Type-Logical Anaphora*

Glyn Morrill
Departament de Llenguatges i Sistemes Informàtics,
Universitat Politècnica de Catalunya,
Mòdul C 5 - Campus Nord,
Jordi Girona Salgado 1-3,
E-08034 Barcelona.

Abstract

We present a type-logical account of anaphora. The principal means is the introduction of type-constructors for 'secondary wrap': wrap around the second of two split points in a discontinuous string. We illustrate in terms of preevaluated proof nets, proof nets in which lexico-syntactic interaction is computed in a lexical compilation rather than at the stage of derivation. We suggest that the account offers an explanation of the delay of principle B effect in the child acquisition of anaphora.

Chomsky (1981) proposes Principles A, B and C governing anaphoric relations. Roughly speaking, Principle A states that a reflexive or reciprocal pronoun must be bound by a local c-commanding antecedent, Principle B states that a personal pronoun must not be bound by a local c-commanding antecedent, and Principle C states that a lexical noun phrase cannot be bound to a c-commanding antecedent. The 'delay of Principle B effect' is the term given to the phenomenon whereby in acquisition there seems to be non-complience with principle B.

Jacobson (1999) discusses the combinatorics of categorial anaphora largely from the point of view of combinatory categorial grammar (Steedman 2000). She takes a step in the type-logical direction by introducing what is essentially a new binary type-constructor (notated as a superfix), however Principle B is not accommodated. Anaphora receives a related type-logical rendering in Jäger (1998). Although Jäger presents a sequent calculus enjoying Cut-elimination for his system, it is not especially elegant, invoking as it does an arbitrary atom in the right inference rule. More importantly, it is not clear how to present the system in natural deduction, perhaps the most transparent format for linguistic purposes, or in terms of proof nets, probably the best basis for computational concerns. Like the proposal of Jacobson, it does not take into account Principle B.

In this paper we present a type-logical account of anaphora taking into account Principle B. The account rests on a generalisation of 'discontinuity calculus' in which wrap is admitted around the second of two split points in a discontinuous string. We illustrate in terms of natural deduction and in terms of proof nets, and we suggest that delay of Principle B can be correlated to memory load in incremental construction of proof nets.

1 Type-logical grammar

Type-logical grammar (Moortgat 1988, Morrill 1994, Moortgat 1997, Carpenter 1998) as it is understood here rests on the methodological assumption that a language model is defined on the basis of a categorial lexicon together with an interpretation of categorial types inducing a consequence relation that projects the language model as the closure of the lexicon under the consequence relation. In such a radical architecture syntax (proof

*E-mail: morrill@lsi.upc.es, http: //www-lsi.upc.es/~morrill/. Work partially supported by CICYT project PB98–0937–C03–03.

theory) has no definitional role and only serves to calculate derivations. It can come in various guises according to convenience. But when processing, i.e. computational, concerns are taken into account, it is relevant to ask what is the essential structure of proofs/derivations, to which question a good answer is provided by proof nets (Girard 1987). In the following we shall make use of both natural deduction and proof nets.

2 Associative Lambek calculus

In the associative Lambek calculus (Lambek 1958) types or category formulas are defined in terms of a set $\mathcal{A}$ of atomic types thus:

$$\mathcal{F} ::= \mathcal{A} \mid \mathcal{F}\backslash\mathcal{F} \mid \mathcal{F}/\mathcal{F} \mid F{\bullet}F \tag{1}$$

The slash operators are called 'under' and 'over' respectively. The dot operator is called 'product'. The standard models for these are such that they are prosodically interpreted by 'residuation' with respect to the concatenative adjunction $\cdot$ of a semigroup $(L, \cdot)$ or a monoid $(L, \cdot, \emptyset)$:

$$\boxed{\alpha}\cdot\boxed{\beta} = \boxed{\alpha}\!\boxed{\beta} \tag{2}$$

$$\begin{array}{rcl} D(A\backslash B) & = & \{s \mid \forall s' \in D(A), s'{\cdot}s \in D(B)\} \\ D(B/A) & = & \{s \mid \forall s' \in D(A), s{\cdot}s' \in D(B)\} \\ D(A{\bullet}B) & = & \{s_1{\cdot}s_2 \mid s_1 \in D(A) \;\&\; s_2 \in D(B)\} \end{array} \tag{3}$$

This interpretation induces a consequence relation such that $A \Rightarrow B$ is valid iff in all interpretations, $D(A) \subseteq D(B)$. Where we write α: A to indicate that expression α is in category A the following Prawitz-style natural deduction rules are valid (letters from the beginning of the Greek alphabet represent prosodic variables; boldface letters from the beginning of the Roman Alphabet, prosodic constants):

$$\frac{\begin{array}{c}\vdots\\ \alpha\colon A\end{array}\quad\begin{array}{c}\vdots\\ \gamma\colon A\backslash B\end{array}}{\alpha{\cdot}\gamma\colon B}\,\backslash\mathrm{E} \qquad \frac{\begin{array}{c}\overline{\mathbf{a}\colon A}^{\,n}\\ \vdots\\ \mathbf{a}{\cdot}\gamma\colon B\end{array}}{\gamma\colon A\backslash B}\,\backslash\mathrm{I}^n \tag{4}$$

$$\frac{\begin{array}{c}\vdots\\ \gamma\colon B/A\end{array}\quad\begin{array}{c}\vdots\\ \alpha\colon A\end{array}}{\gamma{\cdot}\alpha\colon B}\,/\mathrm{E} \qquad \frac{\begin{array}{c}\overline{\mathbf{a}\colon A}^{\,n}\\ \vdots\\ \gamma{\cdot}\mathbf{a}\colon B\end{array}}{\gamma\colon B/A}\,/\mathrm{I}^n \tag{5}$$

$$\frac{\begin{array}{c}\vdots\\ \delta\colon A{\bullet}B\end{array}\qquad\begin{array}{c}\overline{\mathbf{a}\colon A}^{\,n}\quad\overline{\mathbf{a}{\rightharpoondown}\delta\colon B}^{\,n}\\ \vdots\\ \gamma\colon C\end{array}}{\gamma\colon C}\,{\bullet}\mathrm{E}^n \qquad \frac{\begin{array}{c}\vdots\\ \alpha\colon A\end{array}\quad\begin{array}{c}\vdots\\ \beta\colon B\end{array}}{\alpha{\cdot}\beta\colon A{\bullet}B}\,{\bullet}\mathrm{I} \tag{6}$$

The overline in \I,/I and •E indicates cancellation of a hypothesis with the coindexed rule. The prosodic operator invoked in •E satisfies $\alpha{\rightharpoondown}\alpha{\cdot}\beta = \beta$.

Semantically, the slash elimination rules are interpreted as functional application of the functor ($A\backslash B$ or B/A) to the argument A. The slash introduction rules are interpreted as functional abstraction with respect to a semantic variable associated with the cancelled hypothesis. The product elimination and introduction rules are semantically interpreted as projection and pair formation respectively.

Lambek (1958) already observes how the restriction of a nominative pronominal to subject position could be captured.

a. He runs. (7)
b. *John likes he.

He notes that one can assign to the nominative pronominal type S/(N\S). Then (7b) is blocked due to the invalidity of 'lowering': S/(N\S) $\Rightarrow$ N.

