mathcal{T}{\rightarrow}\mathcal{T} \mid \mathcal{T}\&\mathcal{T}$ is a set of semantic types generated from a set $\mathcal{B}$ of primitive semantic types; $D_{\tau\rightarrow\tau'} = D_{\tau'}^{D_\tau}$, the set of all functions from D_τ to $D_{\tau'}$; and $D_{\tau\&\tau'} = D_\tau \times D_{\tau'}$, the Cartesian product of D_τ and $D_{\tau'}$. A *type map* is a function T from category formulas to semantic types such that $T(A\backslash_i B) = T(B/_i A) = T(A){\rightarrow}T(B)$ and $T(A\bullet_i B) = T(A)\&T(B)$. A formula A is semantically interpreted as a subset of $T(A)$. We generalise the prosodic interpretation given earlier to a two-dimensional, prosodic-semantic, interpretation: the interpretation function $[\![\cdot]\!]$ maps each formula A of sort ω to a subset of $L_\omega \times T(A)$ satisfying the following interpretation clauses.

(7)
$$\begin{array}{rcl}
[\![A\backslash_i B]\!] & = & \{\langle s,m\rangle \mid \forall\langle s',m'\rangle\in[\![A]\!], \langle s'{+_i}s, m(m')\rangle\in[\![B]\!]\}\\
[\![B/_i A]\!] & = & \{\langle s,m\rangle \mid \forall\langle s',m'\rangle\in[\![A]\!], \langle s{+_i}s', m(m')\rangle\in[\![B]\!]\}\\
[\![A\bullet_i B]\!] & = & \{\langle s,m\rangle \mid \exists\langle s_1,m_1\rangle\in[\![A]\!], \langle s_2,m_2\rangle\in[\![B]\!],\\
 & & \qquad\qquad s = s_1{+}s_2, m = \langle m_1,m_2\rangle\}
\end{array}$$

To incorporate the semantic dimension in the generic inference schemata we use $\phi,\psi,\chi,\ldots$ as variables over semantic terms, which are terms of typed lambda calculus including, in addition to functional application $(\phi\ \psi)$ and functional abstraction $\lambda x\phi$, pairing (ϕ,ψ) and first and second projections $\pi_1\phi$ and $\pi_2\phi$. A prosodically and semantically labelled formula has the form $\alpha - \phi$: A; the rules are as follows:[5]

(8)
$$\frac{\begin{array}{cc}\vdots & \vdots\\ \alpha - \phi\colon A & \gamma - \chi\colon A\backslash_i B\end{array}}{\alpha{+_i}\gamma - (\chi\ \phi)\colon B}\backslash_i\mathrm{E} \qquad \frac{\begin{array}{c}\overline{\mathbf{a} - x\colon A}^{\,n}\\ \vdots\\ \mathbf{a}{+_i}\gamma - \psi\colon B\end{array}}{\gamma - \lambda x\psi\colon A\backslash_i B}\backslash_i\mathrm{I}^n$$

(9)
$$\frac{\begin{array}{cc}\vdots & \vdots\\ \gamma - \chi\colon B/_iA & \alpha - \phi\colon A\end{array}}{\gamma{+_i}\alpha - (\chi\ \phi)\colon B}/_i\mathrm{E} \qquad \frac{\begin{array}{c}\overline{\mathbf{a} - x\colon A}^{\,n}\\ \vdots\\ \gamma{+_i}\mathbf{a} - \psi\colon B\end{array}}{\gamma - \lambda x\psi\colon B/_iA}/_i\mathrm{I}^n$$

(10)
$$\frac{\begin{array}{cc}\vdots & \vdots\\ \alpha - \phi\colon A & \beta - \psi\colon B\end{array}}{\alpha{+_i}\beta - (\phi, \psi)\colon A\bullet_i B}\bullet_i\mathrm{I}$$

1.2 Sorted discontinuity calculus

We now present the sorted discontinuity calculus, an instance of sorted multimodality. Let us assume a monoid $\langle L, +, \varepsilon\rangle$ comprising the set of strings over some vocabulary, with $+$ the associative operation of concatenation and with ε the empty string. The concatenation adjunction $+$ has functionality $L, L \to L$. We define a juxtaposition adjunction $(\cdot,\cdot)$ which is ordered pair formation over L, of functionality $L, L \to L^2$; $(s_1, s_2) =_{df} \langle s_1, s_2\rangle$. And we further define an interpolation adjunction W of functionality $L^2, L \to L$; $\langle s_1, s_2\rangle W s =_{df} s_1{+}s{+}s_2$. Because these operations are sorted, the categorial types and type-constructors defined with respect to them are correspondingly sorted. We refer to sort L as sort *string*, and sort L^2 as sort *split string*.

The family of concatenation connectives $\{/, \backslash, \bullet\}$ are defined with respect to the concatenation adjunction $+$. The existential conjunction (product) $A{\bullet}B$ (A product B) is the set-wise sum of the concatenation adjunction over A and B; $A\backslash B$ (A under B) and B/A (B over A) are the universal directional implications (divisions). Each of these type-constructors requires its operands to be of sort string and produces a composite type of sort string.

The family of juxtaposition connectives $\{<, >, \diamond\}$ are defined by residuation

[5] The product elimination rule is thus:

(i)
$$\frac{\begin{array}{cc}\vdots & \begin{array}{c}\overline{\mathbf{a}_1 - \pi_1\omega\colon A}^{\,n} \qquad \overline{\mathbf{a}_2 - \pi_2\omega\colon B}^{\,n}\\ \vdots\end{array}\\ \delta - \omega\colon A\bullet_i B & \gamma(\mathbf{a}_1{+}\mathbf{a}_2) - \chi\colon C\end{array}}{\gamma(\delta) - \chi\colon C}\bullet_i\,\mathrm{E}^n$$

with respect to the juxtaposition adjunction $(\cdot,\cdot)$. The product $A\circ B$ is the set-wise sum of the juxtaposition adjunction over A and B; $A{>}B$ (A to B) and $B{<}A$ (B from A) are the directional divisions. Since juxtaposition combines two strings to form a split string, product types are of sort split string with sort string operands; and divisor types are of sort string and have the denominator type of sort string and the numerator type of sort split string.

The family of interpolation connectives $\{\uparrow, \downarrow, \odot\}$ are defined by residuation with respect to the interpolation adjunction W. The product $A\odot B$ is the set-wise sum of the interpolation adjunction over A and B; $A{\downarrow}B$ (A infix B) and $B{\uparrow}A$ (B extract A) are the divisions. Sorting considerations apply in ways similar to those made before. Overall, let us assume that atomic formulas $\mathcal{A}$ are of sort string;[6] then the well-sorted category formulas $\mathcal{F}_n$ of sort L^n, i.e. $\mathcal{F}_1$ of sort string and $\mathcal{F}_2$ of sort split string, are defined by mutual recursion by the rules derived as shown in (2) and summarised in (11).

$$(11)\quad \begin{array}{lll} \mathcal{F}_1 & ::= & \mathcal{A} \mid \mathcal{F}_1/\mathcal{F}_1 \mid \mathcal{F}_1\backslash\mathcal{F}_1 \mid \mathcal{F}_1\bullet\mathcal{F}_1 \mid \mathcal{F}_2{<}\mathcal{F}_1 \mid \mathcal{F}_1{>}\mathcal{F}_2 \mid \mathcal{F}_2{\downarrow}\mathcal{F}_1 \mid \mathcal{F}_2\odot\mathcal{F}_1 \\ \mathcal{F}_2 & ::= & \mathcal{F}_1\circ\mathcal{F}_1 \mid \mathcal{F}_1{\uparrow}\mathcal{F}_1 \end{array}$$

