Parsing Logical Grammar: CatLog3*

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Abstract

CatLog3 is a Prolog parser/theorem-prover for (type) logical (categorial) grammar. In such logical grammar, grammar is *reduced* to logic: a string of words is grammatical if and only if an associated logical statement is a theorem. CalLog3 implements a logic extending displacement calculus, a sublinear fragment including as primitive connectives the continuous (Lambek) and discontinuous wrapping connectives of the displacement calculus, additives, 1st order quantifiers, normal modalities, bracket modalities and subexponentials. In this paper we survey how CatLog3 is implemented on the principles of Andreoli's focusing and a generalisation of van Benthem's count-invariance.

1 Introduction

The linguistics descended from formal grammar as popularised by Chomsky (1957[6]) has renegaded on formalisation, and discrete computational grammar in the genre of the 1980s has given way to statistical NLP. However, there is a venerable older line of linguistics practicing grammar according to the standards of mathematical logic. The seminal paper in this line is Lambek (1958[19]) which defines a syntactic calculus and proves Cut-elimination for it, but the tradition dates back at least to Bar-Hillel (1953[3]) and Ajdukiewicz (1935[1]). The Lambek calculus is a calculus of concatentation which is free of structural rules. The displacement calculus of Morrill et al. (2011[45]) generalises Lambek calculus with intercalation, containing both continuous and discontinuous connective families, while remaining free of structural rules, and preserving Cut-elimination and its good corollaries: the subformula property, decidability, the finite reading property, and the focusing property. There are several monographs and reference articles on this type logical approach: Moortgat (1988[22]; 1997[24])), Morrill (1994[46]; 2010[30]; 2011[47]; 2012[33]), Carpenter (1997[5]), Jäger (2005[13]), and Moot and Retoré (2012[26]). The CatLog program series comprises implementations in Prolog of type logical parser/theorem-provers starting from the basis of logic programming of displacement calculus theorem-proving (Morrill 2011[31]):

- CatLog1 (Morrill 2012[32] was based on uniform proof (Miller et al. 1991[20]), and count-invariance for multiplicatives (van Benthem 1991[50]).
- CatLog2 was based on Andreoli's focusing (Andreoli 1992[2]) , and count-invariance for multiplicatives, additives and bracket modalities (Valentín et al. 2013[49]).

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	cont. mult.	disc. mult.	add.	qu.	norm. mod.	brack. mod.	exp.	lim. contr. & weak.
primary	$/$ $\backslash$ $\bullet$ I	$\uparrow$ $\downarrow$ $\odot$ J	$\&$ $\oplus$	$\bigwedge$ $\bigvee$	$\Box$ $\Diamond$	$[\,]^{-1}$ $\langle\rangle$	$!$ $?$	$\mid$ W
sem. inactive variants	•− −◦ ◦− −• ◐ ◑	⫯ ⫰ ⫯ ⫰ ◓ ◒	$\sqcap$ $\sqcup$	$\forall$ $\exists$	■ ◆			
det. synth.	$\triangleleft^{-1}$ $\triangleright^{-1}$ $\triangleleft$ $\triangleright$	ˇ ˆ						diff.
nondet. synth.	$\div$ $\times$	$\Uparrow$ $\Downarrow$ $\circledcirc$						$-$

Table 1: Categorial connectives

- CatLog3 is based on focalisation and count-invariance for multiplicatives, additives, bracket modalities and (sub)exponentials (Kuznetov et al. 2017[16]).

In this paper we survey the methods on which the implementation of CatLog3 is based. In Section 2 we describe the primitive connectives of the logical fragment for which parsing/theorem-proving is implemented. In Sections 3 and 4 we discuss focusing and count-invariance respectively. In Section 5 we illustrate in relation to the *Montague Test* (Morrill and Valentín 2016[42]): the task of providing a computational grammar of Montague's (1973[21]) fragment.

2 Displacement logic

The formalism used comprises the connectives of Table 1. The heart of the logic is the displacement calculus of Morrill and Valentín (2010[37]) and Morrill, Valentín and Fadda (2011[45]) made up of twin continuous and discontinuous residuated families of connectives having a pure Gentzen sequent calculus —without labels and free of structural rules— and enjoying Cut-elimination (Valentín 2012[48]). Other primary connectives include additives, 1st order quantifiers, normal (i.e. distributive) modalities, bracket (i.e. nondistributive) modalities, and (sub)exponentials.[1] We can draw a clear

[1]Once Cut-elimination is established, the only challenge to decidability comes from nonlinearity: the contraction rule of the universal exponential, and the infinitary left rule of the existential exponential. In this connection, linguistically the existential left rule is not required; and Morrill and Valentín (2015[40]) introduced displacement logics $\mathbf{Db!_b?}$ and $\mathbf{Db!?}$ with a relevant modality ! ,with and without bracket conditioning for contaction. Kanovich et al. (2016[15]) prove the undecidability of $\mathbf{Db}$!? and in unpublished work announce the undecidability of $\mathbf{Db!_b?}$. But Morrill and Valentín (2015[40]) prove the decidability of a linguistically sufficient 'bracket non-negative' special case of $\mathbf{Db!_b?}$.

1.	$\mathcal{F}_i$	::=	$\mathcal{F}_{i+j}/\mathcal{F}_j$	$T(C/B)$	=	$T(B){\to}T(C)$	over
2.	$\mathcal{F}_j$	::=	$\mathcal{F}_i\backslash\mathcal{F}_{i+j}$	$T(A\backslash C)$	=	$T(A){\to}T(C)$	under
3.	$\mathcal{F}_{i+j}$	::=	$\mathcal{F}_i{\bullet}\mathcal{F}_j$	$T(A{\bullet}B)$	=	$T(A)\&T(B)$	continuous product
4.	$\mathcal{F}_0$	::=	I	$T(I)$	=	$\top$	continuous unit
5.	$\mathcal{F}_{i+1}$	::=	$\mathcal{F}_{i+j}{\uparrow_k}\mathcal{F}_j, 1 \leq k \leq i{+}j$	$T(C{\uparrow_k}B)$	=	$T(B){\to}T(C)$	circumfix
6.	$\mathcal{F}_j$	::=	$\mathcal{F}_{i+1}{\downarrow_k}\mathcal{F}_{i+j}, 1 \leq k \leq i{+}1$	$T(A{\downarrow_k}C)$	=	$T(A){\to}T(C)$	infix
7.	$\mathcal{F}_{i+j}$	::=	$\mathcal{F}_{i+1}{\odot_k}\mathcal{F}_j, 1 \leq k \leq i{+}1$	$T(A{\odot_k}B)$	=	$T(A)\&T(B)$	discontinuous product
8.	$\mathcal{F}_1$	::=	J	$T(J)$	=	$\top$	discontinuous unit
9.	$\mathcal{F}_i$	::=	$\mathcal{F}_i\&\mathcal{F}_i$	$T(A\&B)$	=	$T(A)\&T(B)$	additive conjunction
10.	$\mathcal{F}_i$	::=	$\mathcal{F}_i{\oplus}\mathcal{F}_i$	$T(A{\oplus}B)$	=	$T(A){+}T(B)$	additive disjunction
11.	$\mathcal{F}_i$	::=	$\bigwedge V\mathcal{F}_i$	$T(\bigwedge vA)$	=	$F{\to}T(A)$	1st order univ. qu.
12.	$\mathcal{F}_i$	::=	$\bigvee V\mathcal{F}_i$	$T(\bigvee vA)$	=	$F\&T(A)$	1st order exist. qu.
13.	$\mathcal{F}_i$	::=	$\Box\mathcal{F}_i$	$T(\Box A)$	=	$\mathbf{L}T(A)$	universal modality
14.	$\mathcal{F}_i$	::=	$\Diamond\mathcal{F}_i$	$T(\Diamond A)$	=	$\mathbf{M}T(A)$	existential modality
15.	$\mathcal{F}_i$	::=	$[\,]^{-1}\mathcal{F}_i$	$T([\,]^{-1}A)$	=	$T(A)$	univ. bracket modality
16.	$\mathcal{F}_i$	::=	$\langle\rangle\mathcal{F}_i$	$T(\langle\rangle A)$	=	$T(A)$	exist. bracket modality
17.	$\mathcal{F}_0$	::=	$!\mathcal{F}_0$	$T(!A)$	=	$T(A)$	universal exponential
18.	$\mathcal{F}_0^\circ$	::=	$?\mathcal{F}_0^\circ$	$T(?A)$	=	$T(A)^+$	existential exponential