Reinhart (1993) argues for a distinction between syntactically bound anaphora, which is governed by grammar, and discourse anaphora, which is governed by pragmatic principles. We follow this distinction here. In the case of syntactically free pronouns, i.e. pronouns which are syntactically unbound, we propose a lexical assignment citing a metavariable. The way in which these variables are instantiated and interpreted, which depends upon pragmatic considerations of discourse, lies beyond the scope of the present paper.

$$\begin{array}{lll} \textbf{he} & - & \lambda x(x\ \phi) \\ & := & \mathrm{S/(N\backslash S)} \end{array} \tag{8}$$

Then 'He runs' is derived as follows:

$$\dfrac{\textbf{he}\colon \mathrm{S/(N\backslash S)} \qquad \textbf{runs}\colon \mathrm{N\backslash S}}{\textbf{he}{\cdot}\textbf{runs}\colon \mathrm{S}}\ /\mathrm{E} \tag{9}$$

The semantics is given node-for-node thus:

$$\dfrac{\lambda x(x\ y) \qquad run}{(run\ y)}\ /\mathrm{E} \tag{10}$$

In the format of proof nets, type occurrences are distinguished according to polarity input ($^\bullet$) and output ($^\circ$). A derivation $\alpha_1\colon A_1, \ldots, \alpha_n\colon A_n \Rightarrow \alpha_1 \cdot \cdots \cdot \alpha_n\colon A$ is obtained by recursively unfolding the types as $\alpha_1 \cdot \cdots \cdot \alpha_n\colon A^\circ\ \alpha_1\colon A_1{}^\bullet\ \cdots\ \alpha_n\colon A_n{}^\bullet$ according to the following schemata:

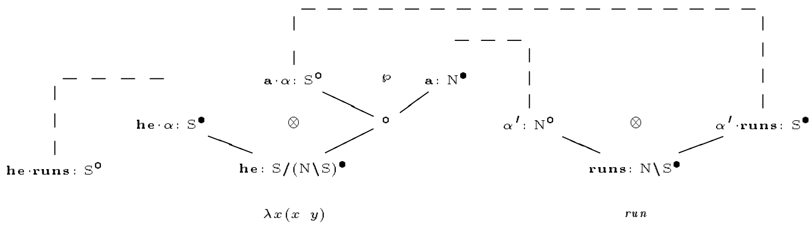
$$\begin{array}{ccc@{\qquad}ccc}
\alpha\colon A^\circ & \otimes & \alpha\cdot\gamma\colon B^\bullet & \mathbf{a}\cdot\gamma\colon B^\circ & \wp & \mathbf{a}\colon A^\bullet \\
 & \gamma\colon A\backslash B^\bullet & & & \gamma\colon A\backslash B^\circ & \\[1ex]
\gamma\cdot\alpha\colon B^\bullet & \otimes & \alpha\colon A^\circ & \mathbf{a}\colon A^\bullet & \wp & \gamma\cdot\mathbf{a}\colon B^\circ \\
 & \gamma\colon B/A^\bullet & & & \gamma\colon B/A^\circ & \\[1ex]
\mathbf{a}\colon A^\bullet & \wp & \mathbf{a}\rightharpoondown\gamma\colon B^\bullet & \alpha\rightharpoondown\gamma\colon B^\circ & \otimes & \alpha\colon A^\circ \\
 & \gamma\colon A\bullet B^\bullet & & & \gamma\colon A\bullet B^\circ &
\end{array} \tag{11}$$

Then a proof net is constructed unifying under associativity of $\cdot$ the expressions of axiom links connecting literals of complementary polarity (Roorda 1991, Moortgat 1996, Morrill 1995a, de Groote 1999):

$$\alpha\colon A \qquad\qquad \alpha\colon \overline{A} \tag{12}$$

'He runs' is represented as follows:

(13)



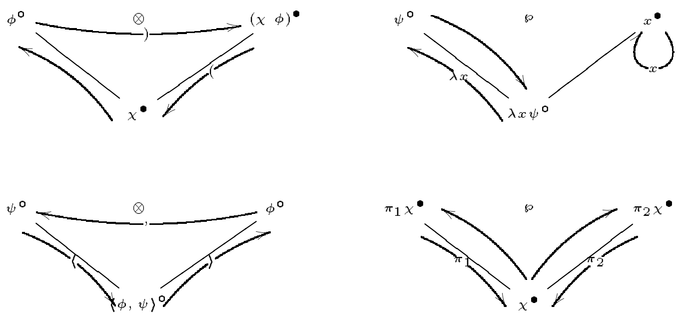
The unification problem induced by the axiom links has solution $\{\mathbf{runs}/\alpha, \mathbf{a}/\alpha'\}$.

To extract a semantic reading from a proof net one first associates distinct variables with each $\wp$-output link with input and output premisses. Then one starts travelling upwards at the output terminal node and continues thus (Lamarche 1995, de Groote and Retoré 1996):

- Going up into the conclusion of a $\wp$ output link with input and output premisses, form a lambda abstraction of the associated variable and continue travelling upwards at the output premise.
- Going down at the input premise of a $\wp$ output link with input and output premisses, return the associated variable and bounce, returning the way you came.
- Going down at the output conclusion of a $\wp$ output link with input and output premisses, continue going down at the conclusion.
- Going up into one conclusion of an identity axiom link, go down at the other.
- Going down into the input premise of a $\otimes$ link with input and output premisses, open a functional application and go down at the conclusion.
- Going up at the input conclusion of a $\otimes$ link with input and output premisses, go up at the output premise.
- Going down into the output premise of a $\otimes$ link with input and output premisses, close the functional application and go up at the other premise.
- Going up at the output conclusion of a $\otimes$ link with output premisses, open a pair formation and go up at the right premise.
- Going down at the right premise of an output $\otimes$ link with output premisses, separate the pair formation and go up at the left premise.
- Going down at the left premise of an output $\otimes$ link with output premisses, close the pair formation and continue going down at the conclusion.
- Going down into the left premise of an input $\otimes$ link with input premises, take the first projection of the result of going down at the conclusion.
- Going down into the right premise of an input $\otimes$ link with input premises, take the second projection of the result of going down at the conclusion.
- Going up into an input $\otimes$ link with input premisses go up at the premise most recently entered.
- Going down at a terminal node, return the associated semantics and bounce, going back the same way.

The semantic travel instructions for links with two premisses and one conclusion are represented graphically in the following (letters, ϕ, ψ, χ represent semantic variables):

(14)



The semantic trip for our example yields the semantic term (15a) which simplifies by λ-conversion to (15b).

a. $(\lambda x(x\ y)\ \lambda z(run\ z))$ (15)
b. $(run\ y)$

De Groote and Retoré (1996) show how lexical semantics can be integrated with derivational syntax by encoding the former also as proof nets, connecting them by a Cut:

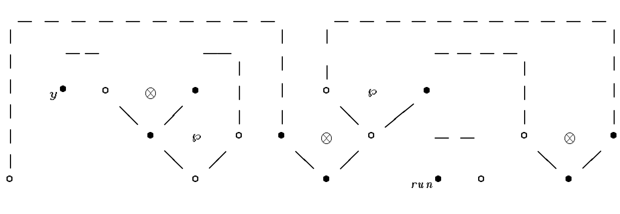
(16)

The semantic travel instruction for Cut is like that for the identity axiom link:

- Going down at one premise of Cut, go up at the other.

Then our example (13) becomes:

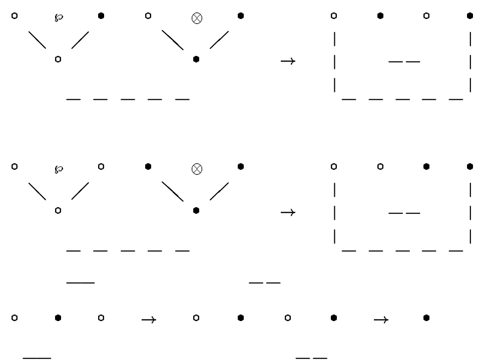
(17)



The semantic trip again yields (15a), which simplifies to (15b).