Each formula A of sort string has a prosodic interpretation $[\![A]\!] \subseteq L$ and each formula A of sort split string has a prosodic interpretation $[\![A]\!] \subseteq L^2$, given by the general scheme (3). The individual cases are as follows:

$$(12)\quad \begin{array}{lll} [\![A\backslash B]\!] & = & \{s \mid \forall s' \in [\![A]\!], s'{+}s \in [\![B]\!]\} \\ [\![B/A]\!] & = & \{s \mid \forall s' \in [\![A]\!], s{+}s' \in [\![B]\!]\} \\ [\![A\bullet B]\!] & = & \{s \mid \exists s_1 \in [\![A]\!], s_2 \in [\![B]\!], s = s_1{+}s_2\} \end{array}$$

$$(13)\quad \begin{array}{lll} [\![A{>}B]\!] & = & \{s \mid \forall s' \in [\![A]\!], \langle s', s\rangle \in [\![B]\!]\} \\ [\![B{<}A]\!] & = & \{s \mid \forall s' \in [\![A]\!], \langle s, s'\rangle \in [\![B]\!]\} \\ [\![A\circ B]\!] & = & \{\langle s_1, s_2\rangle \mid s_1 \in [\![A]\!] \;\&\; s_2 \in [\![B]\!]\} \end{array}$$

$$(14)\quad \begin{array}{lll} [\![A{\downarrow}B]\!] & = & \{s \mid \forall\langle s_1, s_2\rangle \in [\![A]\!], s_1{+}s{+}s_2 \in [\![B[\![\} \\ [\![B{\uparrow}A[\![& = & \{\langle s_1, s_2\rangle \mid \forall s \in [\![A]\!], s_1{+}s{+}s_2 \in [\![B[\![\} \\ [\![A\odot B]\!] & = & \{s \mid \exists\langle s_1, s_2\rangle \in [\![A]\!], s' \in [\![B]\!], s = s_1{+}s'{+}s_2\} \end{array}$$

We also give explicitly the individual inference rules, though leaving out the semantic labelling. The labelled natural deduction rules can be seen as a restatement left-to-right and right-to-left of the interpretation clauses rotated ninety degrees clockwise for the elimination (E) rules and anticlockwise for the introduction (I) rules,[7] with metavariables or Skolem constants according to quantifications in the interpretation clauses and the polarity of their context:[8]

$$(15)\quad \dfrac{\begin{array}{cc}\vdots & \vdots \\ \alpha\colon A & \gamma\colon A\backslash B\end{array}}{\alpha{+}\gamma\colon B}\backslash\mathrm{E} \qquad\qquad \dfrac{\begin{array}{c}\overline{\mathbf{a}\colon A}^{\,n} \\ \vdots \\ \mathbf{a}{+}\gamma\colon B\end{array}}{\gamma\colon A\backslash B}\backslash\mathrm{I}^n$$

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[6] Of course one may also assume, if desired, atomic formulas of sort split string.

[7] Cf. Ranta (1994) for type theory, who attributes the observation to Martin-Löf (1987).

[8] The product elimination rules can be given as follows:

(16)
$$\dfrac{\begin{array}{cc}\vdots & \vdots\\ \gamma\colon B/A & \alpha\colon A\end{array}}{\gamma{+}\alpha\colon B}/\mathrm{E} \qquad \dfrac{\begin{array}{c}\overline{\mathbf{a}\colon A}^{n}\\ \vdots\\ \gamma{+}\mathbf{a}\colon B\end{array}}{\gamma\colon B/A}/\mathrm{I}^{n}$$

(17)
$$\dfrac{\begin{array}{cc}\vdots & \vdots\\ \alpha\colon A & \beta\colon B\end{array}}{\alpha{+}\beta\colon A{\bullet}B}\bullet\mathrm{I}$$

(18)
$$\dfrac{\begin{array}{cc}\vdots & \vdots\\ \alpha\colon A & \gamma\colon A{>}B\end{array}}{(\alpha,\gamma)\colon B}{>}\mathrm{E} \qquad \dfrac{\begin{array}{c}\overline{\mathbf{a}\colon A}^{n}\\ \vdots\\ (\mathbf{a},\gamma)\colon B\end{array}}{\gamma\colon A{>}B}{>}\mathrm{I}^{n}$$

(19)
$$\dfrac{\begin{array}{cc}\vdots & \vdots\\ \gamma\colon B{<}A & \alpha\colon A\end{array}}{(\gamma,\alpha)\colon B}{<}\mathrm{E} \qquad \dfrac{\begin{array}{c}\overline{\mathbf{a}\colon A}^{n}\\ \vdots\\ (\gamma,\mathbf{a})\colon B\end{array}}{\gamma\colon B{<}A}{<}\mathrm{I}^{n}$$

(20)
$$\dfrac{\begin{array}{cc}\vdots & \vdots\\ \alpha\colon A & \beta\colon B\end{array}}{(\alpha,\beta)\colon A{\diamond}B}\diamond\mathrm{I}$$

(i)
$$\dfrac{\begin{array}{cc} & \overline{\mathbf{a}_1\colon A}^{n} \quad \overline{\mathbf{a}_2\colon B}^{n}\\ \vdots & \vdots\\ \delta\colon A{\bullet}B & \gamma(\mathbf{a}_1{+}\mathbf{a}_2)\colon C\end{array}}{\gamma(\delta)\colon C}\bullet\mathrm{E}^{n}$$

(ii)
$$\dfrac{\begin{array}{cc} & \overline{\alpha\colon A}^{n} \quad \overline{\beta\colon B}^{n}\\ \vdots & \vdots\\ (\alpha,\beta)\colon A{\diamond}B & \gamma\colon C\end{array}}{\gamma\colon C}\diamond\mathrm{E}^{n}$$

(iii)
$$\dfrac{\begin{array}{cc} & \overline{(\mathbf{a}_1,\mathbf{a}_2)\colon A}^{n} \quad \overline{\mathbf{b}\colon B}^{n}\\ \vdots & \vdots\\ \delta\colon A{\odot}B & \gamma(\mathbf{a}_1{+}\mathbf{b}{+}\mathbf{a}_2)\colon C\end{array}}{\gamma(\delta)\colon C}\odot\mathrm{E}^{n}$$

(21)
$$\dfrac{\begin{array}{cc}\vdots & \vdots\\ (\alpha_1,\alpha_2)\colon A & \gamma\colon A{\downarrow}B\end{array}}{\alpha_1{+}\gamma{+}\alpha_2\colon B}{\downarrow}E \qquad \dfrac{\begin{array}{c}\overline{(\mathbf{a}_1,\mathbf{a}_2)\colon A}^{\,n}\\ \vdots\\ \mathbf{a}_1{+}\gamma{+}\mathbf{a}_2\colon B\end{array}}{\gamma\colon A{\downarrow}B}{\downarrow}I^n$$

(22)
$$\dfrac{\begin{array}{cc}\vdots & \vdots\\ (\gamma_1,\gamma_2)\colon B{\uparrow}A & \alpha\colon A\end{array}}{\gamma_1{+}\alpha{+}\gamma_2\colon B}{\uparrow}E \qquad \dfrac{\begin{array}{c}\overline{\mathbf{a}\colon A}^{\,n}\\ \vdots\\ \gamma_1{+}\mathbf{a}{+}\gamma_2\colon B\end{array}}{(\gamma_1,\gamma_2)\colon B{\uparrow}A}{\uparrow}I^n$$

(23)
$$\dfrac{\begin{array}{cc}\vdots & \vdots\\ (\alpha_1,\alpha_2)\colon A & \beta\colon B\end{array}}{\alpha_1{+}\beta{+}\alpha_2\colon A{\odot}B}{\odot}I$$

1.3 Examples

The works Morrill (1994a, chs. 4–5, 1995a) discuss applications of discontinuity calculus to, for example, quantification and reflexivisation, following Moortgat (1990, 1996b). A quantifier like ‘everyone’ may be given semantics $\lambda x\forall y(x\ y)$ in category (S↑N)↓S, infixing at the position of a nominal extracted from a sentence, i.e. treated as a sentence-level in situ binder. A subject-oriented reflexive pronoun can be similarly treated as a verb phrase level in situ binder with semantics $\lambda x\lambda y((x\ y)\ y)$ in category (VP↑N)↓VP, where VP=N\S.[9] Rather than just repeat this material here, we exemplify discontinuity calculus by reference to VP ellipsis, not yet mentioned in the literature, and we go on to consider its interaction with reflexivisation and quantification.

We describe our treatment by reference to examples from Dalrymple, Shieber and Pereira (1991), showing how categorial proof construction executes the higher-order inference they attest. Consider the following:

(24) Dan likes golf and George does too.

Bob Carpenter (p.c.) proposes analysis of the auxiliary element in VP ellipsis by analogy with a reflexive, as a VP in situ binder in backwards VP modifiers:[10]

(25) **does+too** – $\lambda x\lambda y((x\ y)\ y)$
:= ((VP\VP)↑VP)↓(VP\VP)

The antecedent VP semantics is supplied at both its own position, and that of the auxiliary element. Thus there is the derivation of ‘Dan likes golf and George does too’ in figure 1.