Table 2: Syntactic types of $\mathbf{DA1S4b!_b?}$

distinction between the primary connectives, the semantically inactive connectives, and the synthetic connectives; the latter two are abbreviatory and are there for convenience, and to simplify derivation. There are semantically inactive variants of the continuous and discontinuous multiplicatives, and semantically inactive variants of the additives, 1st order quantifiers, and normal modalities.[2] Synthetic connectives (Girard 2011[11]) divide into the continuous and discontinuous deterministic (unary) synthetic connectives, and the continuous and discontinuous nondeterministic (binary) synthetic connectives.[3]

2.1 Syntactic types

The syntactic types of displacement logic are sorted $\mathcal{F}_0, \mathcal{F}_1, \mathcal{F}_2, \ldots$ according to the number of points of discontinuity $0, 1, 2, \ldots$ their expressions contain. Each type predicate letter has a sort and an arity which are naturals, and a corresponding semantic type. Assuming ordinary terms to be already given, where P is a type predicate letter of sort i and arity n and $t_1, \ldots, t_n$ are terms, $Pt_1 \ldots t_n$ is an (atomic) type of sort i of the corresponding semantic type. Compound types of $\mathbf{DA1S4b!_b?}$ are formed as illustrated in Table 2, and the structure preserving semantic type map T associates these with semantic types.

2.2 Gentzen sequent calculus

We use a Gentzen sequent presentation standard from Gentzen (1934[9]) and Lambek (1958[19]). In Gentzen sequent antecedents for displacement logic with bracket modalities (structural inhibition) and exponentials (structural facilitation) there are also bracket constructors and 'stoups'.

[2]For example, the semantically inactive additive conjunction $A{\sqcap}B{:}\,\phi$ abbreviates $A\&B{:}\,(\phi,\phi)$.

[3]For example, the nondeterministic continuous division $B{\div}A$ abbreviates $(A\backslash B){\sqcap}(B/A)$.

Stoups (cf. the linear logic of Girard 2011[11]) (ζ) are stores read as multisets for re-usable (non-linear) resources which appear at the left of a configuration marked off by a semicolon (when the stoup is empty the semicolon may be ommited). The stoup of linear logic is for resources which can be contracted (copied) or weakened (deleted). By contrast, our stoup is for a linguistically motivated variant of contraction, and does not allow weakening. Furthermore, whereas linear logic is commutative, our logic is in general noncommutative and here the stoup is used for resources which are also commutative. A configuration together with a stoup is a *zone* (Ξ). The bracket constructor applies not to a configuration alone but to a configuration with a stoup, i.e a zone: reusable resources are specific to their domain. Stoups $\mathcal{S}$ and configurations $\mathcal{O}$ are defined by ($\emptyset$ is the empty stoup; Λ is the empty configuration; the *separator* 1 marks points of discontinuity):[4]

$$(1)\quad \begin{array}{lll} \mathcal{S} & ::= & \emptyset \mid \mathcal{F}_0, \mathcal{S} \\ \mathcal{O} & ::= & \Lambda \mid \mathcal{T}, \mathcal{O} \\ \mathcal{T} & ::= & 1 \mid \mathcal{F}_0 \mid \mathcal{F}_{i>0}\{\underbrace{\mathcal{O} : \ldots : \mathcal{O}}_{i\ \mathcal{O}'\text{s}}\} \mid [\mathcal{S}; \mathcal{O}] \end{array}$$

For a type A, its sort $s(A)$ is the i such that $A \in \mathcal{F}_i$. For a configuration Γ, its sort $s(\Gamma)$ is $|\Gamma|_1$, i.e. the number of points of discontinuity 1 which it contains. Sequents are of the form:

$$(2)\quad \mathcal{S}; \mathcal{O} \Rightarrow \mathcal{F} \text{ such that } s(\mathcal{O}) = s(\mathcal{F})$$

The figure $\overrightarrow{A}$ of a type A is defined by:

$$(3)\quad \overrightarrow{A} = \begin{cases} A & \text{if } s(A) = 0 \\ A\{\underbrace{1 : \ldots : 1}_{s(A)\ 1'\text{s}}\} & \text{if } s(A) > 0 \end{cases}$$

Where Γ is a configuration of sort i and $\Delta_1, \ldots, \Delta_i$ are configurations, the *fold* $\Gamma \otimes \langle \Delta_1 : \ldots : \Delta_i \rangle$ is the result of replacing the successive 1's in Γ by $\Delta_1, \ldots, \Delta_i$ respectively. Where Γ is of sort i, the hyperoccurrence notation $\Delta\langle\Gamma\rangle$ abbreviates $\Delta_0(\Gamma \otimes \langle \Delta_1 : \ldots : \Delta_i \rangle)$, i.e. a context configuration Δ (which is externally Δ_0 and internally $\Delta_1, \ldots, \Delta_i$) with a potentially discontinuous distinguished subconfiguration Γ (continuous if $i = 0$, discontinuous if $i > 0$). Where Δ is a configuration of sort $i > 0$ and Γ is a configuration, the kth *metalinguistic intercalation* $\Delta\,|_k\,\Gamma$, $1 \leq k \leq i$, is given by:

$$(4)\quad \Delta\,|_k\,\Gamma =_{df} \Delta \otimes \langle \underbrace{1 : \ldots : 1}_{k-1\ 1\text{'s}} : \Gamma : \underbrace{1 : \ldots : 1}_{i-k\ 1\text{'s}} \rangle$$

i.e. $\Delta\,|_k\,\Gamma$ is the configuration resulting from replacing by Γ the kth separator in Δ.

2.3 Rules and linguistic applications

A semantically labelled sequent is a sequent in which the antecedent type occurrences $A_1, \ldots, A_n$ are labelled by distinct variables $x_1, \ldots, x_n$ of types $T(A_1), \ldots, T(A_n)$ respectively, and the succedent type A is labelled by a term of type $T(A)$ with free variables drawn from $x_1, \ldots, x_n$. In this section we give the semantically labelled Gentzen sequent rules for the connectives of $\mathbf{DA1S4b!_b?}$, and indicate some linguistic applications.

The continuous multiplicatives, the Lambek connectives of Lambek (1958[19]; 1988[18]), Figure 1, defined in relation to concatenation/appending, are the basic means of categorial categorization and subcategorization. Note that here and throughout the active types in antecedents are figures

[4] Note that only types of sort 0 can go into the stoup; reusable types of other sorts would not preserve the sequent antecedent-succedent sort equality under contraction or expansion: $0 + 0 = 0$, but $i + i \neq i$ for $i > 0$.