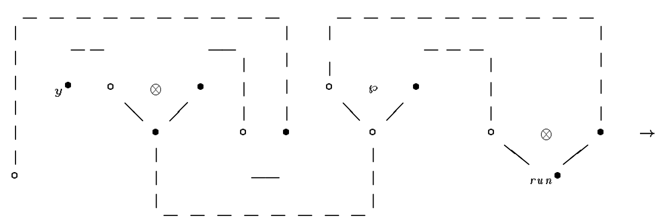
As de Groote and Retoré (1996) note, the simplification can be made on the representation as proof nets by reductions such as the following:

(18)

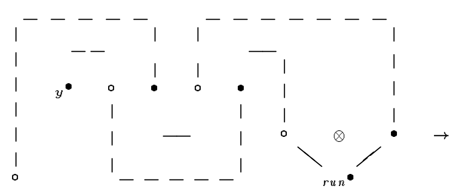


Thus (17) reduces to the following:

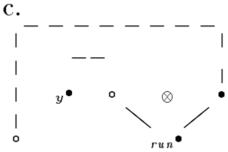
a. (19)



b.



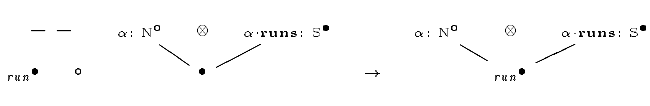
c.



Making the semantic trip on this reduced proof net yields the simplified semantics (15b).

However, Morrill (1999) proposes that the proof net reduction be performed not after syntactic derivation (placement of axiom links) but before, in a preevaluation of the partial proof nets (modules) resulting from Cutting lexical semantics into lexical categories. Thus the lexical entry for 'runs' is preevaluated thus:

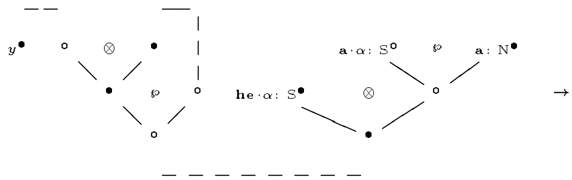
(20)



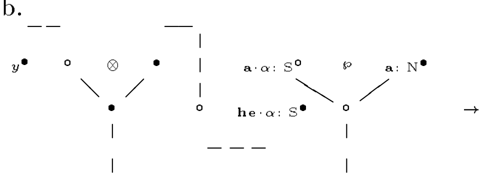
And the lexical entry for 'he' is preevaluated thus:

(21)

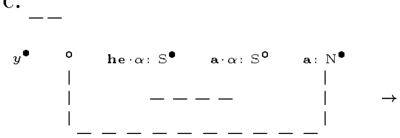
a.



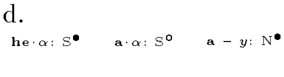
b.



c.

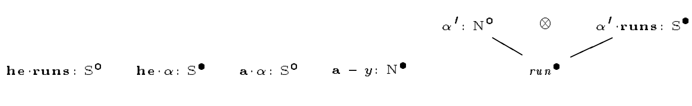


d.



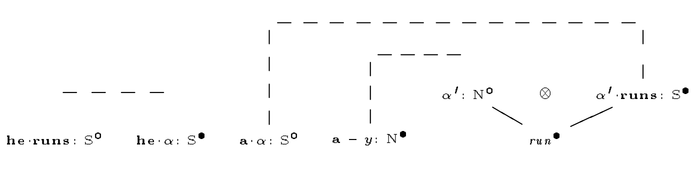
The derivation of 'He runs' is then performed on the basis of assembling the evaluated lexical modules as follows:

(22)



The derivation is completed by placing the axiom links:

(23)



The unification problem induced has solution $\{\mathbf{runs}/\alpha, \mathbf{a}/\alpha'\}$. The result of the semantic trip is the simplified semantic form (15b).

3 Discontinuity Calculus

Associative Lambek calculus was presented as a logic of concatenation, which role is strengthened by the result of Pentus (1994) improving to free semigroups the completeness result of Buszkowski (1986) for semigroups. However, natural grammar includes non-concatenative phenomena, leading to the search for calculi of discontinuity serving for non-concatenative adjunction as the Lambek calculus serves for concatenative adjunction. Thus Versmissen (1991) proposes to treat discontinuity via split strings. Algebraic approaches include the following:

- Solias (1992): $(L, \cdot, \emptyset, \langle\cdot,\cdot\rangle)$ where $(L, \cdot, \emptyset)$ is a free monoid and $(L, \langle\cdot,\cdot\rangle)$ is a free groupoid. Wrap is a partial operation defined by $\langle s_1, s_2\rangle W s = s_1{\cdot}s{\cdot}s_2$.
- Morrill and Solias (1993): $(L, \cdot, \emptyset, \langle\cdot,\cdot\rangle, 1, 2)$ where $(L, \cdot, \emptyset)$ is a monoid, $(L, \langle\cdot,\cdot\rangle)$ is a groupoid and $1\langle s_1, s_2\rangle = s_1, 2\langle s_1, s_2\rangle = s_2$ and $\langle 1s, 2s\rangle = s$. Wrap is a total operation defined by $sWs' = 1s{\cdot}s'{\cdot}2s$.
- Morrill (1994, 1995b): $(L, \cdot, \emptyset, (\cdot,\cdot), W)$ where $(L, \cdot, \emptyset)$ is a monoid, $(L, (\cdot,\cdot))$ and (L, W) are groupoids and $(s_1, s_2)Ws = s_1{\cdot}s{\cdot}s_2$. Wrap is a primitive total operation.

See also Moortgat (1996). The format adopted here is that of Morrill and Merenciano (1996) in which a monoid $(L, \cdot, \emptyset)$ is extended to the sorted algebra $(L, L^2, \cdot, \emptyset, W)$ where wrapping W, of functionality $L^2, L \rightarrow L$, is defined by $\langle s_1, s_2\rangle W s = s_1{\cdot}s{\cdot}s_2$:

$$\boxed{\alpha}\cdots\boxed{\gamma}\ W\ \boxed{\beta} = \boxed{\alpha}\boxed{\beta}\boxed{\gamma} \tag{24}$$

Category formulas become sorted into two kinds according to their prosodic type, string or split string:

$$\begin{array}{lcl}\mathcal{F} & ::= & \mathcal{A} \mid \mathcal{F}\backslash\mathcal{F} \mid \mathcal{F}/\mathcal{F} \mid F{\bullet}F \mid \mathcal{F}^2{\downarrow}\mathcal{F} \mid \mathcal{F}^2{\odot}\mathcal{F}\\ \mathcal{F}^2 & ::= & \mathcal{F}{\uparrow}\mathcal{F}\end{array} \tag{25}$$

The Lambek connectives are interpreted as before by residuation with respect $\cdot$ while the new connectives $\downarrow$ ('infix'), $\uparrow$ ('extract') and $\odot$ ('discontinuous product') are interpreted by residuation with respect to W:

$$\begin{array}{lcl}D(A{\downarrow}B) & = & \{s \mid \forall\langle s_1, s_2\rangle \in D(A), s_1{\cdot}s{\cdot}s_2 \in D(B)\}\\ D(B{\uparrow}A) & = & \{\langle s_1, s_2\rangle \mid \forall s \in D(A), s_1{\cdot}s{\cdot}s_2 \in D(B)\}\\ D(A{\odot}B) & = & \{s_1{\cdot}s{\cdot}s_2 \mid \langle s_1, s_2\rangle \in D(A)\ \&\ s \in D(B)\}\end{array} \tag{26}$$