[9] For locality constraints see also Morrill (1992, 1994a, ch. 7, 1994b).
[10] We need the discontinuity apparatus since the auxiliary element may be medial, e.g. ‘Dan likes golf and George does too probably’, where the adverb may scope over just the second conjunct.

$$
\dfrac{\mathbf{Dan}: \mathrm{N} \qquad \dfrac{\mathbf{likes{+}golf}: \mathrm{VP} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\overline{a: \mathrm{N}}^{1} \quad \overline{b: \mathrm{VP}}^{2}}{a{+}b: \mathrm{S}}\backslash \mathrm{E} \qquad \dfrac{\mathbf{and}: (\mathrm{S}\backslash \mathrm{S})/\mathrm{S} \qquad \dfrac{\mathbf{George}: \mathrm{N} \quad \overline{c: \mathrm{VP}}^{3}}{\mathbf{George}{+}c: \mathrm{S}}\backslash \mathrm{E}}{\mathbf{and{+}George}{+}c: \mathrm{S}\backslash \mathrm{S}}/\mathrm{E}}{a{+}b{+}\mathbf{and{+}George}{+}c: \mathrm{S}}\backslash \mathrm{E}}{b{+}\mathbf{and{+}George}{+}c: \mathrm{VP}}\backslash \mathrm{I}^{1}}{\mathbf{and{+}George}{+}c: \mathrm{VP}\backslash \mathrm{VP}}\backslash \mathrm{I}^{2}}{(\mathbf{and{+}George}, \varepsilon): (\mathrm{VP}\backslash \mathrm{VP})\uparrow \mathrm{VP}}\uparrow \mathrm{I}^{3} \qquad \text{does too}}{\mathbf{and{+}George{+}does{+}too}: \mathrm{VP}\backslash \mathrm{VP}}\downarrow \mathrm{E}}{\mathbf{likes{+}golf{+}and{+}George{+}does{+}too}: \mathrm{VP}}\backslash \mathrm{E}}{\mathbf{Dan{+}likes{+}golf{+}and{+}George{+}does{+}too}: \mathrm{S}}\backslash \mathrm{E}
$$

Figure 1. VP ellipsis

$$
\dfrac{\mathbf{d} \qquad \dfrac{\mathbf{lg} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\overline{x}^{1} \quad \overline{y}^{2}}{(y\ x)}\backslash \mathrm{E} \qquad \dfrac{\lambda v \lambda u[v \wedge u] \qquad \dfrac{\mathbf{g} \quad \overline{z}^{3}}{(z\ \mathbf{g})}\backslash \mathrm{E}}{\lambda u[(z\ \mathbf{g}) \wedge u]}/\mathrm{E}}{[(z\ \mathbf{g}) \wedge (y\ x)]}\backslash \mathrm{E}}{\lambda x[(z\ \mathbf{g}) \wedge (y\ x)]}\backslash \mathrm{I}^{1}}{\lambda y \lambda x[(z\ \mathbf{g}) \wedge (y\ x)]}\backslash \mathrm{I}^{2}}{\lambda z \lambda y \lambda x[(z\ \mathbf{g}) \wedge (y\ x)]}\uparrow \mathrm{I}^{3} \qquad \lambda s \lambda t((s\ t)\ t)}{\lambda t \lambda x[(t\ \mathbf{g}) \wedge (t\ x)]}\downarrow \mathrm{E}}{\lambda x[(\mathbf{lg}\ \mathbf{g}) \wedge (\mathbf{lg}\ x)]}\backslash \mathrm{E}}{[(\mathbf{lg}\ \mathbf{g}) \wedge (\mathbf{lg}\ \mathbf{d})]}\backslash \mathrm{E}
$$

Figure 2. Semantics of VP ellipsis

$$
\begin{prooftree}
\AxiomC{$\mathbf{John}$: N}
\AxiomC{$\mathbf{likes}$: VP/N}
\AxiomC{}
\RightLabel{^{1}}
\UnaryInfC{a: N}
\RightLabel{/E}
\BinaryInfC{$\mathbf{likes}{+}a$: VP}
\RightLabel{$\uparrow\mathrm{I}^1$}
\UnaryInfC{$(\mathbf{likes}, \varepsilon)$: VP$\uparrow$N}
\AxiomC{$\mathbf{himself}$: (VP$\uparrow$N)$\downarrow$VP}
\RightLabel{$\downarrow$E}
\BinaryInfC{$\mathbf{likes{+}himself}$: VP}
\AxiomC{$\mathbf{and{+}Bill{+}does{+}too}$: VP$\backslash$VP}
\RightLabel{$\backslash$E}
\BinaryInfC{$\mathbf{likes{+}himself{+}and{+}Bill{+}does{+}too}$: VP}
\RightLabel{$\backslash$E}
\BinaryInfC{$\mathbf{John{+}likes{+}himself{+}and{+}Bill{+}does{+}too}$: VP}
\end{prooftree}
$$

$$
\begin{prooftree}
\AxiomC{$\mathbf{j}$}
\AxiomC{$\mathbf{like}$}
\AxiomC{}
\RightLabel{^{1}}
\UnaryInfC{x}
\RightLabel{/E}
\BinaryInfC{$(\mathbf{like}\ x)$}
\RightLabel{$\uparrow\mathrm{I}^1$}
\UnaryInfC{$\lambda x(\mathbf{like}\ x)$}
\AxiomC{$\lambda p\lambda q((p\ q)\ q)$}
\RightLabel{$\downarrow$E}
\BinaryInfC{$\lambda q((\mathbf{like}\ q)\ q)$}
\AxiomC{$\lambda t\lambda x[(t\ \mathbf{b}) \wedge (t\ x)]$}
\RightLabel{$\backslash$E}
\BinaryInfC{$\lambda x[((\mathbf{like}\ \mathbf{b})\ \mathbf{b}) \wedge ((\mathbf{like}\ x)\ x)]$}
\RightLabel{$\backslash$E}
\BinaryInfC{$[((\mathbf{like}\ \mathbf{b})\ \mathbf{b}) \wedge ((\mathbf{like}\ \mathbf{j})\ \mathbf{j})]$}
\end{prooftree}
$$

Figure 3. Interaction of VP ellipsis and reflexivisation: sloppy reading

Semantics is derived as described above: functional application for rules of implicational use (elimination) and functional abstraction for rules of implicational proof (introduction). In figure 2 we show the normalised semantic terms for each node of figure 1.

Consider now the interaction of VP ellipsis and reflexivisation in (26).

(26) John likes himself and Bill does too.

There is both a strict reading, according to which Bill likes John, and a sloppy one, according to which Bill likes himself. The analyses of VP ellipsis and reflexivisation interact in such a way as to predict precisely this effect. The point is that the in situ reflexive binder may take effect at either a verb phrase formed over 'likes himself', which gives rise to a sloppy reading, or one formed by 'likes himself and Bill does too', giving a strict reading: see the derivations in figures 3 and 4. The constituent 'and Bill does too' is derived as in figure 1.

We also find the correct interaction with quantification in examples like the following.

(27) John greeted everyone when Bill did.

The sentence has two interpretations: "When Bill greeted everyone, John greeted everyone" (with the quantifier scoping in 'greeted everyone' of category N\S), and "for each person, when Bill greeted him/her, so did John" (with the quantifier scoping over the entire sentence). We give the derivations in figure 5. Note that 'when ... did' is analysed in just the same way as 'and ... does too'.

The following two sections generalise the calculus that we have described and exemplified.

$$
\dfrac{\textbf{John}\text{: N} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\textbf{likes}\text{: VP/N} \qquad \dfrac{}{a\text{: N}}{}^{1}}{\textbf{likes}+a\text{: VP}}/\text{E} \qquad \textbf{and+Bill+does+too}\text{: VP}\backslash\text{VP}}{\textbf{likes}+a+\textbf{and+Bill+does+too}\text{: VP}}\backslash\text{E}}{(\textbf{likes}, \textbf{and+Bill+does+too})\text{: VP}\uparrow\text{N}}\uparrow\text{I}^{1} \qquad \textbf{himself}\text{: (VP}\uparrow\text{N)}\downarrow\text{VP}}{\textbf{likes+himself+and+Bill+does+too}\text{: VP}}\downarrow\text{E}}{\textbf{John+likes+himself+and+Bill+does+too}\text{: VP}}\backslash\text{E}
$$

$$
\dfrac{\mathbf{j} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\textbf{like} \qquad \dfrac{}{x}{}^{1}}{(\textbf{like}\ x)}/\text{E} \qquad \lambda t\lambda y[(t\ \mathbf{b}) \wedge (t\ y)]}{\lambda y[((\textbf{like}\ x)\ \mathbf{b}) \wedge ((\textbf{like}\ x)\ y)]}\backslash\text{E}}{\lambda x\lambda y[((\textbf{like}\ x)\ \mathbf{b}) \wedge ((\textbf{like}\ x)\ y)]}\uparrow\text{I}^{1} \qquad \lambda p\lambda q((p\ q)\ q)}{\lambda q[((\textbf{like}\ q)\ \mathbf{b}) \wedge ((\textbf{like}\ q)\ q)]}\downarrow\text{E}}{[((\textbf{like}\ \mathbf{j})\ \mathbf{b}) \wedge ((\textbf{like}\ \mathbf{j})\ \mathbf{j})]}\backslash\text{E}
$$