1. $$\frac{\zeta_1; \Gamma \Rightarrow B\colon \psi \qquad \Xi(\zeta_2; \Delta\langle \overrightarrow{C}\colon z\rangle) \Rightarrow D\colon \omega}{\Xi(\zeta_1 \uplus \zeta_2; \Delta\langle \overrightarrow{C/B}\colon x, \Gamma\rangle) \Rightarrow D\colon \omega\{(x\ \psi)/z\}} /L \qquad \frac{\zeta; \Gamma, \overrightarrow{B}\colon y \Rightarrow C\colon \chi}{\zeta; \Gamma \Rightarrow C/B\colon \lambda y \chi} /R$$

2. $$\frac{\zeta_1; \Gamma \Rightarrow A\colon \phi \qquad \Xi(\zeta_2; \Delta\langle \overrightarrow{C}\colon z\rangle) \Rightarrow D\colon \omega}{\Xi(\zeta_1 \uplus \zeta_2; \Delta\langle \Gamma, \overrightarrow{A\backslash C}\colon y\rangle) \Rightarrow D\colon \omega\{(y\ \phi)/z\}} \backslash L \qquad \frac{\zeta; \overrightarrow{A}\colon x, \Gamma \Rightarrow C\colon \chi}{\zeta; \Gamma \Rightarrow A\backslash C\colon \lambda x \chi} \backslash R$$

3. $$\frac{\Xi\langle \overrightarrow{A}\colon x, \overrightarrow{B}\colon y\rangle \Rightarrow D\colon \omega}{\Xi\langle \overrightarrow{A{\bullet}B}\colon z\rangle \Rightarrow D\colon \omega\{\pi_1 z/x, \pi_2 z/y\}} \bullet L \qquad \frac{\zeta_1; \Delta \Rightarrow A\colon \phi \qquad \zeta_2; \Gamma \Rightarrow B\colon \psi}{\zeta_1 \uplus \zeta_2; \Delta, \Gamma \Rightarrow A{\bullet}B\colon (\phi, \psi)} \bullet R$$

4. $$\frac{\Xi\langle \Lambda\rangle \Rightarrow A\colon \phi}{\Xi\langle \overrightarrow{I}\colon x\rangle \Rightarrow A\colon \phi} IL \qquad \frac{}{\emptyset; \Lambda \Rightarrow I\colon 0} IR$$

Figure 1: Lambek multiplicatives

(vectorial) whereas those in succedents are not; intuitively this is because antecedents are structured but succedents are not. The directional divisions over, /, and under, \, are exemplified by assignments such as *the*: N/CN for *the man*: N and *sings*: $N\backslash S$ for *John sings*: S, and *loves*: $(N\backslash S)/N$ for *John loves Mary*: S.

5. $$\frac{\zeta_1; \Gamma \Rightarrow B\colon \psi \qquad \Xi(\zeta_2; \Delta\langle \overrightarrow{C}\colon z\rangle) \Rightarrow D\colon \omega}{\Xi(\zeta_1 \uplus \zeta_2; \Delta\langle \overrightarrow{C{\uparrow_k}B}\colon x \,|_k\, \Gamma\rangle) \Rightarrow D\colon \omega\{(x\ \psi)/z\}} \uparrow_k L \qquad \frac{\zeta; \Gamma \,|_k\, \overrightarrow{B}\colon y \Rightarrow C\colon \chi}{\zeta; \Gamma \Rightarrow C{\uparrow_k}B\colon \lambda y \chi} \uparrow_k R$$

6. $$\frac{\zeta_1; \Gamma \Rightarrow A\colon \phi \qquad \Xi(\zeta_2; \Delta\langle \overrightarrow{C}\colon z\rangle) \Rightarrow D\colon \omega}{\Xi(\zeta_1 \uplus \zeta_2; \Delta\langle \Gamma \,|_k\, \overrightarrow{A{\downarrow_k}C}\colon y\rangle) \Rightarrow D\colon \omega\{(y\ \phi)/z\}} \downarrow_k L \qquad \frac{\zeta; \overrightarrow{A}\colon x \,|_k\, \Gamma \Rightarrow C\colon \chi}{\zeta; \Gamma \Rightarrow A{\downarrow_k}C\colon \lambda x \chi} \downarrow_k R$$

7. $$\frac{\Xi\langle \overrightarrow{A}\colon x \,|_k\, \overrightarrow{B}\colon y\rangle \Rightarrow D\colon \omega}{\Xi\langle \overrightarrow{A{\odot_k}B}\colon z\rangle \Rightarrow D\colon \omega\{\pi_1 z/x, \pi_2 z/y\}} \odot_k L \qquad \frac{\zeta_1; \Delta \Rightarrow A\colon \phi \qquad \zeta_2; \Gamma \Rightarrow B\colon \psi}{\zeta_1 \uplus \zeta_2; \Delta \,|_k\, \Gamma \Rightarrow A{\odot_k}B\colon (\phi, \psi)} \odot_k R$$

8. $$\frac{\Xi\langle 1\rangle \Rightarrow A\colon \phi}{\Xi\langle \overrightarrow{J}\colon x\rangle \Rightarrow A\colon \phi} JL \qquad \frac{}{\emptyset; 1 \Rightarrow J\colon 0} JR$$

Figure 2: Displacement multiplicatives

The discontinuous multiplicatives of Figure 2, the displacement connectives, Morrill and Valentín (2010[37]), Morrill et al. (2011[45]), are defined in relation to intercalation/plugging. When the value of the k subindex indicates the first (leftmost) point of discontinuity it may be omitted, i.e. it defaults to 1. Circumfixation, ↑, is exemplified by a discontinuous particle verb assignment *calls*+1+*up*: $(N\backslash S){\uparrow}N$ for *Mary calls John up*: S, and infixation, ↓, and circumfixation together are exemplified by a quantifier phrase assignment *everyone*: $(S{\uparrow}N){\downarrow}S$ simulating Montague's S14 treatment of quantifying in; see Section 5.

In relation to the multiplicative rules, notice how the stoup is distributed reading bottom-up from conclusions to premise: it is partitioned between the two premises in the case of binary rules, copied to the premise in the case of unary rules, and empty in the case of nullary rules (axioms).

9. $$\frac{\Xi\langle\overrightarrow{A}\colon x\rangle \Rightarrow C\colon\chi}{\Xi\langle\overrightarrow{A\&B}\colon z\rangle \Rightarrow C\colon\chi\{\pi_1 z/x\}}\&L_1 \qquad \frac{\Xi\langle\overrightarrow{B}\colon y\rangle \Rightarrow C\colon\chi}{\Xi\langle\overrightarrow{A\&B}\colon z\rangle \Rightarrow C\colon\chi\{\pi_2 z/y\}}\&L_2$$

$$\frac{\Xi \Rightarrow A\colon\phi \qquad \Xi \Rightarrow B\colon\psi}{\Xi \Rightarrow A\&B\colon(\phi,\psi)}\&R$$

10. $$\frac{\Xi\langle\overrightarrow{A}\colon x\rangle \Rightarrow C\colon\chi_1 \qquad \Xi\langle\overrightarrow{B}\colon y\rangle \Rightarrow C\colon\chi_2}{\Xi\langle\overrightarrow{A\oplus B}\colon z\rangle \Rightarrow C\colon z \to x.\chi_1; y.\chi_2}\oplus L$$

$$\frac{\Xi \Rightarrow A\colon\phi}{\Xi \Rightarrow A\oplus B\colon\iota_1\phi}\oplus R_1 \qquad \frac{\Xi \Rightarrow B\colon\psi}{\Xi \Rightarrow A\oplus B\colon\iota_2\psi}\oplus R_2$$

Figure 3: Additives

The additives of Figure 3, Lambek (1961[17]), Morrill (1990[27]), Kanazawa (1992[14]), have application to polymorphism. For example the additive conjunction & can be used for *rice*: $N\&CN$ as in *rice grows*: S and *the rice grows*: S,[5] and the additive disjunction $\oplus$ can be used for *is*: $(N\backslash S)/(N\oplus(CN/CN))$ as in *Tully is Cicero*: S and *Tully is humanist*: S. The additive disjunction can be used together with the continuous unit to express the optionality of a complement as in *eats*: $(N\backslash S)/(N\oplus I)$ for *John eats fish*: S and *John eats*: S.[6]

Notice how the stoup is identical in conclusions and premises of additive rules.

11. $$\frac{\Xi\langle\overrightarrow{A[t/v]}\colon x\rangle \Rightarrow B\colon\psi}{\Xi\langle\overrightarrow{\bigwedge vA}\colon z\rangle \Rightarrow B\colon\psi\{(z\ t)/x\}}\bigwedge L \qquad \frac{\Xi \Rightarrow A[a/v]\colon\phi}{\Xi \Rightarrow \bigwedge vA\colon\lambda v\phi}\bigwedge R^\dagger$$

12. $$\frac{\Xi\langle\overrightarrow{A[a/v]}\colon x\rangle \Rightarrow B\colon\psi}{\Xi\langle\overrightarrow{\bigvee vA}\colon z\rangle \Rightarrow B\colon\psi\{\pi_2 z/x\}}\bigvee L^\dagger \qquad \frac{\Xi \Rightarrow A[t/v]\colon\phi}{\Xi \Rightarrow \bigvee vA\colon(t,\phi)}\bigvee R$$

Figure 4: Quantifiers, where † indicates that there is no a in the conclusion

The quantifiers of Figure 4, Morrill (1994[46]), have application to features. For example, singular and plural number in *sheep*: $\bigwedge nCNn$ for *the sheep grazes*: S and *the sheep graze*: S. And for a past, present or future tense finite sentence complement we can have *said*: $(N\backslash S)/\bigvee tSf(t)$ in *John said Mary walked*: S, *John said Mary walks*: S and *John said Mary will walk*: S.