The following Prawitz-style natural deduction rules are valid:

$$\frac{\begin{array}{cc}\vdots & \vdots\\ (\alpha_1, \alpha_2)\colon A & \gamma\colon A{\downarrow}B\end{array}}{\alpha_1{\cdot}\gamma{\cdot}\alpha_2\colon B}\,{\downarrow}\mathrm{E} \qquad \frac{\begin{array}{c}\overline{(\mathbf{a}_1, \mathbf{a}_2)\colon A}^{\,n}\\ \vdots\\ \mathbf{a}_1{\cdot}\gamma{\cdot}\mathbf{a}_2\colon B\end{array}}{\gamma\colon A{\downarrow}B}\,{\downarrow}\mathrm{I}^n \tag{27}$$

$$\frac{\begin{array}{cc}\vdots & \vdots\\ (\gamma_1, \gamma_2)\colon B{\uparrow}A & \alpha\colon A\end{array}}{\gamma_1{\cdot}\alpha{\cdot}\gamma_2\colon B}\,{\uparrow}\mathrm{E} \qquad \frac{\begin{array}{c}\overline{\mathbf{a}}^{\,n}\\ \vdots\\ \gamma_1{\cdot}\mathbf{a}{\cdot}\gamma_2\colon B\end{array}}{(\gamma_1, \gamma_2)\colon B{\uparrow}A}\,{\uparrow}\mathrm{I}^n \tag{28}$$

$$\frac{\begin{array}{cc}\vdots & \begin{array}{c}\overline{(\mathbf{a}_1, \mathbf{a}_2)\colon A}^{\,n}\quad \overline{\mathbf{a}_1{\rightharpoondown}\delta{\leftharpoondown}\mathbf{a}_2\colon B}^{\,n}\\ \vdots\end{array}\\ \delta\colon A{\odot}B & \gamma\colon C\end{array}}{\gamma\colon C}\,{\odot}\mathrm{E}^n \qquad \frac{\begin{array}{cc}\vdots & \vdots\\ (\alpha_1, \alpha_2)\colon A & \beta\colon B\end{array}}{\alpha_1{\cdot}\beta{\cdot}\alpha_2\colon A{\odot}B}\,{\odot}\mathrm{I} \tag{29}$$

The prosodic operator invoked in $\odot$E satisfies $\alpha_1{\cdot}(\alpha_1{\rightharpoondown}\beta{\leftharpoondown}\alpha_2){\cdot}\alpha_2 = \beta$. As before, the functor elimination and introduction rules are semantically interpreted by functional application and functional abstraction, and the product elimination and introduction rules by projection and pairing, respectively.

Consider now subject-oriented reflexives:

a. John buys Fido for himself. (30)
b. John buys himself Fido.

These can be characterised by the lexical assignment (31) (see Moortgat 1996).[1]

$$\begin{array}{lll} \textbf{himself} & - & \lambda x \lambda y((x\ y)\ y) \\ & := & ((\mathrm{N}\backslash\mathrm{S})\uparrow\mathrm{N})\downarrow(\mathrm{N}\backslash\mathrm{S}) \end{array} \tag{31}$$

(Note that the best that could be done in the Lambek calculus is to assign type ((N\S)/N)\(N\S), which would derive (30a) but not (30b).) Then for example, (30b) is derived thus:

$$\dfrac{\textbf{J}\!: \mathrm{N} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\textbf{buys}\!: ((\mathrm{N}\backslash\mathrm{S})/\mathrm{N})/\mathrm{N} \qquad \dfrac{}{\textbf{a}\!: \mathrm{N}}\,1}{\textbf{buys}{\cdot}\textbf{a}\!: (\mathrm{N}\backslash\mathrm{S})/\mathrm{N}}\,/\mathrm{E} \qquad \textbf{F}\!: \mathrm{N}}{\textbf{buys}{\cdot}\textbf{a}{\cdot}\textbf{F}\!: \mathrm{N}\backslash\mathrm{S}}\,/\mathrm{E}}{(\textbf{buys}, \textbf{F})\!: (\mathrm{N}\backslash\mathrm{S})\uparrow\mathrm{N}}\,\uparrow\mathrm{I}^1 \qquad \textbf{himself}\!: ((\mathrm{N}\backslash\mathrm{S})\uparrow\mathrm{N})\downarrow(\mathrm{N}\backslash\mathrm{S})}{\textbf{buys}{\cdot}\textbf{himself}{\cdot}\textbf{F}\!: \mathrm{N}\backslash\mathrm{S}}\,\downarrow\mathrm{E}}{\textbf{J}{\cdot}\textbf{buys}{\cdot}\textbf{himself}{\cdot}\textbf{F}\!: \mathrm{S}}\,\backslash\mathrm{E} \tag{32}$$

The corresponding semantics given node for node is (33).

$$\dfrac{j \qquad \dfrac{\dfrac{\dfrac{\dfrac{buy \qquad \dfrac{}{x}\,1}{(buy\ x)}\,/\mathrm{E} \qquad f}{((buy\ x)\ f)}\,/\mathrm{E}}{\lambda x((buy\ x)\ f)}\,\uparrow\mathrm{I}^1 \qquad \lambda x \lambda y((x\ y)\ y)}{\lambda y(((buy\ y)\ f)\ y)}\,\downarrow\mathrm{E}}{(((buy\ j)\ f)\ j)}\,\backslash\mathrm{E} \tag{33}$$

The proof net links for the discontinuity connectives are as follows:

$$\begin{array}{cc}
\dfrac{(\alpha_1, \alpha_2)\!: A^{\circ} \qquad \otimes \qquad \alpha_1{\cdot}\gamma{\cdot}\alpha_2\!: B^{\bullet}}{\gamma\!: A{\downarrow}B^{\bullet}} & \dfrac{\mathbf{a}_1{\cdot}\gamma{\cdot}\mathbf{a}_2\!: B^{\circ} \qquad \wp \qquad (\mathbf{a}_1, \mathbf{a}_2)\!: A^{\bullet}}{\gamma\!: A{\downarrow}B^{\circ}} \\[2ex]
\dfrac{\gamma_1{\cdot}\alpha{\cdot}\gamma_2\!: B^{\bullet} \qquad \otimes \qquad \alpha\!: A^{\circ}}{(\gamma_1, \gamma_2)\!: B{\uparrow}A^{\bullet}} & \dfrac{\mathbf{a}\!: A^{\bullet} \qquad \wp \qquad \gamma_1{\cdot}\mathbf{a}{\cdot}\gamma_2\!: B^{\circ}}{(\gamma_1, \gamma_2)\!: B{\uparrow}A^{\circ}} \\[2ex]
\dfrac{(\mathbf{a}_1, \mathbf{a}_2)\!: A^{\bullet} \qquad \wp \qquad \mathbf{a}_1{\rightharpoondown}\gamma{\leftharpoondown}\mathbf{a}_2\!: B^{\bullet}}{\gamma\!: A{\odot}B^{\bullet}} & \dfrac{\alpha_1{\rightharpoondown}\gamma{\leftharpoondown}\alpha_2\!: B^{\circ} \qquad \otimes \qquad (\alpha_1, \alpha_2)\!: A^{\circ}}{\gamma\!: A{\odot}B^{\circ}}
\end{array} \tag{34}$$

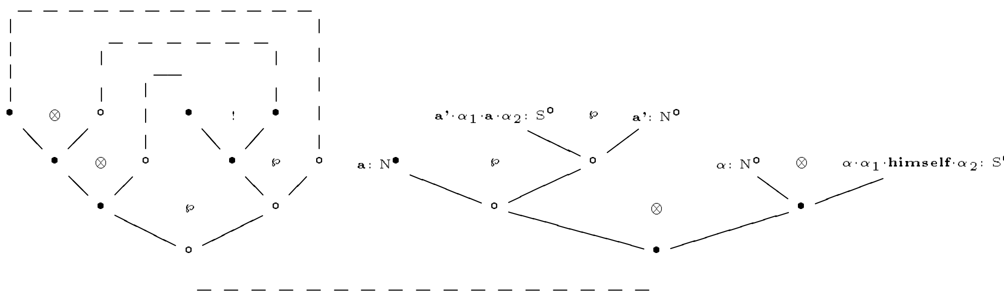
To represent in proof nets the multiple binding in the lexical semantics of 'himself', we use a simplified version of !-links (see de Groote and Retoré 1996). The relevant travel instructions are:

[1] The assignment (31) does not block examples such as ∗'John says himself runs' or ∗'John says Mary saw himself'. However, Morrill (1990, 1994) argues that intensionality, for example temporal domains, provide the appropriate term of reference for locality effects. Thus in a modal logic of intensional types, 'John says himself runs' would be $\Box$N, $\Box$((N\S)/$\Box$S), $\Box$(((N\S)↑N)↓(N\S)), $\Box$(N\S) $\Rightarrow$ S which is not modally valid. Similarly, for 'John says Mary saw himself', the reflexive cannot occupy an N position in the superior modal domain. In this way Principle A effects are imposed as expounded in the references.