Figure 4. Interaction of VP ellipsis and reflexivisation: strict reading

$$\dfrac{\mathbf{John}: \mathrm{N} \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\overline{b: \mathrm{N}}^{2}\quad \mathbf{greeted}: \mathrm{VP/N} \quad \overline{a: \mathrm{N}}^{1}}{b+\mathbf{greeted}+a: \mathrm{S}}}{(b+\mathbf{greeted}, \varepsilon): \mathrm{S}\uparrow\mathrm{N}}\uparrow \mathrm{I}^1 \quad \mathbf{everyone}: (\mathrm{S}\uparrow\mathrm{N})\downarrow\mathrm{S}}{b+\mathbf{greeted}+\mathbf{everyone}: \mathrm{S}}\downarrow \mathrm{E}}{\mathbf{greeted}+\mathbf{everyone}: \mathrm{VP}}\backslash \mathrm{I}^2 \quad \mathbf{when}+\mathbf{Bill}+\mathbf{did}: \mathrm{VP}\backslash\mathrm{VP}}{\mathbf{greeted}+\mathbf{everyone}+\mathbf{when}+\mathbf{Bill}+\mathbf{did}: \mathrm{VP}}\backslash \mathrm{E}}{\mathbf{John}+\mathbf{greeted}+\mathbf{everyone}+\mathbf{when}+\mathbf{Bill}+\mathbf{did}: \mathrm{VP}}\backslash \mathrm{E}$$

$$\dfrac{\dfrac{\dfrac{\mathbf{John}: \mathrm{N} \quad \dfrac{\dfrac{\mathbf{greeted}: \mathrm{VP/N} \quad \overline{d: \mathrm{N}}^{4}}{\mathbf{greeted}+d: \mathrm{VP}}/\mathrm{E} \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\overline{a: \mathrm{N}}^{1}\quad \overline{b: \mathrm{VP}}^{2}}{a+b: \mathrm{S}}\backslash \mathrm{E} \quad \dfrac{\mathbf{when}: (\mathrm{S}\backslash\mathrm{S})/\mathrm{S} \quad \dfrac{\mathbf{Bill}: \mathrm{N} \quad \overline{c: \mathrm{VP}}^{3}}{\mathbf{Bill}+c: \mathrm{S}}\backslash \mathrm{E}}{\mathbf{when}+\mathbf{Bill}+c: \mathrm{S}\backslash\mathrm{S}}/\mathrm{E}}{a+b+\mathbf{when}+\mathbf{Bill}+c: \mathrm{S}}\backslash \mathrm{E}}{b+\mathbf{when}+\mathbf{Bill}+c: \mathrm{VP}}\backslash \mathrm{I}^1}{\mathbf{when}+\mathbf{Bill}+c: \mathrm{VP}\backslash\mathrm{VP}}\backslash \mathrm{I}^2}{(\mathbf{when}+\mathbf{Bill}, \varepsilon): (\mathrm{VP}\backslash\mathrm{VP})\uparrow\mathrm{VP}}\uparrow \mathrm{I}^3 \quad \mathrm{did}}{\mathbf{when}+\mathbf{Bill}+\mathbf{did}: \mathrm{VP}\backslash\mathrm{VP}}\downarrow \mathrm{E}}{\mathbf{greeted}+d+\mathbf{when}+\mathbf{Bill}+\mathbf{did}: \mathrm{VP}}\backslash \mathrm{E}}{\mathbf{John}+\mathbf{greeted}+d+\mathbf{when}+\mathbf{Bill}+\mathbf{did}: \mathrm{S}}\backslash \mathrm{E}}{(\mathbf{John}+\mathbf{greeted}, \mathbf{when}+\mathbf{Bill}+\mathbf{did}): \mathrm{S}\uparrow\mathrm{N}}\uparrow \mathrm{I}^1 \quad \mathrm{everyone}}{\mathbf{John}+\mathbf{greeted}+\mathbf{everyone}+\mathbf{when}+\mathbf{Bill}+\mathbf{did}: \mathrm{S}}\downarrow \mathrm{E}$$

Figure 5. Interaction of VP ellipsis and quantification

2 BRIDGE AND SPLIT OPERATORS

We propose here to enrich the discontinuity calculus of the previous section with two unary operators $\hat{}$ (*bridge*) and $\check{}$ (*split*) which relate continuous and split strings. Bridging is to map split strings into strings and splitting is to map strings into split strings, so the formulas are extended as follows.

(28)
$$\begin{array}{lll} \mathcal{F}_1 & ::= & \mathcal{A} \mid \mathcal{F}_1/\mathcal{F}_1 \mid \mathcal{F}_1\backslash\mathcal{F}_1 \mid \mathcal{F}_1{\bullet}\mathcal{F}_1 \mid \mathcal{F}_2{<}\mathcal{F}_1 \mid \mathcal{F}_1{>}\mathcal{F}_2 \mid \mathcal{F}_2{\downarrow}\mathcal{F}_1 \mid \mathcal{F}_2{\odot}\mathcal{F}_1 \mid {}^{\wedge}\mathcal{F}_2 \\ \mathcal{F}_2 & ::= & \mathcal{F}_1{\circ}\mathcal{F}_1 \mid \mathcal{F}_2{\uparrow}\mathcal{F}_1 \mid {}^{\vee}\mathcal{F}_1 \end{array}$$

Signs in ${}^{\vee}A$ are split strings which once fused give a string in A; signs in ${}^{\wedge}A$ are strings which are the result of fusing some split string in A:[11]

(29)
$$\begin{array}{lll} [\![{}^{\vee}A]\!] & = & \{\langle s_1, s_2\rangle \mid s_1{+}s_2 \in [\![A]\!]\} \\ [\![{}^{\wedge}A]\!] & = & \{s \mid \exists\langle s_1, s_2\rangle \in [\![A]\!], s = s_1{+}s_2\} \end{array}$$

The operators are identity mappings with respect to the semantic dimension of signs.[12] Labelled natural deduction rules are read off the interpretation clauses as before:[13]

(30)
$$\dfrac{\begin{array}{c}\vdots\\(\alpha_1, \alpha_2)\colon {}^{\vee}A\end{array}}{\alpha_1{+}\alpha_2\colon A}{}^{\vee}\mathrm{E} \qquad\qquad \dfrac{\begin{array}{c}\vdots\\ \alpha_1{+}\alpha_2\colon A\end{array}}{(\alpha_1, \alpha_2)\colon {}^{\vee}A}{}^{\vee}\mathrm{I}$$

(31)
$$\dfrac{\begin{array}{c}\vdots\\(\alpha_1, \alpha_2)\colon A\end{array}}{\alpha_1{+}\alpha_2\colon {}^{\wedge}A}{}^{\wedge}\mathrm{I}$$

The bridge and split type-constructors form a conjugate pair of respectively existential and universal modal operators (see Moortgat 1995, 1996d) and thus satisfy laws such as $A \Rightarrow {}^{\vee\wedge}A$ (but not $A \Rightarrow {}^{\wedge\vee}A$):

[11] With a product unit type I such that $[\![I]\!] = \{\varepsilon\}$ we could define ${}^{\vee}A$ as $A{\uparrow}I$ and ${}^{\wedge}A$ as $A{\odot}I$. The more cautious approach in the main text illustrates application that does not require the full power of the product unit in the type language.

[12] I.e. $T({}^{\vee}A) = T({}^{\wedge}A) = T(A)$, $[\![{}^{\vee}A]\!] = \{\langle\langle s_1, s_2\rangle, m\rangle \mid \langle s_1{+}s_2, m\rangle \in [\![A]\!]\}$ and $[\![{}^{\wedge}A]\!] = \{\langle s, m\rangle \mid \exists\langle\langle s_1, s_2\rangle, m\rangle \in [\![A]\!], s = s_1{+}s_2\}$.