Notice how the stoup is identical in conclusion and premise in each quantifier rule.

With respect to the (**S4**) normal modalities of Figure 5, the universal (Morrill 1990[28]) has application to intensionality. For example, for a propositional attitude verb such as *believes* we can

[5]Note the computational advantage of this approach over assuming an empty determiner: if empty operators were allowed they could occur any number of times in any positions.

[6]Note the advantage of this over simply listing intransitive and transitive lexical entries: empirically the latter does not capture the generalisation that in both cases the verb *eats* combines with a subject to the left, and computationally every lexical ambiguity doubles the lexical insertion search space. Appeal to lexical ambiguity constitutes resignation from the capture of generalisations and is at best a promissory solution, unless there is true ambiguity.

13. $$\frac{\Xi\langle\overrightarrow{A}: x\rangle \Rightarrow B: \psi}{\Xi\langle\overrightarrow{\Box A}: z\rangle \Rightarrow B: \psi\{^{\vee}z/x\}}\ \Box L \qquad \frac{\boxtimes\Xi \Rightarrow A: \phi}{\boxtimes\Xi \Rightarrow \Box A: {}^{\wedge}\phi}\ \Box R$$

14. $$\frac{\boxtimes\Xi\langle\overrightarrow{A}: x\rangle \Rightarrow \oplus B: \psi}{\boxtimes\Xi\langle\overrightarrow{\Diamond A}: z\rangle \Rightarrow \oplus B: \psi\{^{\cup}z/x\}}\ \Diamond L \qquad \frac{\Xi \Rightarrow A: \phi}{\Xi \Rightarrow \Diamond A: {}^{\cap}\phi}\ \Diamond R$$

Figure 5: Normal modalities, where $\boxtimes/\oplus$ marks a structure all the types of which have main connective a box/diamond

assign type $\Box((N\backslash S)/\Box S)$ with a modality outermost since the word has a sense, and a modality on the first argument but not the second, since the sentential complement is an intensional domain, but not the subject. The modalities are in the categorial type, distinctly from, but in relation to, the logical interpretation of the propositional attitude verb. The $\Box$ Right rule is semantically interpreted by intensionalisation $^{\wedge}$ and the $\Box$ Left rule is semantically interpreted by extensionalisation $^{\vee}$ in such a way that the Curry-Howard correspondence for the modality yields the law of down-up cancellation (Dowty et al. 1981[7]): $^{\vee\wedge}\phi = \phi$.

Notice how the stoup is identical in conclusion and premise in each normal modality rule.

15. $$\frac{\Xi\langle\overrightarrow{A}: x\rangle \Rightarrow B: \psi}{\Xi\langle[\overrightarrow{[\,]^{-1}A}: x]\rangle \Rightarrow B: \psi}\ [\,]^{-1}L \qquad \frac{[\Xi] \Rightarrow A: \phi}{\Xi \Rightarrow [\,]^{-1}A: \phi}\ [\,]^{-1}R$$

16. $$\frac{\Xi\langle[\overrightarrow{A}: x]\rangle \Rightarrow B: \psi}{\Xi\langle\overrightarrow{\langle\rangle A}: x\rangle \Rightarrow B: \psi}\ \langle\rangle L \qquad \frac{\Xi \Rightarrow A: \phi}{[\Xi] \Rightarrow \langle\rangle A: \phi}\ \langle\rangle R$$

Figure 6: Bracket modalities

The bracket modalities of Figure 6, Morrill (1992[29]) and Moortgat 1995[23]), have application to nonassociativity and syntactical domains such as prosodic phrases and extraction islands. For example, single bracketing for weak islands: *walks*: $\langle\rangle N\backslash S$ for the subject condition, and *without*: $[\,]^{-1}(VP\backslash VP)/VP$ for the adverbial island constraint; and double bracketing for strong islands such as *and*: $(S\backslash[\,]^{-1}[\,]^{-1}S)/S$ for the coordinate structure constraint.

Notice how the stoup is identical in conclusions and premises of bracket modality rules.

Finally, there is nonlinearity. The universal exponential of Figure 7, Girard (1987[10]), Barry, Hepple, Leslie and Morrill (1991[4]), Morrill (1994[46]), Morrill and Valentín (2015[40]), and Morrill (2017[34]), has application to extraction including parasitic extraction. In the formulation here $!L$ moves the operand of a universal exponential (e.g. the hypothetical subtype of relativisation) into the stoup, where it will percolate as commented for the above rules. From there it can be copied into the stoup of a newly-created bracketed domain by the contraction rule $!C$ (producing a parasitic gap), and it can be moved into any position in the matrix configuration of its zone by $!P$ (producing a normal nonparasitic or host gap).

Using the universal exponential, !, for which contraction induces island brackets, we can assign a relative pronoun type *that*: $(CN\backslash CN)/(S/!N)$ allowing parasitic extraction such as *paper that John filed without reading*: CN, where parasitic gaps can appear only in (weak) islands, but can be iterated in subislands, for example, *man who the fact that the friends of admire without praising surprises*. Cru-

17. $$\frac{\Xi(\zeta \uplus \{A\colon x\}; \Gamma_1, \Gamma_2) \Rightarrow B\colon \psi}{\Xi(\zeta; \Gamma_1, !A\colon x, \Gamma_2) \Rightarrow B\colon \psi}\, !L \qquad \frac{\zeta; \Lambda \Rightarrow A\colon \phi}{\zeta; \Lambda \Rightarrow !A\colon \phi}\, !R$$

$$\frac{\Xi(\zeta; \Gamma_1, A\colon x, \Gamma_2) \Rightarrow B\colon \psi}{\Xi(\zeta \uplus \{A\colon x\}; \Gamma_1, \Gamma_2) \Rightarrow B\colon \psi}\, !P$$

$$\frac{\Xi(\zeta_1 \uplus \zeta_2 \uplus \{A\colon x\}; \Gamma_1, [\zeta_2 \uplus \{A\colon y\}; \Gamma_2], \Gamma_3) \Rightarrow B\colon \psi}{\Xi(\zeta_1 \uplus \zeta_2 \uplus \{A\colon x\}; \Gamma_1, \Gamma_2, \Gamma_3) \Rightarrow B\colon \psi\{x/y\}}\, !C$$

18. $$\frac{\Xi(A\colon x_1) \Rightarrow B\colon \psi([x_1]) \qquad \Xi(A\colon x_1, A\colon x_2) \Rightarrow B\colon \psi([x_1, x_2]) \qquad \ldots}{\Xi(?A\colon x) \Rightarrow B\colon \psi(x)}\, ?L$$

$$\frac{\Xi \Rightarrow A\colon \phi}{\Xi \Rightarrow ?A\colon [\phi]}\, ?R \qquad \frac{\zeta; \Gamma \Rightarrow A\colon \phi \qquad \zeta'; \Delta \Rightarrow ?A\colon \psi}{\zeta \uplus \zeta'; \Gamma, \Delta \Rightarrow ?A\colon [\phi|\psi]}\, ?M$$

Figure 7: Exponentials

cially, in the linguistic formulation ! does not have weakening, i.e. deletion, since, e.g., the body of a relative clause *must* contain a gap: **man who John loves Mary*.

The existential exponential ? has application to iterated coordination (Morrill 1994[46]; Morrill and Valentín 2015[40]) and (unboundedly iterated) *respectively* (Morrill and Valentín 2016[43]). Using the existential exponential, ?, we can assign a coordinator type *and*: $(?N\backslash N)/N$ allowing iterated coordination as in *John, Bill, Mary and Suzy*: N, or *and*: $(?(S/N)\backslash(S/N))/(S/N)$ for *John likes Mary dislikes, and Bill hates, London* (iterated right node raising), and so on.