- Going down at a premise of an input !-link, go down at the conclusion.
- Going up at the conclusion of an input !-link, go up at the premise most recently entered.

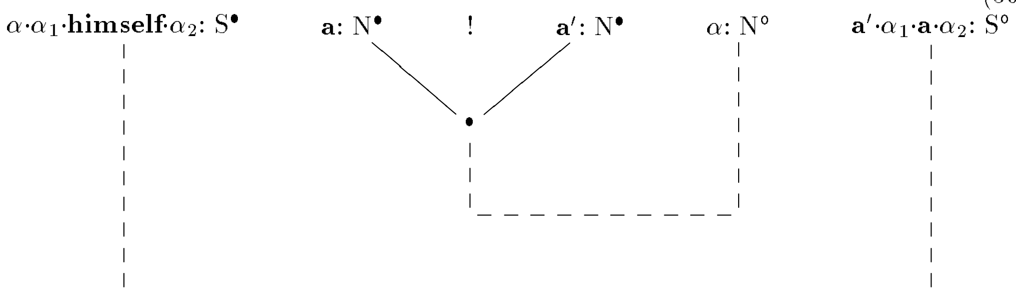
The unevaluated lexical module for 'himself' is (35).

(35)



This evaluates to (36).

(36)



Thus (30b) has the proof net analysis given in figure 1. The unification problem induced by the axiom links has solution $\{\mathbf{J}/\beta, \mathbf{buys}/\beta_1, \mathbf{F}/\beta_2, \mathbf{b}'/\alpha'', \mathbf{F}/\alpha', \mathbf{b}/\alpha\}$. The semantic trip yields the simplified semantics of (33).

For a syntactically free accusative pronoun we can make the lexical categorisation (37).[2]

$$\begin{array}{lll} \mathbf{him} & - & \lambda x\lambda y((x\ \phi)\ y) \\ & := & ((N\backslash S)\uparrow N)\downarrow(N\backslash S) \end{array} \tag{37}$$

In terms of natural deduction, 'Mary saw him' is derived as in (38).

$$\dfrac{\mathbf{M}\text{: N} \qquad \dfrac{\dfrac{\dfrac{\mathbf{saw}\text{: (N\textbackslash S)/N} \qquad \overline{\mathbf{a}\text{: N}}^{\,1}}{\mathbf{saw}\cdot\mathbf{a}\text{: N\textbackslash S}}/E}{(\mathbf{saw}, \emptyset)\text{: (N\textbackslash S)}\uparrow\text{N}}\uparrow I^1 \qquad \mathbf{him}\text{: ((N\textbackslash S)}\uparrow\text{N)}\downarrow\text{(N\textbackslash S)}}{\mathbf{saw}\cdot\mathbf{him}\text{: N\textbackslash S}}\downarrow E}{\mathbf{M}\cdot\mathbf{saw}\cdot\mathbf{him}\text{: S}}\backslash E \tag{38}$$

The semantics is thus:

[2] Again, (37) does not block *'John$_i$ says him$_i$ runs', but in a modal logic of intensional types, assignment to $\square(((N\backslash S)\uparrow N)\downarrow(N\backslash S))$ does block this.

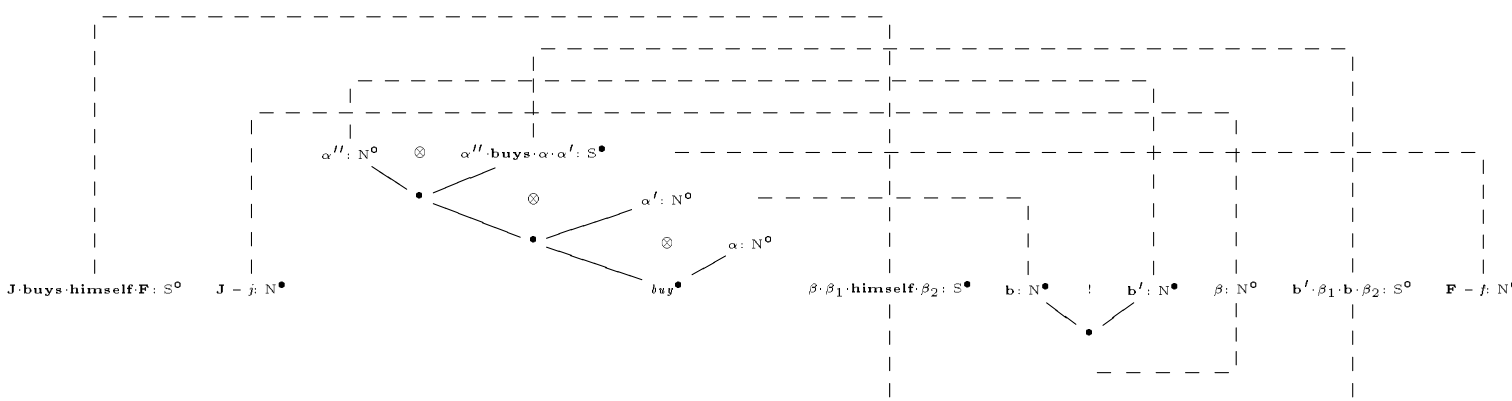


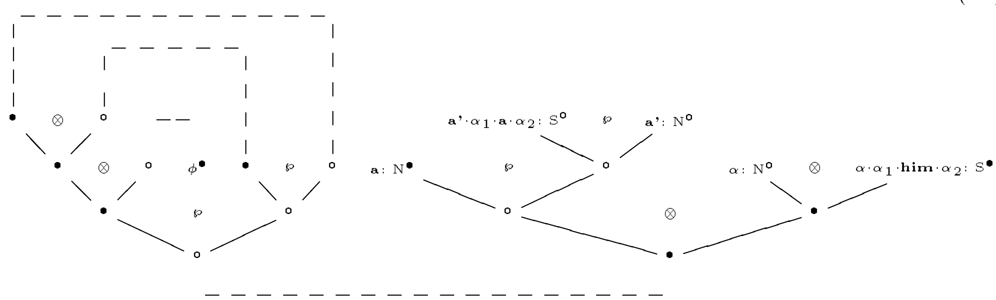
Figure 1: Preevaluated proof net for ‘John buys himself Fido’

(39)

$$\dfrac{m \qquad \dfrac{\dfrac{\dfrac{see \qquad \overline{x}^{\,1}}{(see\ x)}/\mathrm{E}}{\lambda x(see\ x)}\uparrow\mathrm{I}^1 \qquad \lambda x\lambda y((x\ z)\ y)}{\lambda y((see\ z)\ y)}\downarrow\mathrm{E}}{((see\ z)\ m)}\backslash\mathrm{E}$$

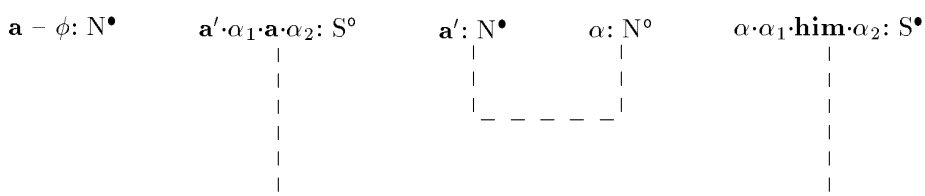
In terms of proof nets, the unevaluated lexical module for a syntactically free accusative pronoun is that in (40).