[13] The bridge elimination rule is thus:

(i)
$$\dfrac{\begin{array}{c}\\ \vdots\\ \beta\colon {}^{\wedge}A\end{array} \quad \begin{array}{c}\overline{(\mathbf{a}_1, \mathbf{a}_2)\colon A}^{\,n}\\ \vdots\\ \gamma(\mathbf{a}_1{+}\mathbf{a}_2)\colon C\end{array}}{\gamma(\beta)\colon C}{}^{\wedge}\mathrm{E}^n$$

$$\dfrac{\dfrac{\textbf{John+does+not+resemble+Jimmy}\text{: S}}{(\textbf{John+does+not},\ \textbf{resemble+Jimmy})\text{: }{}^{\vee}\text{S}}{}^{\vee}\text{I} \qquad \textbf{of+course}\text{: }{}^{\vee}\text{S}{\downarrow}\text{S}}{\textbf{John+does+not+of+course+resemble+Jimmy}\text{: S}}{\downarrow}\text{E}$$

Figure 6. Parenthetical infixation

$$\dfrac{\textbf{that}\text{: (CN\\CN)/}{}^{\wedge}\text{(S}{\uparrow}\text{N)} \qquad \dfrac{\dfrac{\dfrac{\textbf{Mary}\text{: N} \qquad \dfrac{\dfrac{\textbf{sent}\text{: ((N\\S)/PP)/N} \qquad \dfrac{}{\textbf{b}\text{: N}}1}{\textbf{sent+b}\text{: (N\\S)/PP}}/\text{E} \qquad \textbf{to+John}\text{: PP}}{\textbf{sent+b+to+John}\text{: N\\S}}/\text{E}}{\textbf{Mary+sent+b+to+John}\text{: S}}\backslash\text{E}}{(\textbf{Mary+sent},\ \textbf{to+John})\text{: S}{\uparrow}\text{N}}{\uparrow}\text{I}^{1}}{\textbf{Mary+sent+to+John}\text{: }{}^{\wedge}\text{(S}{\uparrow}\text{N)}}{}^{\wedge}\text{I}}{\textbf{that+Mary+sent+to+John}\text{: CN\\CN}}/\text{E}$$

Figure 7. Medial relativisation

(32)
$$\dfrac{\dfrac{(\alpha_1, \alpha_2)\text{: } A}{\alpha_1{+}\alpha_2\text{: } {}^{\wedge}A}{}^{\wedge}\text{I}}{(\alpha_1, \alpha_2)\text{: } {}^{\vee\wedge}A}{}^{\vee}\text{I}$$

By way of linguistic illustration of splitting, let us observe that, although very many questions of rhythm and intonation (and their relation to information structure) are implicated, *parentheticals* may, to a rough first approximation, be considered as infixing at arbitrary positions in a sentence:

(33) a. John, of course, does not resemble Jimmy.
b. John does, of course, not resemble Jimmy.
c. John does not, of course, resemble Jimmy.
d. John does not resemble, of course, Jimmy.

Such an effect is achieved by assignment of a parenthetical element like 'of course' to ${}^{\vee}\text{S}{\downarrow}\text{S}$. Thus we obtain, for example, (33c) in the manner shown in figure 6, but in view of the associativity of concatenation we likewise obtain all the other cases.[14]

By way of illustration of bridging, consider the fact that relativisation may be medial, e.g. in 'that Mary sent to John' the extraction site is the direct object position left of the indirect object. We observe that assignment of a type

[14] For partial non-associativity, restricting free variation of prosodic phrasing, see e.g. Morrill (1994a, ch. 7).

(CN\CN)/^(S↑N) to a relative pronoun allows medial relativisation to be generated as a continuous string as shown in figure 7.[15]

3 GENERALISED SORTED DISCONTINUITY CALCULUS

In this section we address the fact that although the classical discontinuity calculus has a range of applications, technically it uses only a small number of the possible binary operations on strings and split strings. Here we illustrate the wider space that exists, and consider some linguistic applications.

3.1 Binary discontinuity connectives

The concatenation, juxtaposition, and interpolation adjunctions of the discontinuity calculus can be represented as follows:

(34)
$$\boxed{\alpha} + \boxed{\beta} \quad = \quad \boxed{\alpha}\boxed{\beta}$$
$$(\boxed{\alpha}, \boxed{\beta}) \quad = \quad \boxed{\alpha}\cdots\boxed{\beta}$$
$$\boxed{\alpha}\cdots\boxed{\gamma}\ W\ \boxed{\beta} \quad = \quad \boxed{\alpha}\boxed{\beta}\boxed{\gamma}$$

These are natural operations in the realm of strings and split strings, but others are imaginable, and linguistically motivated. We shall present some of these (but only some of these), organised by the functionality of their adjunctions.

First, there are versions of concatenation which take as one operand a split string and which fuse this operand before concatenating with the other.[16] There is a left-handed version of functionality $L^2, L \to L$ and a right-handed one of functionality $L, L^2 \to L$:

(35)
$$\boxed{\alpha}\cdots\boxed{\beta} +_l \boxed{\gamma} \quad = \quad \boxed{\alpha}\boxed{\beta}\boxed{\gamma}$$
$$\boxed{\alpha} +_r \boxed{\beta}\cdots\boxed{\gamma} \quad = \quad \boxed{\alpha}\boxed{\beta}\boxed{\gamma}$$

Next, there are left and right inheriting juxtapositions of functionality $L^2, L \to L^2$ and $L, L^2 \to L^2$ respectively. One operand is a split string, and its split point is

[15] The compound $^\wedge(B{\uparrow}A)$ is equivalent to (the existential version) of Moortgat's (1988) extractor, for which an elimination rule could not be given. We can see now that such a rule could be derived from those for $^\wedge$ and $\uparrow$, but that it would be in terms of a determinate split point, and concealment of the split point. A similar point can be made regarding $^\vee A{\downarrow}B$ and Moortgat's universal infixation. Likewise, it is the determinacy of juxtaposition which enables us to define in situ binding in terms of extraction and infixation. Thus, the proposals we offer provide decompositions of Moortgat's operators which resolve technical anomilies.

[16] Many of the operations we list are interdefinable given fusing. Indeed, given juxtaposition the associative concatenation adjunction can be defined as the fusion of juxtaposition, e.g. $A\backslash B = A{>}^\vee B$. We do not pursue these questions of definability here.

inherited in the output:

(36) $$(\boxed{\alpha}\cdots\boxed{\beta},_l \boxed{\gamma}) \quad = \quad \boxed{\alpha}\cdots\boxed{\beta}\boxed{\gamma}$$

$$(\boxed{\alpha},_r \boxed{\beta}\cdots\boxed{\gamma}) \quad = \quad \boxed{\alpha}\boxed{\beta}\cdots\boxed{\gamma}$$

Similarly, there are left and right interior interpolations of functionality $L^2, L \rightarrow L^2$ in which the handedness indicates that a split position is kept open at one or other side of the infix:

(37) $$(\boxed{\alpha}\cdots\boxed{\gamma}\ W_l\ \boxed{\beta}) \quad = \quad \boxed{\alpha}\cdots\boxed{\beta}\boxed{\gamma}$$

$$(\boxed{\alpha}\cdots\boxed{\gamma}\ W_r\ \boxed{\beta}) \quad = \quad \boxed{\alpha}\boxed{\beta}\cdots\boxed{\gamma}$$

In $L^2, L^2 \rightarrow L$, with both operands split, we can find "concatenation squared", "juxtaposition squared" and "wrap squared" which close all split points:

(38) $$\boxed{\alpha}\cdots\boxed{\beta} +^2 \boxed{\gamma}\cdots\boxed{\delta} = \boxed{\alpha}\boxed{\gamma}\boxed{\beta}\boxed{\delta}$$

$$(\boxed{\alpha}\cdots\boxed{\beta},^2 \boxed{\gamma}\cdots\boxed{\delta}) = \boxed{\alpha}\boxed{\beta}\boxed{\gamma}\boxed{\delta}$$

$$\boxed{\alpha}\cdots\boxed{\beta}\ W^2\ \boxed{\gamma}\cdots\boxed{\delta} = \boxed{\alpha}\boxed{\gamma}\boxed{\delta}\boxed{\beta}$$

In $L^2, L^2 \rightarrow L^2$ we find variants of each of these which inherit a split point to the left, in the middle, or to the right:

(39) $$\boxed{\alpha}\cdots\boxed{\beta} +_l^2 \boxed{\gamma}\cdots\boxed{\delta} = \boxed{\alpha}\cdots\boxed{\gamma}\boxed{\beta}\boxed{\delta}$$

$$\boxed{\alpha}\cdots\boxed{\beta} +_m^2 \boxed{\gamma}\cdots\boxed{\delta} = \boxed{\alpha}\boxed{\gamma}\cdots\boxed{\beta}\boxed{\delta}$$

$$\boxed{\alpha}\cdots\boxed{\beta} +_r^2 \boxed{\gamma}\cdots\boxed{\delta} = \boxed{\alpha}\boxed{\gamma}\boxed{\beta}\cdots\boxed{\delta}$$