In relation to the rest of the primary connectives: the limited contraction | of Jäger (2005[13]) has application to anaphora and the limited weakening W of Morrill and Valentín (2014[39]) has application to words as types. The remaining, semantically inactive, connectives listed here were introduced as follows. Semantically inactive multiplicatives {•−, −∘, ∘−, −•, ◐, ◑, ⫯, ⫰, ⫯, ⫰, ◓, ◒}: Morrill and Valentín (2014[39]). Semantically inactive additives {⊓, ⊔}: Morrill (1994[46]). Semantically inactive first-order quantifiers {∀, ∃}: Morrill (1994[46]). Semantically inactive normal modalities {■, ♦}: Hepple (1990[12]), Morrill (1994[46]). The rules for semantically inactive variants are the same as those for the semantically active versions syntactically, but have the same label on premises and conclusions semantically.[7]

3 Focusing

Spurious ambiguity is the phenomenon whereby distinct derivations in grammar may assign the same structural reading, resulting in redundancy in the parse search space and inefficiency in parsing. Understanding the problem depends on identifying the essential mathematical structure of derivations. This is trivial in the case of context free grammar, where the parse structures are ordered trees; in the case of type logical categorial grammar, the parse structures are proof nets. However, with respect to

[7]The synthetic connectives are: left and right projection and injection $\{\triangleleft^{-1}, \triangleright^{-1}, \triangleleft, \triangleright\}$, Morrill, Fadda and Valentín (2009[44]); split and bridge {˘, ˆ}, Morrill and Merenciano (1996[36]); continuous and discontinuous nondeterministic multiplicatives $\{\div, \times, \Uparrow, \Downarrow, \circledcirc\}$, Morrill, Valentín and Fadda (2011[45]). The difference operator − of Morrill and Valentín (2014[38]) has application to linguistic exceptions.

$$\frac{\overrightarrow{A}: x, \Gamma \Rightarrow C: \chi \quad \neg\ \mathbf{foc}}{\Gamma \Rightarrow A\backslash C: \lambda x\chi \quad \neg\ \mathbf{foc} \wedge\ \mathbf{rev}}\backslash R \qquad \frac{\Gamma, \overrightarrow{B}: y \Rightarrow C: \chi \quad \neg\ \mathbf{foc}}{\Gamma \Rightarrow C/B: \lambda y\chi \quad \neg\ \mathbf{foc} \wedge\ \mathbf{rev}}/R$$

$$\frac{\Delta\langle \overrightarrow{A}: x, \overrightarrow{B}: y\rangle \Rightarrow D: \omega \quad \neg\ \mathbf{foc}}{\Delta\langle \overrightarrow{A\bullet B}: z\rangle \Rightarrow D: \omega\{\pi_1 z/x, \pi_2 z/y\} \quad \neg\ \mathbf{foc} \wedge\ \mathbf{rev}}\bullet L$$

$$\frac{\Delta\langle \Lambda\rangle \Rightarrow A: \phi \quad \neg\ \mathbf{foc}}{\Delta\langle \overrightarrow{I}: x\rangle \Rightarrow A: \phi \quad \neg\ \mathbf{foc} \wedge\ \mathbf{rev}} IL$$

$$\frac{\overrightarrow{A}: x\,|_k\,\Gamma \Rightarrow C: \chi \quad \neg\ \mathbf{foc}}{\Gamma \Rightarrow A{\downarrow_k}C: \lambda x\chi \quad \neg\ \mathbf{foc} \wedge\ \mathbf{rev}}{\downarrow_k}R \qquad \frac{\Gamma\,|_k\,\overrightarrow{B}: y \Rightarrow C: \chi \quad \neg\ \mathbf{foc}}{\Gamma \Rightarrow C{\uparrow_k}B: \lambda y\chi \quad \neg\ \mathbf{foc} \wedge\ \mathbf{rev}}{\uparrow_k}R$$

$$\frac{\Delta\langle \overrightarrow{A}: x\,|_k\,\overrightarrow{B}: y\rangle \Rightarrow D: \omega \quad \neg\ \mathbf{foc}}{\Delta\langle \overrightarrow{A\odot_k B}: z\rangle \Rightarrow D: \omega\{\pi_1 z/x, \pi_2 z/y\} \quad \neg\ \mathbf{foc} \wedge\ \mathbf{rev}}\odot_k L$$

$$\frac{\Delta\langle 1\rangle \Rightarrow A: \phi \quad \neg\ \mathbf{foc}}{\Delta\langle \overrightarrow{J}: x\rangle \Rightarrow A: \phi \quad \neg\ \mathbf{foc} \wedge\ \mathbf{rev}} JL$$

Figure 8: Reversible multiplicative rules

multiplicatives intrinsic proof nets have not yet been given for displacement calculus (but see Morrill and Fadda (2008[35], Fadda 2010[8], and Moot 2014[25]) In this context CatLog3 approaches spurious ambiguity by means of Andreoli's (1982[2]) proof-theoretic technique of focalisation, which engenders a substantial reduction of spurious ambiguity.

In focalisation, *situated* (in the antecedent of a sequent, input, $^{\bullet}$/ in the succedent of a sequent, output, $^{\circ}$) non-atomic types are classified as of *reversible/negative* or *irreversible/positive polarity* according as their associated rule is reversible or not. There are alternating phases of don't-care nondeterministic negative rule application, and positive rule application locking on to *focalised* formulas. Given a sequent with no occurrences of negative formulas, one chooses a positive formula as principal formula (which is boxed; we say it is focalised) and applies proof search to its subformulas while these remain positive. When one finds a negative formula or a literal, invertible rules are applied in a don't care nondeterminitic fashion until no longer possible, when another positive formula is chosen, and so on. CatLog3 can be set to focus all atoms in the input (as in the example at the end) or in the output, i.e. it implements uniform *bias*.

A sequent is either unfocused and as before, or else focused and has exactly one type boxed. This is the focused type. The focalised logical rules for displacement calculus are given in Figures 8–12. Sequents are accompanied by *judgements*: focalised or not focalised and reversible or not reversible. The completeness of this focalisation, together with additives, is proved in Morrill and Valentín (2015[41]). The completeness of focalisation for other connectives of CatLog3 is a topic of ongoing research.

$$\frac{\Gamma \Rightarrow \boxed{P}\colon \phi \quad \textbf{foc} \wedge \neg\, \textbf{rev} \qquad \Delta\langle \overrightarrow{\boxed{Q}}\colon z\rangle \Rightarrow D\colon \omega \quad \textbf{foc} \wedge \neg\, \textbf{rev}}{\Delta\langle \Gamma, \overrightarrow{\boxed{P\backslash Q}}\colon y\rangle \Rightarrow D\colon \omega\{(y\ \phi)/z\} \quad \textbf{foc} \wedge \neg\, \textbf{rev}}\ \backslash L$$

$$\frac{\Gamma \Rightarrow \boxed{P_1}\colon \phi \quad \textbf{foc} \wedge \neg\, \textbf{rev} \qquad \Delta\langle \overrightarrow{P_2}\colon z\rangle \Rightarrow D\colon \omega \quad \neg\, \textbf{foc} \wedge ?P_2\ \textbf{rev}}{\Delta\langle \Gamma, \overrightarrow{\boxed{P_1\backslash P_2}}\colon y\rangle \Rightarrow D\colon \omega\{(y\ \phi)/z\} \quad \textbf{foc} \wedge \neg\, \textbf{rev}}\ \backslash L$$

$$\frac{\Gamma \Rightarrow Q_1\colon \phi \quad \neg\, \textbf{foc} \wedge ?Q_1\ \textbf{rev} \qquad \Delta\langle \overrightarrow{\boxed{Q_2}}\colon z\rangle \Rightarrow D\colon \omega \quad \textbf{foc} \wedge \neg\, \textbf{rev}}{\Delta\langle \Gamma, \overrightarrow{\boxed{Q_1\backslash Q_2}}\colon y\rangle \Rightarrow D\colon \omega\{(y\ \phi)/z\} \quad \textbf{foc} \wedge \neg\, \textbf{rev}}\ \backslash L$$