(40)



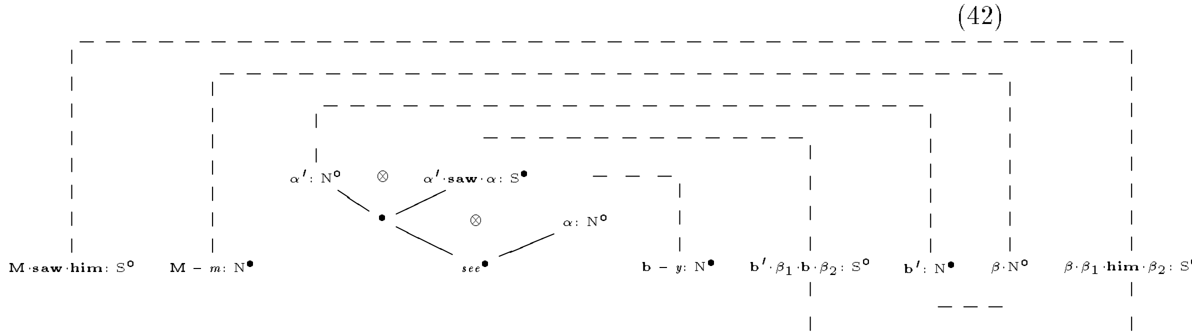
This evaluates to (41).

(41)



Thus the preevaluated proof net derivation for 'Mary saw him' is (42).

(42)



The unification problem induced by the axiom limks has solution $\{\emptyset/\beta_2, \mathbf{M}/\beta, \mathbf{saw}/\beta_1, \mathbf{b}/\alpha, \mathbf{b}'/\alpha'\}$ and the semantic trip yields the simplified result of (39).

4 Generalised discontinuity

More expressivity is required for bound pronouns if we are to capture cases such as those in (43) where the antecedent does not c-command the pronoun.

a. The mother of every boy/John said he won. (43)
b. The mother of every boy/John praised him.

On the analysis pursued here, pronouns involve wrap around the second of two split points in a doubly split string (secondary wrap). We generalise the sorted algebra $(L, L^2, \cdot, \emptyset, W)$ of the discontinuity calculus to $(L, L^2, L^3, \cdot, \emptyset, W, W_2)$ where secondary wrapping W_2, of functionality $L^3, L \to L^2$, is defined by $(s_1, s_2, s_3)W_2 s = (s_1, s_2{\cdot}s{\cdot}s_3)$:

$$\boxed{\alpha}\cdots\boxed{\beta}\cdots\boxed{\delta}\; W_2\; \boxed{\gamma} = \boxed{\alpha}\cdots\boxed{\beta}\boxed{\gamma}\boxed{\delta} \tag{44}$$

Category formulas become sorted into three kinds according to their prosodic type, string, split string, or doubly split string:

$$\begin{array}{lcl} \mathcal{F} & ::= & \mathcal{A} \mid \mathcal{F}\backslash\mathcal{F} \mid \mathcal{F}/\mathcal{F} \mid F{\bullet}F \mid \mathcal{F}^2{\downarrow}\mathcal{F} \mid \mathcal{F}^2{\odot}\mathcal{F} \mid \mathcal{F}^3{\downarrow_2}\mathcal{F}^2 \\ \mathcal{F}^2 & ::= & \mathcal{F}{\uparrow}\mathcal{F} \mid \mathcal{F}^3{\odot_2}\mathcal{F} \\ \mathcal{F}^3 & ::= & \mathcal{F}^2{\uparrow_2}\mathcal{F} \end{array} \tag{45}$$

The new connectives $\downarrow_2$ ('secondary infix'), $\uparrow_2$ ('secondary extract') and $\odot_2$ ('secondary discontinuous product') are interpreted by residuation with respect to W_2:

$$\begin{array}{lcl} D(A{\downarrow_2}B) & = & \{s \mid \forall\langle s_1, s_2, s_3\rangle \in D(A), \langle s_1, s_2{\cdot}s{\cdot}s_3\rangle \in D(B)\} \\ D(B{\uparrow_2}A) & = & \{\langle s_1, s_2, s_3\rangle \mid \forall s \in D(A), \langle s_1, s_2{\cdot}s{\cdot}s_3\rangle \in D(B)\} \\ D(A{\odot_2}B) & = & \{\langle s_1, s_2{\cdot}s{\cdot}s_3\rangle \mid \langle s_1, s_2, s_3\rangle \in D(A)\ \&\ s \in D(B)\} \end{array} \tag{46}$$

The following Prawitz-style natural deduction rules are valid:

(47)

$$\dfrac{\begin{array}{cc}\vdots & \vdots \\ (\alpha_1, \alpha_2, \alpha_3)\colon A & \gamma\colon A{\downarrow_2}B\end{array}}{(\alpha_1, \alpha_2{\cdot}\gamma{\cdot}\alpha_3)\colon B}\,{\downarrow_2}\mathrm{E} \qquad \dfrac{\begin{array}{c}\overline{(\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3)\colon A}^{\,n} \\ \vdots \\ (\mathbf{a}_1, \mathbf{a}_2{\cdot}\gamma{\cdot}\mathbf{a}_3)\colon B\end{array}}{\gamma\colon A{\downarrow_2}B}\,{\downarrow_2}\mathrm{I}^n$$

(48)

$$\dfrac{\begin{array}{cc}\vdots & \vdots \\ (\gamma_1, \gamma_2, \gamma_3)\colon B{\uparrow_2}A & \alpha\colon A\end{array}}{(\gamma_1, \gamma_2{\cdot}\alpha{\cdot}\gamma_3)\colon B}\,{\uparrow_2}\mathrm{E} \qquad \dfrac{\begin{array}{c}\overline{\mathbf{a}}^{\,n} \\ \vdots \\ (\gamma_1, \gamma_2{\cdot}\mathbf{a}{\cdot}\gamma_3)\colon B\end{array}}{(\gamma_1, \gamma_2, \gamma_3)\colon B{\uparrow_2}A}\,{\uparrow_2}\mathrm{I}^n$$

(49)

$$\dfrac{\begin{array}{ccc}\vdots & \overline{(\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3)\colon A}^{\,n} & \overline{(\mathbf{a}_1, \mathbf{a}_2{\rightharpoondown}\delta{\leftharpoondown}\mathbf{a}_3)\colon B}^{\,n} \\ & & \vdots \\ \delta\colon A{\odot_2}B & & \gamma\colon C\end{array}}{\gamma\colon C}\,{\odot_2}\mathrm{E}^n \qquad \dfrac{\begin{array}{cc}\vdots & \vdots \\ (\alpha_1, \alpha_2, \alpha_3)\colon A & \beta\colon B\end{array}}{(\alpha_1, \alpha_2{\cdot}\beta{\cdot}\alpha_3)\colon A{\odot}B}\,{\odot}\mathrm{I}$$

As before, the functor elimination and introduction rules are semantically interpreted by functional application and functional abstraction, and the product elimination and introduction rules by projection and pairing, respectively.