(40) $$(\boxed{\alpha}\cdots\boxed{\beta},_l^2 \boxed{\gamma}\cdots\boxed{\delta}) = \boxed{\alpha}\cdots\boxed{\beta}\boxed{\gamma}\boxed{\delta}$$

$$(\boxed{\alpha}\cdots\boxed{\beta},_m^2 \boxed{\gamma}\cdots\boxed{\delta}) = \boxed{\alpha}\boxed{\beta}\cdots\boxed{\gamma}\boxed{\delta}$$

$$(\boxed{\alpha}\cdots\boxed{\beta},_r^2 \boxed{\gamma}\cdots\boxed{\delta}) = \boxed{\alpha}\boxed{\beta}\boxed{\gamma}\cdots\boxed{\delta}$$

(41) $$\boxed{\alpha}\cdots\boxed{\beta}\ W_l^2\ \boxed{\gamma}\cdots\boxed{\delta} = \boxed{\alpha}\cdots\boxed{\gamma}\boxed{\delta}\boxed{\beta}$$

$$\boxed{\alpha}\cdots\boxed{\beta}\ W_m^2\ \boxed{\gamma}\cdots\boxed{\delta} = \boxed{\alpha}\boxed{\gamma}\cdots\boxed{\delta}\boxed{\beta}$$

$$\boxed{\alpha}\cdots\boxed{\beta}\ W_r^2\ \boxed{\gamma}\cdots\boxed{\delta} = \boxed{\alpha}\boxed{\gamma}\boxed{\delta}\cdots\boxed{\beta}$$

$+_j$	$\backslash_j$	under (left, right)	$/_j$	over (left, right)	$\bullet_j$	concatenate (left, right)
$(\cdot,_j \cdot)$	$>_j$	to (left, right)	$<_j$	from (left, right)	$\diamond_j$	juxtapose (left, right)
W_j	$\uparrow_j$	extract (left, right)	$\downarrow_j$	infix (left, right)	$\odot_j$	wrap (left, right)
$+^2_k$	$\backslash^2_k$	under squared	$/^2_k$	over squared	$\bullet^2_k$	concatenate squared
$(\cdot,^2_k \cdot)$	$>^2_k$	to squared	$<^2_k$	from squared	$\diamond^2_k$	juxtapose squared
W^2_k	$\uparrow^2_k$	extract squared	$\downarrow^2_k$	infix squared	$\odot^2_k$	wrap squared

Figure 8. Generalised discontinuity connectives

Adding these adjunctions, the community of discontinuity connectives becomes generalised to that shown in figure 8, where $j \in \{0, l, r\}$ and $k \in \{0, l, m, r\}$. (Above, the zero variants are written with subscripts omitted, and they continue to be so below.)

Interpretation is made by residuation in exactly the way defined earlier. By way of example, for the concatenation squared family we have:

(42)
$$\begin{array}{lcl} [\![A\backslash^2 B]\!] & = & \{\langle s_3, s_4\rangle |\ \forall\langle s_1, s_2\rangle \in [\![A]\!], s_1{+}s_3{+}s_2{+}s_4 \in [\![B]\!]\} \\ [\![B/^2 A]\!] & = & \{\langle s_1, s_2\rangle |\ \forall\langle s_3, s_4\rangle \in [\![A]\!], s_1{+}s_3{+}s_2{+}s_4 \in [\![B]\!]\} \\ [\![A\bullet^2 B]\!] & = & \{s |\ \exists\langle s_1, s_2\rangle \in [\![A]\!], \langle s_3, s_4\rangle \in [\![B]\!], s = s_1{+}s_3{+}s_2{+}s_4\} \end{array}$$

We do not list labelled deduction rules since these are entirely predictable, being obtained in just the same way as those for the original discontinuity calculus. To mention but a single case, under squared elimination is (43).

(43)
$$\frac{\begin{array}{cc} \vdots & \vdots \\ (\alpha_1, \alpha_2)\colon A & (\beta_1, \beta_2)\colon A\backslash^2 B \end{array}}{\alpha_1{+}\beta_1{+}\alpha_2{+}\beta_2\colon B}\backslash^2\mathrm{E}$$

3.2 Examples

3.2.1 Gapping as Like-Category Coordination

We present a characterisation of gapping as like-category coordination. We take our inspiration from Hendriks (1995a, 1995b), but our generalisation of the discontinuity calculus is different from hers; in particular our generalised discontinuity calculus has the computationally convenient sorted formulation which the generalisation of Hendriks necessarily lacks,[17] but our analysis can be said to be borrowed. Essentially, the analysis is that the coordinator combines with conjuncts which are sentences lacking some medial element, and that the coordinate structure combines with one such element, interpolating it in the left conjunct (cf. Steedman 1990). We treat the example 'John likes Paris and Mary, Rome' by assigning 'and' the like-category coordinator type $(X{>_l}X)/_rX$ where X is S$\uparrow$TV and TV is (N\S)/N, with semantics $\lambda x\lambda y\lambda z[(y\ z) \wedge (x\ z)]$; see figure 9. 'Mary

[17] The problem is that her interaction principle [MA], (1995b p.113) requires Δ_2 to be of split string sort *qua* the split operand of wrap (her notation is swapped relative to ours); but then since the new g-mode gets a string sort left operand in the top line, it cannot take first operand Δ_2, of split string sort, in the second line.

$$\dfrac{\dfrac{(\mathbf{John},\ \mathbf{Paris})\colon \mathrm{S}{\uparrow}\mathrm{TV} \qquad \dfrac{\mathrm{and} \qquad \dfrac{\dfrac{\mathbf{Mary}\colon \mathrm{N} \quad \overline{\mathbf{a}\colon \mathrm{TV}}^{1} \quad \mathbf{Rome}\colon \mathrm{N}}{\mathbf{Mary{+}a{+}Rome}\colon \mathrm{S}}}{(\mathbf{Mary}, \mathbf{Rome})\colon \mathrm{S}{\uparrow}\mathrm{TV}}{\uparrow}\mathrm{I}^1}{\mathbf{and{+}Mary{+}Rome}\colon \mathrm{X}{>_l}\mathrm{X}}/_r\mathrm{E}}{(\mathbf{John}, \mathbf{Paris{+}and{+}Mary{+}Rome})\colon \mathrm{S}{\uparrow}\mathrm{TV}}{>_r}\mathrm{E} \qquad \mathbf{likes}\colon \mathrm{TV}}{\mathbf{John{+}likes{+}Paris{+}and{+}Mary{+}Rome}\colon \mathrm{S}}{\uparrow}\mathrm{E}$$

$$\dfrac{\dfrac{\lambda x((x\ \mathbf{p})\ \mathbf{j}) \qquad \dfrac{\lambda x\lambda y\lambda z[(y\ z) \wedge (x\ z)] \qquad \dfrac{\dfrac{\mathbf{m} \quad \overline{x}^{1} \quad \mathbf{r}}{((x\ \mathbf{r})\ \mathbf{m})}}{\lambda x((x\ \mathbf{r})\ \mathbf{m})}{\uparrow}\mathrm{I}^1}{\lambda y\lambda z[(y\ z) \wedge ((z\ \mathbf{r})\ \mathbf{m})]}/_r\mathrm{E}}{\lambda z[((z\ \mathbf{p})\ \mathbf{j}) \wedge ((z\ \mathbf{r})\ \mathbf{m})]}{>_r}\mathrm{E} \qquad \mathbf{like}}{[((\mathbf{like}\ \mathbf{p})\ \mathbf{j}) \wedge ((\mathbf{like}\ \mathbf{r})\ \mathbf{m})]}{\uparrow}\mathrm{E}$$

Figure 9. Gapping as like-category coordination

a Rome' is derived straightforwardly as a sentence from the hypothetical transitive verb **a**. The hypothetical can be withdrawn to yield a split form which wants to wrap around a transitive verb to form a sentence. The coordinator combines with this by over right elimination to form a continuous string. The left-hand conjunct 'John Paris' is also derivable as S↑TV, in just the same way as 'Mary Rome'; when the coordinator combines with this conjunct, by to left elimination, the split marking of *this* conjunct is inherited by the result, again in type S↑TV. So this wraps around the transitive verb interpolating it in the first conjunct, and distributes its semantics over the conjuncts. This example illustrates "first-order" variants of the classical adjunctions. Our next example will illustrate concatenation squared and wrap squared.