$$\frac{\Gamma \Rightarrow Q\colon \phi \quad \neg\, \textbf{foc} \wedge ?Q\ \textbf{rev} \qquad \Delta\langle \overrightarrow{P}\colon z\rangle \Rightarrow D\colon \omega \quad \neg\, \textbf{foc} \wedge ?P\ \textbf{rev}}{\Delta\langle \Gamma, \overrightarrow{\boxed{Q\backslash P}}\colon y\rangle \Rightarrow D\colon \omega\{(y\ \phi)/z\} \quad \textbf{foc} \wedge \neg\, \textbf{rev}}\ \backslash L$$

$$\frac{\Gamma \Rightarrow \boxed{P}\colon \psi \quad \textbf{foc} \wedge \neg\, \textbf{rev} \qquad \Delta\langle \overrightarrow{\boxed{Q}}\colon z\rangle \Rightarrow D\colon \omega \quad \textbf{foc} \wedge \neg\, \textbf{rev}}{\Delta\langle \overrightarrow{\boxed{Q/P}}\colon x, \Gamma\rangle \Rightarrow D\colon \omega\{(x\ \psi)/z\} \quad \textbf{foc} \wedge \neg\, \textbf{rev}}\ /L$$

$$\frac{\Gamma \Rightarrow Q_1\colon \psi \quad \neg\, \textbf{foc} \wedge ?Q_1\ \textbf{rev} \qquad \Delta\langle \overrightarrow{\boxed{Q_2}}\colon z\rangle \Rightarrow D\colon \omega \quad \textbf{foc} \wedge \neg\, \textbf{rev}}{\Delta\langle \overrightarrow{\boxed{Q_2/Q_1}}\colon x, \Gamma\rangle \Rightarrow D\colon \omega\{(x\ \psi)/z\} \quad \textbf{foc} \wedge \neg\, \textbf{rev}}\ /L$$

$$\frac{\Gamma \Rightarrow \boxed{P_1}\colon \psi \quad \textbf{foc} \wedge \neg\, \textbf{rev} \qquad \Delta\langle \overrightarrow{P_2}\colon z\rangle \Rightarrow D\colon \omega \quad \neg\, \textbf{foc} \wedge ?P_2\ \textbf{rev}}{\Delta\langle \overrightarrow{\boxed{P_2/P_1}}\colon x, \Gamma\rangle \Rightarrow D\colon \omega\{(x\ \psi)/z\} \quad \textbf{foc} \wedge \neg\, \textbf{rev}}\ /L$$

$$\frac{\Gamma \Rightarrow Q\colon \psi \quad \neg\, \textbf{foc} \wedge ?Q\ \textbf{rev} \qquad \Delta\langle \overrightarrow{P}\colon z\rangle \Rightarrow D\colon \omega \quad \neg\, \textbf{foc} \wedge ?P\ \textbf{rev}}{\Delta\langle \overrightarrow{\boxed{P/Q}}\colon x, \Gamma\rangle \Rightarrow D\colon \omega\{(x\ \psi)/z\} \quad \textbf{foc} \wedge \neg\, \textbf{rev}}\ /L$$

Figure 9: Left irreversible continuous multiplicative rules

$$\frac{\Gamma \Rightarrow \boxed{P}\colon \phi \quad \mathbf{foc} \wedge \neg\, \mathbf{rev} \qquad \Delta\langle \overrightarrow{\boxed{Q}}\colon z\rangle \Rightarrow D\colon \omega \quad \mathbf{foc} \wedge \neg\, \mathbf{rev}}{\Delta\langle \Gamma\,|_k\, \overrightarrow{\boxed{P{\downarrow_k}Q}}\colon y\rangle \Rightarrow D\colon \omega\{(y\ \phi)/z\} \quad \mathbf{foc} \wedge \neg\, \mathbf{rev}}\ {\downarrow_k}L$$

$$\frac{\Gamma \Rightarrow \boxed{P_1}\colon \phi \quad \mathbf{foc} \wedge \neg\, \mathbf{rev} \qquad \Delta\langle \overrightarrow{P_2}\colon z\rangle \Rightarrow D\colon \omega \quad \neg\, \mathbf{foc} \wedge ?P_2\ \mathbf{rev}}{\Delta\langle \Gamma\,|_k\, \overrightarrow{\boxed{P_1{\downarrow_k}P_2}}\colon y\rangle \Rightarrow D\colon \omega\{(y\ \phi)/z\} \quad \mathbf{foc} \wedge \neg\, \mathbf{rev}}\ {\downarrow_k}L$$

$$\frac{\Gamma \Rightarrow Q_1\colon \phi \quad \neg\, \mathbf{foc} \wedge ?Q_1\ \mathbf{rev} \qquad \Delta\langle \overrightarrow{\boxed{Q_2}}\colon z\rangle \Rightarrow D\colon \omega \quad \mathbf{foc} \wedge \neg\, \mathbf{rev}}{\Delta\langle \Gamma\,|_k\, \overrightarrow{\boxed{Q_1{\downarrow_k}Q_2}}\colon y\rangle \Rightarrow D\colon \omega\{(y\ \phi)/z\} \quad \mathbf{foc} \wedge \neg\, \mathbf{rev}}\ {\downarrow_k}L$$

$$\frac{\Gamma \Rightarrow Q\colon \phi \quad \neg\, \mathbf{foc} \wedge ?Q\ \mathbf{rev} \qquad \Delta\langle \overrightarrow{P}\colon z\rangle \Rightarrow D\colon \omega \quad \neg\, \mathbf{foc} \wedge ?P\ \mathbf{rev}}{\Delta\langle \Gamma\,|_k\, \overrightarrow{\boxed{Q{\downarrow_k}P}}\colon y\rangle \Rightarrow D\colon \omega\{(y\ \phi)/z\} \quad \mathbf{foc} \wedge \neg\, \mathbf{rev}}\ {\downarrow_k}L$$

$$\frac{\Gamma \Rightarrow \boxed{P}\colon \psi \quad \mathbf{foc} \wedge \neg\, \mathbf{rev} \qquad \Delta\langle \overrightarrow{\boxed{Q}}\colon z\rangle \Rightarrow D\colon \omega \quad \mathbf{foc} \wedge \neg\, \mathbf{rev}}{\Delta\langle \overrightarrow{\boxed{Q{\uparrow_k}P}}\colon x\,|_k\, \Gamma\rangle \Rightarrow D\colon \omega\{(x\ \psi)/z\} \quad \mathbf{foc} \wedge \neg\, \mathbf{rev}}\ {\uparrow_k}L$$

$$\frac{\Gamma \Rightarrow Q_1\colon \psi \quad \neg\, \mathbf{foc} \wedge ?Q_1\ \mathbf{rev} \qquad \Delta\langle \overrightarrow{\boxed{Q_2}}\colon z\rangle \Rightarrow D\colon \omega \quad \mathbf{foc} \wedge \neg\, \mathbf{rev}}{\Delta\langle \overrightarrow{\boxed{Q_2{\uparrow_k}Q_1}}\colon x\,|_k\, \Gamma\rangle \Rightarrow D\colon \omega\{(x\ \psi)/z\} \quad \mathbf{foc} \wedge \neg\, \mathbf{rev}}\ {\uparrow_k}L$$

$$\frac{\Gamma \Rightarrow \boxed{P_1}\colon \psi \quad \mathbf{foc} \wedge \neg\, \mathbf{rev} \qquad \Delta\langle \overrightarrow{P_2}\colon z\rangle \Rightarrow D\colon \omega \quad \neg\, \mathbf{foc} \wedge ?P_2\ \mathbf{rev}}{\Delta\langle \overrightarrow{\boxed{P_2{\uparrow_k}P_1}}\colon x\,|_k\, \Gamma\rangle \Rightarrow D\colon \omega\{(x\ \psi)/z\} \quad \mathbf{foc} \wedge \neg\, \mathbf{rev}}\ {\uparrow_k}L$$

$$\frac{\Gamma \Rightarrow Q\colon \psi \quad \neg\, \mathbf{foc} \wedge ?Q\ \mathbf{rev} \qquad \Delta\langle \overrightarrow{P}\colon z\rangle \Rightarrow D\colon \omega \quad \neg\, \mathbf{foc} \wedge ?P\ \mathbf{rev}}{\Delta\langle \overrightarrow{\boxed{P{\uparrow_k}Q}}\colon x\,|_k\, \Gamma\rangle \Rightarrow D\colon \omega\{(x\ \psi)/z\} \quad \mathbf{foc} \wedge \neg\, \mathbf{rev}}\ {\uparrow_k}L$$