Bound pronouns can be categorised thus:

$$
\begin{array}{lll}
\textbf{he} & - & \lambda x\lambda y\lambda z((y\ (x\ z))\ z) \\
 & := & (((\mathrm{S}{\uparrow}\mathrm{N}){\uparrow}_2\mathrm{S}){\downarrow}_2(\mathrm{S}{\uparrow}\mathrm{N}))/(\mathrm{N}\backslash\mathrm{S}) \\
\textbf{him} & - & \lambda x\lambda y\lambda z\lambda w((z\ ((x\ w)\ y))\ w) \\
 & := & ((\mathrm{N}\backslash\mathrm{S}){\uparrow}\mathrm{N}){\downarrow}(\mathrm{N}\backslash(((\mathrm{S}{\uparrow}\mathrm{N}){\uparrow}_2\ \mathrm{S}){\downarrow}_2(\mathrm{S}{\uparrow}\mathrm{N}))
\end{array}
\tag{50}
$$

So ‘John$_i$ said he$_i$ won’ can be derived in natural deduction as follows:

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\overline{\mathbf{b}\colon \mathrm{N}}^{\,1} \quad \dfrac{\mathbf{said}\colon (\mathrm{N}\backslash\mathrm{S})/\mathrm{S} \quad \overline{\mathbf{a}\colon \mathrm{S}}^{\,2}}{\mathbf{said{\cdot}a}\colon \mathrm{N}\backslash\mathrm{S}}/\mathrm{E}}{\mathbf{b{\cdot}said{\cdot}a}\colon \mathrm{S}}\backslash\mathrm{E}}{(\emptyset, \mathbf{said{\cdot}a})\colon \mathrm{S}{\uparrow}\mathrm{N}}{\uparrow}\mathrm{I}^1}{(\emptyset, \mathbf{said}, \emptyset)\colon (\mathrm{S}{\uparrow}\mathrm{N}){\uparrow}_2\mathrm{S}}{\uparrow}_2\mathrm{I}^2 \quad \dfrac{\mathbf{he}\colon (((\mathrm{S}{\uparrow}\mathrm{N}){\uparrow}_2\mathrm{S}){\downarrow}_2(\mathrm{S}{\uparrow}\mathrm{N}))/(\mathrm{N}\backslash\mathrm{S}) \quad \mathbf{won}\colon \mathrm{N}\backslash\mathrm{S}}{\mathbf{he{\cdot}won}\colon ((\mathrm{S}{\uparrow}\mathrm{N}){\uparrow}_2\mathrm{S}){\downarrow}_2(\mathrm{S}{\uparrow}\mathrm{N})}/\mathrm{E}}{(\emptyset, \mathbf{said{\cdot}he{\cdot}won})\colon \mathrm{S}{\uparrow}\mathrm{N}}{\downarrow}_2\mathrm{E} \quad \mathbf{J}\colon \mathrm{N}}{\mathbf{J{\cdot}said{\cdot}he{\cdot}won}\colon \mathrm{S}}{\uparrow}\mathrm{E}
\tag{51}
$$

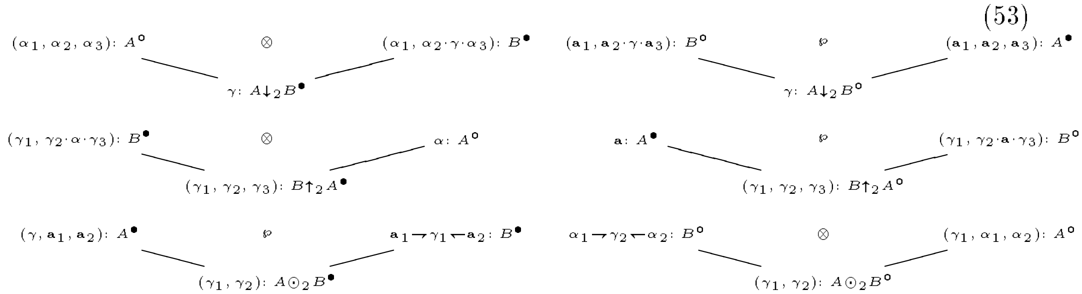
The node-for-node semantics is as follows:

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\overline{u}^{\,1} \quad \dfrac{say \quad \overline{w}^{\,2}}{(say\ w)}/\mathrm{E}}{((say\ w)\ u)}\backslash\mathrm{E}}{\lambda u((say\ w)\ u)}{\uparrow}\mathrm{I}^1}{\lambda w\lambda u((say\ w)\ u)}{\uparrow}_2\mathrm{I}^2 \quad \dfrac{\lambda x\lambda y\lambda z((y\ (x\ z))\ z) \quad win}{\lambda y\lambda z((y\ (win\ z))\ z)}}{\lambda z((say\ (win\ z))\ z)} \quad j}{((say\ (win\ j))\ j)}}{}
\tag{52}
$$

5 Delay of Principle B effect

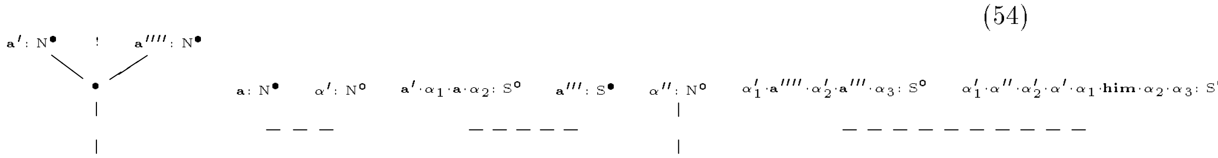
The proof links for the generalisation of the discontinuity calculus given in the previous section are as follows:

(53)



The evaluated module for syntactically bound ‘him’ is as follows:

(54)



Chien and Wexler (1990) reports an experiment (experiment 1) in which children were presented with sentences of the form (55).

a. Kitty says that Sarah should point to herself (55)
b. Kitty says that Sarah should point to her

In (55a) the reflexive pronoun has the local antecedent 'Sarah'; in (55b) the personal pronoun has the non-local antecedent 'Kitty'. Children's understanding of the sentences, i.e. whether Kitty or Sarah is chosen as antecedent, was tested by requiring them to act out what is said. In a few cases children took the pronouns to have a sentence-external referent, but in the main they chose one of the two sentence-internal referents.

Children's performance on the locality property of reflexives increased continuously from about 13% at age 2;6 to about 90% at 6;0, i.e. children older than 6;0 knew the major properties of reflexives. By contrast, children's performance on the non-locality property of personal pronouns stayed roughly flat from 2;6 to 6;6 with only a slight improvement to 64%. That is, children between the ages of 6;0 and 6;6 demonstrated knowledge of Principle A but appeared to allow violation of Principle B.

Consider now the proof nets for (55a) and (55b) given in figures 2 and 3 respectively. The number of axiom links crossing in between any two words is the number of unresolved dependencies or valencies at that point in an incremental analysis. It is argued in Johnson (1998) and Morrill (2000) that this number reflects the processing load on memory in the course of time. Thus we can read off from the proof nets complexity profiles showing the incremental burden on memory. For our examples these are as follows:

(56)

7									
6					b	b			
5			b	b			ab	ab	
4									
3									
2		ab			a	a			
1	ab		a	a					
0									ab
a.	Kitty	says	that	Sarah	should	point	to	herself	
b.	Kitty	says	that	Sarah	should	point	to	her	

We see that the complexity curve for the b case is notably higher: in this sense it is predicted that a child with cognitively limited processing capacity may manifest difficulties in processing the long dependencies of personal pronouns. Thus we can maintain a strong innateness hypothesis: that children know principle B, but that they exhibit a delay of Principle B effect due to processing limitations.

References

Buszkowski, Wojciech: 1986, 'Completeness results for Lambek syntactic calculus', *Zeitschrift für mathematische Logik und Grundlage der Mathematik* **32**, 13–28.

Carpenter, Bob: 1998, *Type-Logical Semantics*, MIT Press, Cambridge, Massachusetts.

Chien, Yu-Chin and Kenneth Wexler: 1990 'Children's Knowledge of Locality Conditions in Binding as Evidence for the Modularity of Syntax and Pragmatics', *Language Aquisition*, **1**, *3*, 225–295.

Chomsky, Noam: 1981, *Lectures on Government and Binding*, Foris, Dordrecht.