3.2.2 Comparative Subdeletion

We make the second illustration of generalised discontinuity by reference to comparative subdeletion. Again the initial treatment is inspired by Hendriks (1995a, 1995b), but it uses the present sorted calculus. Our analytical perspective is that in examples such as (44), 'more ... than' combines with two sentences each lacking one quantifier; 'more' occupies the determiner gap in the first, and the two sentences are conjoined by 'than'.

(44) John ate more bagels than Mary ate donuts.

Semantically there is a comparison between the cardinality of the set of bagels that John ate, and the cardinality of the set of donuts that Mary ate. As a first approximation, let us assume the following assignment, where D abbreviates the

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathbf{John}: \mathrm{N} \quad \mathbf{ate}: \mathrm{TV} \quad \overline{\mathbf{a}: \mathrm{D}}^{1} \quad \mathbf{bagels}: \mathrm{CN}}{\mathbf{John{+}ate{+}a{+}bagels}: \mathrm{S}}}{(\mathbf{John{+}ate}, \mathbf{bagels}): \mathrm{S}{\uparrow}\mathrm{D}}{\uparrow}\mathrm{I}^1 \qquad \text{more than}}{\mathbf{John{+}ate{+}more{+}bagels{+}than}: \mathrm{S}/_r(\mathrm{S}{\uparrow}\mathrm{D})}\backslash^2\mathrm{E} \qquad (\mathbf{Mary{+}ate}, \mathbf{donuts}): \mathrm{S}{\uparrow}\mathrm{D}}{\mathbf{John{+}ate{+}more{+}bagels{+}than{+}Mary{+}ate{+}donuts}: \mathrm{S}}/_r\mathrm{E}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathbf{j} \quad \mathbf{eat} \quad \overline{w}^{1} \quad \mathbf{bagel}}{((w\ \mathbf{bagel})\ \lambda u((\mathbf{eat}\ u)\ \mathbf{j}))}}{\lambda w((w\ \mathbf{bagel})\ \lambda u((\mathbf{eat}\ u)\ \mathbf{j}))}{\uparrow}\mathrm{I}^1 \qquad \text{more than}}{\lambda y[\lambda z[(\mathbf{bagel}\ z) \wedge ((\mathbf{eat}\ z)\ \mathbf{j})] > \lambda z(y\ \lambda p\lambda q[(p\ z) \wedge (q\ z)])]}\backslash^2\mathrm{E} \qquad \lambda w((w\ \mathbf{donut})\ \lambda u((\mathbf{eat}\ u)\ \mathbf{m}))}{[\lambda z[(\mathbf{bagel}\ z) \wedge ((\mathbf{eat}\ z)\ \mathbf{j})] > \lambda z[(\mathbf{donut}\ z) \wedge ((\mathbf{eat}\ z)\ \mathbf{m})]]}/_r\mathrm{E}$$

Figure 10. Comparative subdeletion

determiner type $((\mathrm{S}{\uparrow}\mathrm{N}){\downarrow}\mathrm{S})/\mathrm{CN}$.

(45) (**more**, **than**) – $\lambda x\lambda y[\lambda z(x\ \lambda p\lambda q[(p\ z) \wedge (q\ z)]) > \lambda z(y\ \lambda p\lambda q[(p\ z) \wedge (q\ z)])]$
$:= (\mathrm{S}{\uparrow}\mathrm{D})\backslash^2(\mathrm{S}/_r(\mathrm{S}{\uparrow}\mathrm{D}))$

Then there is the derivation in figure 10 of (44), where TV again abbreviates $(\mathrm{N}\backslash\mathrm{S})/\mathrm{N}$. The relevant semantic comparison of cardinalities is indeed made.

In this example two object determiner positions are related. But the same assignment (45) can also compare subject determiner positions as in (46a) and even make subject-object comparisons as in (46), since the type $\mathrm{S}{\uparrow}\mathrm{D}$ is indifferent to the determiner position.

(46) a. More sheep ran than fish swam.
b. More candidates turned up than John had received applications.

However, one might also hope that a treatment could be given which even extended to include such cases as (47), where various kinds of gapping-like ellipsis accompany the comparison.

(47) a. John ate more bagels than Mary donuts.
b. John ate more bagels than Mary.
c. John ate more bagels than donuts.

This is achieved by the following polymorphic lexical type assignment.

(48) (**more**, **than**) – $\lambda x\lambda y[\lambda u(\pi_1 x\ (\pi_2 x\ \lambda p\lambda q[(p\ u) \wedge (q\ u)])) > \lambda u(y\ (\pi_2 x\ \lambda p\lambda q[(p\ u) \wedge (q\ u)]))]$
$:= ((\mathrm{S}{\uparrow}X)\odot^2_m(X{\uparrow}\mathrm{D}))\backslash^2(\mathrm{S}/_r(\mathrm{S}{\uparrow}X))$

$$\dfrac{\dfrac{\dfrac{}{(\mathbf{a}_1, \mathbf{a}_2)\colon \mathrm{S}{\uparrow}\mathrm{D}}1 \qquad \dfrac{\dfrac{}{\mathbf{b}\colon \mathrm{D}}2}{(\varepsilon, \varepsilon)\colon \mathrm{D}{\uparrow}\mathrm{D}}\mathrm{I}{\uparrow}^2}{(\mathbf{a}_1, \mathbf{a}_2)\colon (\mathrm{S}{\uparrow}\mathrm{D})\odot^2_m(\mathrm{D}{\uparrow}\mathrm{D}) \quad \text{more than}}\odot^2_m\mathrm{I}}{\dfrac{\mathbf{a}_1\mathbf{+more+a}_2\mathbf{+than}\colon \mathrm{S}/_r(\mathrm{S}{\uparrow}\mathrm{D})}{(\mathbf{more}, \mathbf{than})\colon (\mathrm{S}{\uparrow}\mathrm{D})\backslash^2(\mathrm{S}/_r(\mathrm{S}{\uparrow}\mathrm{D}))}\backslash^2\mathrm{I}^1}\backslash^2\mathrm{E}$$

b.

$$\dfrac{\dfrac{\dfrac{}{x}1 \qquad \dfrac{\dfrac{}{y}2}{\lambda yy}\mathrm{I}{\uparrow}^2}{(x, \lambda yy) \quad \text{more than}}\odot^2_m\mathrm{I}}{\dfrac{\phi}{\lambda x\phi}\backslash^2\mathrm{I}^1}\backslash^2\mathrm{E}$$

Figure 11. Derivability of the basic comparative subdeletion assignment

The variable X must be allowed to take values over a range of types including D, S/CN, VP, and VP/CN.[18] These are the categories of the element that may be missing from the right-hand sentence. The left-hand sentence comprises the (middle) wrap squared of a sentence missing an X and an X missing a determiner, thus the ellipsis antecedent X is identified. Semantically, these two components $\pi_1 x$ and $\pi_2 x$ are composed for the left-hand sentence, and in the right-hand sentence the ellipsis context y is composed with the antecedent $\pi_2 x$. First, we show in figure 11 that the assignment (45) is *derivable* from (48) when X=D (ϕ abbreviates $\lambda y[\lambda z(x\ \lambda p\lambda q[(p\ z) \wedge (q\ z)]) > \lambda z(y\ \lambda p\lambda q[(p\ z) \wedge (q\ z)])]$). Thus, we do not need to assume (45) lexically at all: the single lexical assignment (48) will be sufficient not only for the gapping-like cases, but also to yield as a special case the analysis considered until now. Let us further consider 'John ate more bagels than Mary donuts': there is the derivation in figure 12 where X=VP/CN. As is seen, we obtain the same semantics as in figure 10, in accordance with the fact that (44) and (47a) are paraphrases.

In the analysis of 'John ate more bagels than donuts' the category X of ellipsis is S/CN: in the left-hand sentence, (ε, **bagels**) is analysed as S↑(S/CN), and the ellipsis antecedent (**John+ate**, ε) is analysed as (S/CN)↑D, which is composed semantically with the right-hand element. Similarly, in 'John ate more bagels than Mary', X is VP: (**John**, ε) is analysed as S↑VP, and the ellipsis antecedent (**ate**, **bagels**) as VP↑D. Thus, in the generalised discontinuity calculus, (44) and (47) receive a unified account.