Figure 10: Left irreversible discontinuous multiplicative rules

$$\frac{\Delta \Rightarrow \boxed{P_1}: \phi \quad \textbf{foc} \wedge \neg\, \textbf{rev} \qquad \Gamma \Rightarrow \boxed{P_2}: \psi \quad \textbf{foc} \wedge \neg\, \textbf{rev}}{\Delta, \Gamma \Rightarrow \boxed{P_1 \bullet P_2}: (\phi, \psi) \quad \textbf{foc} \wedge \neg\, \textbf{rev}} \bullet R$$

$$\frac{\Delta \Rightarrow \boxed{P}: \phi \quad \textbf{foc} \wedge \neg\, \textbf{rev} \qquad \Gamma \Rightarrow Q: \psi \quad \neg\, \textbf{foc} \wedge ?Q\ \textbf{rev}}{\Delta, \Gamma \Rightarrow \boxed{P \bullet Q}: (\phi, \psi) \quad \textbf{foc} \wedge \neg\, \textbf{rev}} \bullet R$$

$$\frac{\Delta \Rightarrow N: \phi \quad \neg\, \textbf{foc} \wedge ?N\ \textbf{rev} \qquad \Gamma \Rightarrow \boxed{P}: \psi \quad \textbf{foc} \wedge \neg\, \textbf{rev}}{\Delta, \Gamma \Rightarrow \boxed{N \bullet P}: (\phi, \psi) \quad \textbf{foc} \wedge \neg\, \textbf{rev}} \bullet R$$

$$\frac{\Delta \Rightarrow N_1: \phi \neg\, \textbf{foc} \wedge ?N_1\ \textbf{rev} \qquad \Gamma \Rightarrow N_2: \psi \quad \textbf{foc} \wedge ?N_2\ \textbf{rev}}{\Delta, \Gamma \Rightarrow \boxed{N_1 \bullet N_2}: (\phi, \psi) \quad \textbf{foc} \wedge \neg\, \textbf{rev}} \bullet R$$

$$\frac{}{\Lambda \Rightarrow \boxed{I}: 0\ \textbf{foc} \wedge \neg\, \textbf{rev}} IR$$

Figure 11: Right irreversible continuous multiplicative rules

$$\frac{\Delta \Rightarrow \boxed{P_1}: \phi \quad \textbf{foc} \wedge \neg\, \textbf{rev} \qquad \Gamma \Rightarrow \boxed{P_2}: \psi \quad \textbf{foc} \wedge \neg\, \textbf{rev}}{\Delta\,|_k\, \Gamma \Rightarrow \boxed{P_1 \odot_k P_2}: (\phi, \psi) \quad \textbf{foc} \wedge \neg\, \textbf{rev}} \odot_k R$$

$$\frac{\Delta \Rightarrow \boxed{P}: \phi \quad \textbf{foc} \wedge \neg\, \textbf{rev} \qquad \Gamma \Rightarrow Q: \psi \quad \neg\, \textbf{foc} \wedge ?N\ \textbf{rev}}{\Delta\,|_k\, \Gamma \Rightarrow \boxed{P \odot_k Q}: (\phi, \psi) \quad \textbf{foc} \wedge \neg\, \textbf{rev}} \odot_k R$$

$$\frac{\Delta \Rightarrow Q: \phi \quad \neg\, \textbf{foc} \wedge ?N\ \textbf{rev} \qquad \Gamma \Rightarrow \boxed{P}: \psi \quad \textbf{foc} \wedge \neg\, \textbf{rev}}{\Delta\,|_k\, \Gamma \Rightarrow \boxed{Q \odot_k P}: (\phi, \psi) \quad \textbf{foc} \wedge \neg\, \textbf{rev}} \odot_k R$$

$$\frac{\Delta \Rightarrow Q_1: \phi \quad \neg\, \textbf{foc} \wedge ?Q_1\ \textbf{rev} \qquad \Gamma \Rightarrow Q_2: \psi \quad \neg\, \textbf{foc} \wedge ?Q_2\ \textbf{rev}}{\Delta\,|_k\, \Gamma \Rightarrow \boxed{Q_1 \odot_k Q_2}: (\phi, \psi) \quad \textbf{foc} \wedge \neg\, \textbf{rev}} \odot_k R$$

$$\frac{}{1 \Rightarrow \boxed{J}: 0\ \textbf{foc} \wedge \neg\, \textbf{rev}} JR$$

Figure 12: Right irreversible discontinuous multiplicative rules

4 Count-invariance

We define infinitary count invariance for categorial logic extending count invariance for multiplicatives (van Benthem 1991[50]) and additives and bracket modalities (Valentín et al. 2013[49]) to include exponentials. This affords effective pruning of proof search in categorial parsing/theorem-proving.

Count invariance for multiplicatives in (sub)linear logic is introduced in van Benthem (1991[50]). This involves simply checking the number of positive and negative occurrences of each atom in a sequent. Thus where $\#(\Sigma)$ is a count of the sequent Σ we have:

(5) $\vdash \Sigma \Longrightarrow \#(\Sigma) = 0$

I.e. the numbers of positive and negative occurrences of each atom must exactly balance. This provides a necessary, but of course not sufficient, criterion for theoremhood, and it can be checked rapidly. It can be used as a filter in proof search: if backward chaining proof search generates a goal which does not satisfy the count invariant, the goal can be safely made to fail immediately. This notion of count for multiplicatives was included in the categorial parser/theorem-prover CatLog1 (Morrill 2012[32]).

In Valentín et al. (2013[49]) the idea is extended to additives (and bracket modalities). Instead of a single count for each atom of a sequent Σ we have a minimum count $\#_{\mathrm{min}}(\Sigma)$ and a maximum count $\#_{\mathrm{max}}(\Sigma)$ and for a sequent to be a theorem it must satisfy two inequations:

(6) $\vdash \Sigma \Longrightarrow \#_{\mathrm{min}}(\Sigma) \leq 0 \leq \#_{\mathrm{max}}(\Sigma)$

I.e. the count functions $\#_{\mathrm{min}}$ and $\#_{\mathrm{max}}$ define an interval which must include the point of balance 0; for the multiplicatives, $\#_{\mathrm{min}} = \#_{\mathrm{max}} = \#$ and (6) reduces to the special case (5). This count-invariance is included in the categorial parser/theorem-prover CatLog2. Here we describe the count-invariance of CatLog3 which includes further infinitary count functions for exponentials (Kuznetsov et al. 2017[16]).