Girard, Jean-Yves: 1987, 'Linear Logic', *Theoretical Computer Science* **50**, 1–102.

de Groote, Philippe: 1999, 'An algebraic correctness criterion for intuitionistic multiplicative proofnets', *Theoretical Computer Science*, 224, 115–134.

de Groote, Philippe and Christian Retoré: 1996, 'On the Semantic Readings of Proof-Nets', in G.-J. Kruijff, G. Morrill and R. T. Oehrle (eds.) *Proceedings of formal Grammar 1996*, Prague, 57–70.

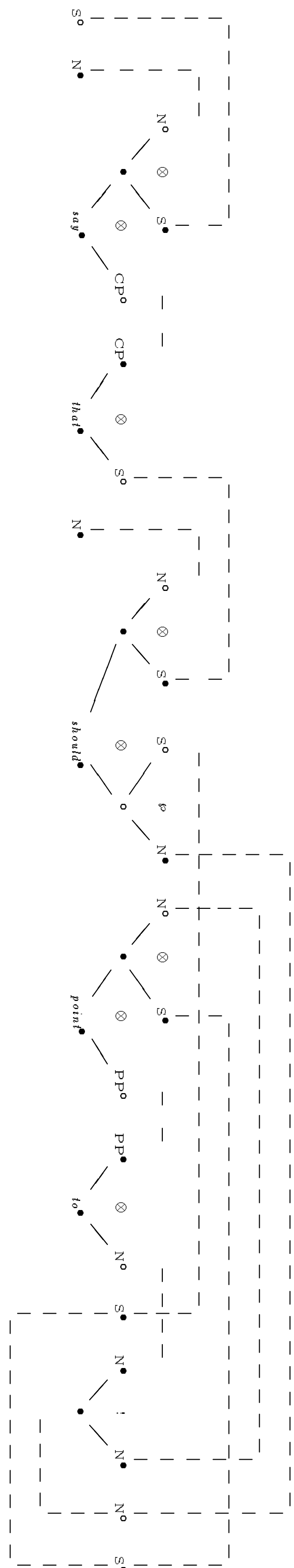


Figure 2: Proof net for 'Kitty says that Sarah should point to herself'

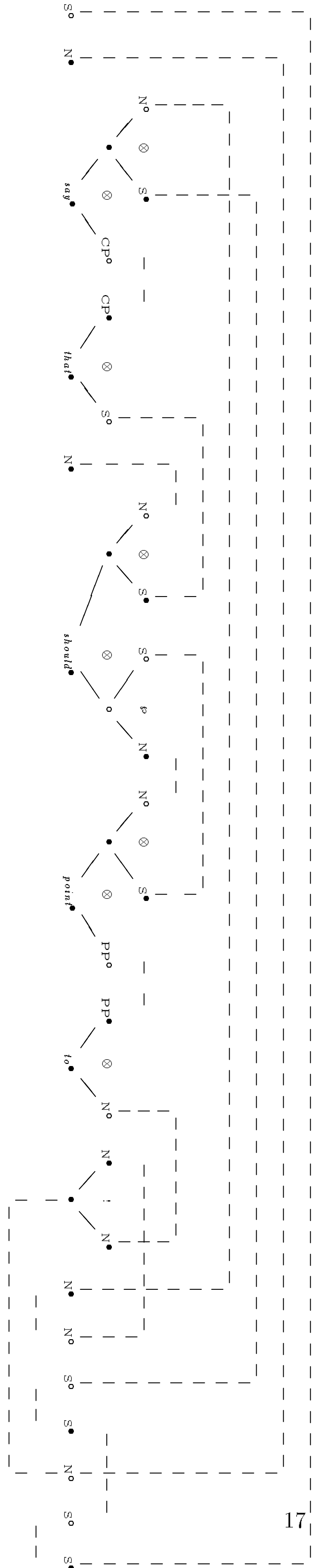


Figure 3: Proof net for ‘Kitty says that Sarah should point to her’

Jacobson, Pauline: 1999, 'Towards a Variable-Free Semantics', *Linguistics and Philosophy* **22** *2*, 117–184.

Jäger, Gerhard: 1998, 'Anaphora and Quantification in Categorial Grammar', to appear in Alain Lecomte and Michael Moortgat (eds.), *Proceedings of Logical Aspects of Computational Linguistics '98*, Springer-Verlag, Berlin.

Johnson, Mark: 1998, 'Proof Nets and the Complexity of Processing Center Embedded Constructions', *Journal of Logic, Language and Information* **7**, 433–447.

Lamarche, F. 1995, 'Games semantics for full proposicional linear logic', *Ninth Annual IEEE Symposium on Logic in Computer Science*, IEEE Press.

Lambek, J.: 1958, 'The mathematics of sentence structure', *American Mathematical Monthly* **65**, 154–170, also in Buszkowski, W., W. Marciszewski, and J. van Benthem (eds.): 1988, *Categorial Grammar*, Linguistic & Literary Studies in Eastern Europe Volume 25, John Benjamins, Amsterdam, 153–172.

Moortgat, Michael: 1988, *Categorial Investigations: Logical and Linguistic Aspects of the Lambek Calculus*, Foris, Dordrecht.

Moortgat, Michael: 1996, 'Generalised Quantification and Discontinuous type constructors', in Bunt and van Horck: 1996 (eds.) *Discontinuous Constituency*, Mouton de Gruyter, Berlin, 181–208.

Moortgat, Michael: 1997, 'Categorial Type Logics', in J. van Benthem and A. ter Meulen (eds.) *Handbook of Logic and Language*, Elsevier, Amsterdam, 93–177.

Morrill, Glyn: 1990, 'Intensionality and Boundedness', *Linguistics and Philosophy* **13** *6*, 699–726.

Morrill, Glyn: 1994, *Type Logical Grammar: Categorial Logic of Signs*, Kluwer Academic Publishers, Dordrecht.

Morrill, Glyn: 1995a, 'Clausal Proofs and Discontinuity', *Bulletin of the Interest Group in Pure and Applied Logics* **3**, *2, 3*, 403–427.

Morrill, Glyn: 1995b, 'Discontinuity in Categorial Grammar', *Linguistics and Philosophy* **18**, 175–219.

Morrill, Glyn: 1999, 'Geometry of Lexico-Syntactic Interaction', Proceedings *Meeting of the European Chapter of the Association for Computational Linguistics*, Bergen, 61–70.

Morrill, Glyn: 2000, 'Incremental Processing and Acceptability', to appear in *Computational Linguistics* **16** *3*.

Morrill, Glyn and Josep Maria Merenciano: 1996, 'Generalising Discontinuity', *Traitement Automatique des Langues* **37**, *2*, 119–143.

Morrill, Glyn and Teresa Solias: 1993, 'Tuples, Discontinuity and Gapping', Proceedings *Meeting of the European Chapter of the Association for Computational Linguistics*, Utrecht, 287–297.

Pentus, M.: 1994, 'Language completeness of the Lambek calculus', *Proceedings of the Eighth Annual IEEE Symposium on Logic in Computer Science*, Paris.

Reinhart, Tanya: 1983, 'Coreference and Bound Anaphora: a Restatement of the Anaphora Questions', *Linguistics and Philosophy*, **6**, 47–88.

Roorda, Dirk: 1991, *Resource Logics: proof-theoretical investigations*, Ph.D. dissertation, Universiteit van Amsterdam.

Solias, Teresa: 1992, *Gramáticas Categoriales, Coordinación Generalizada y Elisión*, Ph.D. dissertation, Universidad Autónoma de Madrid.

Steedman, Mark: 2000, *The Syntactic Process*, to appear, MIT Press, Cambridge, Massachusetts.

Versmissen, Koen: 1991, 'Discontinuous Type Constructors in Categorial Grammar', Masters dissertation, Universiteit Utrecht.