[18] As for gapping and coordination in general, there are open questions as to the category classes implicated empirically and the technical recourses appropriate to their characterisation.

$$\dfrac{\overline{a\colon \mathrm{N}}^{1} \qquad \mathbf{ate}\colon \mathrm{TV} \qquad \overline{d\colon \mathrm{N}}^{4}}{a+\mathbf{ate}+d\colon \mathrm{S}} \qquad \dfrac{a+\mathbf{ate}+d\colon \mathrm{S}}{(a+\mathbf{ate}, \varepsilon)\colon \mathrm{S}{\uparrow}\mathrm{N}}{\uparrow}\mathrm{I}^{4} \qquad \dfrac{\overline{b\colon \mathrm{D}}^{2} \qquad \overline{c\colon \mathrm{CN}}^{3}}{b+c\colon (\mathrm{S}{\uparrow}\mathrm{N}){\downarrow}\mathrm{S}}/\mathrm{E}$$

$$\dfrac{(a+\mathbf{ate}, \varepsilon)\colon \mathrm{S}{\uparrow}\mathrm{N} \qquad b+c\colon (\mathrm{S}{\uparrow}\mathrm{N}){\downarrow}\mathrm{S}}{a+\mathbf{ate}+b+c\colon \mathrm{S}}{\downarrow}\mathrm{E} \qquad \dfrac{a+\mathbf{ate}+b+c\colon \mathrm{S}}{\mathbf{ate}+b+c\colon \mathrm{VP}}\backslash\mathrm{I}^{1} \qquad \dfrac{\mathbf{ate}+b+c\colon \mathrm{VP}}{\mathbf{ate}+b\colon \mathrm{X}}/\mathrm{I}^{3} \qquad \dfrac{\mathbf{ate}+b\colon \mathrm{X}}{(\mathbf{ate}, \varepsilon)\colon \mathrm{X}{\uparrow}\mathrm{D}}/\mathrm{I}^{2}$$

$$\dfrac{\mathbf{John}\colon \mathrm{S} \qquad \overline{e\colon \mathrm{X}}^{5} \qquad \mathbf{bagels}\colon \mathrm{CN}}{\mathbf{John}+e+\mathbf{bagels}\colon \mathrm{S}} \qquad \dfrac{\mathbf{John}+e+\mathbf{bagels}\colon \mathrm{S}}{(\mathbf{John}, \mathbf{bagels})\colon \mathrm{S}{\uparrow}\mathrm{X}}{\uparrow}\mathrm{I}^{5}$$

$$\dfrac{(\mathbf{John}, \mathbf{bagels})\colon \mathrm{S}{\uparrow}\mathrm{X} \qquad (\mathbf{ate}, \varepsilon)\colon \mathrm{X}{\uparrow}\mathrm{D}}{(\mathbf{John}+\mathbf{ate}, \mathbf{bagels})\colon (\mathrm{S}{\uparrow}\mathrm{X})\odot^{2}_{m}(\mathrm{X}{\uparrow}\mathrm{D})}\odot^{2}_{m}\mathrm{I}$$

$$\dfrac{(\mathbf{John}+\mathbf{ate}, \mathbf{bagels})\colon (\mathrm{S}{\uparrow}\mathrm{X})\odot^{2}_{m}(\mathrm{X}{\uparrow}\mathrm{D}) \qquad \text{more than}}{\mathbf{John}+\mathbf{ate}+\mathbf{more}+\mathbf{bagels}+\mathbf{than}\colon \mathrm{S}/_{r}(\mathrm{S}{\uparrow}\mathrm{X})}\backslash^{2}\mathrm{E} \qquad \dfrac{\text{Mary donuts}}{(\mathbf{Mary}, \mathbf{donuts})\colon \mathrm{S}{\uparrow}\mathrm{X}}$$

$$\dfrac{\mathbf{John}+\mathbf{ate}+\mathbf{more}+\mathbf{bagels}+\mathbf{than}\colon \mathrm{S}/_{r}(\mathrm{S}{\uparrow}\mathrm{X}) \qquad (\mathbf{Mary}, \mathbf{donuts})\colon \mathrm{S}{\uparrow}\mathrm{X}}{\mathbf{John}+\mathbf{ate}+\mathbf{more}+\mathbf{bagels}+\mathbf{than}+\mathbf{Mary}+\mathbf{donuts}\colon \mathrm{S}}/_{r}\mathrm{E}$$

$$\dfrac{\overline{x}^{1} \qquad \mathbf{eat} \qquad \overline{w}^{4}}{((\mathbf{eat}\ w)\ x)} \qquad \dfrac{((\mathbf{eat}\ w)\ x)}{\lambda w((\mathbf{eat}\ w)\ x)}{\uparrow}\mathrm{I}^{4} \qquad \dfrac{\overline{y}^{2} \qquad \overline{z}^{3}}{(y\ z)}/\mathrm{E}$$

$$\dfrac{\lambda w((\mathbf{eat}\ w)\ x) \qquad (y\ z)}{((y\ z)\ \lambda w((\mathbf{eat}\ w)\ x))}{\downarrow}\mathrm{E} \qquad \dfrac{((y\ z)\ \lambda w((\mathbf{eat}\ w)\ x))}{\lambda x((y\ z)\ \lambda w((\mathbf{eat}\ w)\ x))}\backslash\mathrm{I}^{1} \qquad \dfrac{\lambda x((y\ z)\ \lambda w((\mathbf{eat}\ w)\ x))}{\lambda z\lambda x((y\ z)\ \lambda w((\mathbf{eat}\ w)\ x))}/\mathrm{I}^{3}$$

$$\dfrac{\lambda z\lambda x((y\ z)\ \lambda w((\mathbf{eat}\ w)\ x))}{\lambda y\lambda z\lambda x((y\ z)\ \lambda w((\mathbf{eat}\ w)\ x))}/\mathrm{I}^{2}$$

$$\dfrac{\mathbf{j} \qquad \overline{v}^{5} \qquad \mathbf{bagel}}{((v\ \mathbf{bagel})\ \mathbf{j})} \qquad \dfrac{((v\ \mathbf{bagel})\ \mathbf{j})}{\lambda v((v\ \mathbf{bagel})\ \mathbf{j})}{\uparrow}\mathrm{I}^{5}$$

$$\dfrac{\lambda v((v\ \mathbf{bagel})\ \mathbf{j}) \qquad \lambda y\lambda z\lambda x((y\ z)\ \lambda w((\mathbf{eat}\ w)\ x))}{(\lambda v((v\ \mathbf{bagel})\ \mathbf{j}), (\lambda y\lambda z\lambda x((y\ z)\ \lambda w((\mathbf{eat}\ w)\ x))))}\odot^{2}_{m}\mathrm{I}$$

$$\dfrac{(\lambda v((v\ \mathbf{bagel})\ \mathbf{j}), (\lambda y\lambda z\lambda x((y\ z)\ \lambda w((\mathbf{eat}\ w)\ x)))) \qquad \text{more than}}{\lambda y[\lambda u[(\mathbf{bagel}\ u) \wedge ((\mathbf{eat}\ u)\ \mathbf{j})] > \lambda u(y\ \lambda z\lambda x[(z\ u) \wedge ((\mathbf{eat}\ u)\ x)])]}\backslash^{2}\mathrm{E} \qquad \dfrac{\text{Mary donuts}}{\lambda v((v\ \mathbf{donut})\ \mathbf{m})}$$

$$\dfrac{\lambda y[\lambda u[(\mathbf{bagel}\ u) \wedge ((\mathbf{eat}\ u)\ \mathbf{j})] > \lambda u(y\ \lambda z\lambda x[(z\ u) \wedge ((\mathbf{eat}\ u)\ x)])] \qquad \lambda v((v\ \mathbf{donut})\ \mathbf{m})}{\lambda u[(\mathbf{bagel}\ u) \wedge ((\mathbf{eat}\ u)\ \mathbf{j})] > \lambda u[(\mathbf{donut}\ u) \wedge ((\mathbf{eat}\ u)\ \mathbf{m})]}/_{r}\mathrm{E}$$

Figure 12. Comparative subdeletion with TV ellipsis

Conclusion

We indicate above that there is nothing essential in our choice of a sorted framework for the generalisations of discontinuity that we present here: they can easily be reformulated as interaction postulates in an unsorted multimodal categorial grammar. Furthermore, both sorted and unsorted formulations have a clausal implementation. However, the implementation of the sorted version dispells with the need for unification under theory, and the sorted version allows us to think in terms of concrete models such as strings and split strings, whereas the unsorted version requires us to assume more abstract objects. There arises an interesting question here as to realism and ontological committment of one or other design for grammar, a question to which we draw attention, but which we do not begin to address.

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