We consider terms built over constants 0, 1, $\bot$ ($-\infty$, minus infinity), and $\top$ ($+\infty$, plus infinity) by operations plus (+), minus (−), minimum (min) and maximum (max), and infinitary step functions X and Y thus; $i, j \in \mathcal{Z}$ and $n \in \mathcal{Z}^+$:

$+$	j	$\bot$	$\top$
i	$i+j$	$\bot$	$\top$
$\bot$	$\bot$	$\bot$	$*$
$\top$	$\top$	$*$	$\top$

$-$	j	$\bot$	$\top$
i	$i-j$	$\top$	$\bot$
$\bot$	$\bot$	$*$	$\bot$
$\top$	$\top$	$\top$	$*$

min	j	$\bot$	$\top$
i	$\frac{\lvert i+j\rvert-\lvert i-j\rvert}{2}$	$\bot$	i
$\bot$	$\bot$	$\bot$	$\bot$
$\top$	j	$\bot$	$\top$

max	j	$\bot$	$\top$
i	$\frac{\lvert i+j\rvert+\lvert i-j\rvert}{2}$	i	$\top$
$\bot$	j	$\bot$	$\top$
$\top$	$\top$	$\top$	$\top$

$$X(i) = \begin{cases} \top & \text{if } i > 0 \\ i & \text{if } i \leq 0 \end{cases} \qquad Y(i) = \begin{cases} i & \text{if } i \geq 0 \\ \bot & \text{if } i < 0 \end{cases}$$

Where $\mathcal{P}$ is the set of primitive types, $P \in \mathcal{P}$, $Q \in \mathcal{P} \cup \{[]\}$, $p \in \{^\bullet, ^\circ\}$, and $\overline{^\bullet} = {}^\circ$ and $\overline{^\circ} = {}^\bullet$ we define the count functions for **DA1S4b!$_\mathbf{b}$?** as shown in Figure 13.

$$
\begin{array}{rcll}
\#^{p}_{m,Q}(P) & = & \begin{array}{ll} 1 & \text{if } Q = P \\ 0 & \text{if } Q \neq P \end{array} & \\
\#^{p}_{m,Q}(A\backslash C) & = & \#^{p}_{m,Q}(A{\downarrow_k}C) & = \#^{p}_{m,Q}(C) - \#^{\overline{p}}_{\overline{m},Q}(A) \\
\#^{p}_{m,Q}(C/B) & = & \#^{p}_{m,Q}(C{\uparrow_k}B) & = \#^{p}_{m,Q}(C) - \#^{\overline{p}}_{\overline{m},Q}(B) \\
\#^{p}_{m,Q}(A{\bullet}B) & = & \#^{p}_{m,Q}(A{\odot_k}B) & = \#^{p}_{m,Q}(A) + \#^{p}_{m,Q}(B) \\
\#^{p}_{m,Q}(I) & = & \#^{p}_{m,Q}(J) & = 0 \\
\#^{\circ}_{m,Q}(A\&B) & = & \overline{m}(\#^{\circ}_{m,Q}(A), \#^{\circ}_{m,Q}(B)) & \\
\#^{\bullet}_{m,Q}(A\&B) & = & m(\#^{\bullet}_{m,Q}(A), \#^{\bullet}_{m,Q}(B)) & \\
\#^{\circ}_{m,Q}(A\oplus B) & = & m(\#^{\circ}_{m,Q}(A), \#^{\circ}_{m,Q}(B)) & \\
\#^{\bullet}_{m,Q}(A\oplus B) & = & \overline{m}(\#^{\bullet}_{m,Q}(A), \#^{\bullet}_{m,Q}(B)) & \\
\#^{p}_{m,Q}(\bigwedge xA) & = & \#^{p}_{m,Q}(\bigvee xA) & = \#^{p}_{m,Q}(A) \\
\#^{p}_{m,Q}(\Box A) & = & \#^{p}_{m,Q}(\Diamond A) & = \#^{p}_{m,Q}(A) \\
\#^{p}_{m,P}([\,]^{-1}A) & = & \#^{p}_{m,P}(A) & \\
\#^{p}_{m,[]}([\,]^{-1}A) & = & \#^{p}_{m,[]}(A) - 1 & \\
\#^{p}_{m,P}(\langle\rangle A) & = & \#^{p}_{m,P}(A) & \\
\#^{p}_{m,[]}(\langle\rangle A) & = & \#^{p}_{m,[]}(A) + 1 & \\
\#^{\bullet}_{\min,Q}(!A) & = & Y(\#^{\bullet}_{\min,Q}(A)) & \\
\#^{\bullet}_{\max,P}(!A) & = & X(\#^{\bullet}_{\max,P}(A)) & \\
\#^{\bullet}_{\max,[]}(!A) & = & \top & \\
\#^{\circ}_{m,Q}(!A) & = & \#^{\circ}_{m,Q}(A) & \\
\#^{\circ}_{\max,Q}(?A) & = & X(\#^{\circ}_{\max,Q}(A)) & \\
\#^{\circ}_{\min,Q}(?A) & = & Y(\#^{\circ}_{\min,Q}(A)) & \\
\#^{\bullet}_{m,Q}(?A) & = & \#^{\bullet}_{m,Q}(A) &
\end{array}
$$

Figure 13: Count function

For zones, stoups, tree terms and configurations, counts are as follows:

$$
\begin{array}{rcl}
\#_{m,Q}(\mathcal{S};\mathcal{O}) & = & \#_{m,Q}(\mathcal{S}) + \#_{m,Q}(\mathcal{O}) \\
\#_{m,Q}(\emptyset) & = & 0 \\
\#_{m,Q}(\mathcal{F},\mathcal{S}) & = & \#_{m,Q}(\mathcal{F}) + \#_{m,Q}(\mathcal{S}) \\
\#_{m,Q}(\Lambda) & = & 0 \\
\#_{m,Q}(\mathcal{T},\mathcal{O}) & = & \#_{m,Q}(\mathcal{T}) + \#_{m,Q}(\mathcal{O}) \\
\#_{m,Q}(1) & = & 0 \\
\#_{m,Q}(\mathcal{F}) & = & \overset{\bullet}{\#}_{m,Q}(\mathcal{F}) \\
\#_{m,Q}(\mathcal{F}\{\mathcal{O}_1 : \ldots : \mathcal{O}_i\}) & = & \overset{\bullet}{\#}_{m,Q}(\mathcal{F}) + \Sigma_{n=1}^{i}\#_{m,Q}(\mathcal{O}_n) \\
\#_{m,[]}([\mathcal{Z}]) & = & \#_{m,[]}(\mathcal{Z}) + 1 \\
\#_{m,P}([\mathcal{Z}]) & = & \#_{m,P}(\mathcal{Z})
\end{array}
$$

The count-invariance theorem is:

(7) **Theorem**.

$\vdash \Xi \Rightarrow A \Longrightarrow \forall Q \in \mathcal{P} \cup \{[]\}, \#_{\mathrm{min},Q}(\Xi \Rightarrow A) \leq 0 \leq \#_{\mathrm{max},Q}(\Xi \Rightarrow A)$

where, $\#_{m,Q}(\Xi \Rightarrow A) = \overset{\circ}{\#}_{m,Q}(A) - \#_{\overline{m},Q}(\Xi)$.

Relativisation including medial and parasitic extraction is obtained by assigning a relative pronoun a type $(CN\backslash CN)/(!N\backslash S)$ whereby the body of a relative clause is analysed as $!N\backslash S$. By way of example of count-invariance, we show how it discards $N, N\backslash S \Rightarrow\, !N\backslash S$ corresponding to the ungrammaticality of a relative clause without a gap: **paper that John walks*. We have the max N-count: $\#_{\mathrm{max},N}(N, N\backslash S \Rightarrow\, !N\backslash S) = \overset{\circ}{\#}_{\mathrm{max},N}(!N\backslash S) - \overset{\bullet}{\#}_{\mathrm{min},N}(N, N\backslash S) = \overset{\circ}{\#}_{\mathrm{max},N}(S) - \overset{\bullet}{\#}_{\mathrm{min},N}(!N) - \overset{\bullet}{\#}_{\mathrm{min},N}(N) - \overset{\bullet}{\#}_{\mathrm{min},N}(N\backslash S) = 0 - Y(\overset{\bullet}{\#}_{\mathrm{min},N}(N)) - 1 - \overset{\bullet}{\#}_{\mathrm{min},N}(S) + \overset{\circ}{\#}_{\mathrm{min},N}(N) = -Y(1) - 1 - 0 + 1 = -1 - 1 + 1 = -1 \not\geq 0$ which means that the count-invariance is not satisfied.

Iterated sentential coordination is obtained by assigning a coordinator the type $(?S\backslash S)/S$. By way of a second example we show how count-invariance discards $N, N, N\backslash S \Rightarrow\, ?S$ corresponding to the ungrammaticality of unequilibrated coordination: **John Mary walks and Suzy talks*. Max N-count is: $\#_{\mathrm{max},N}(N, N, N\backslash S \Rightarrow\, ?S) = \overset{\circ}{\#}_{\mathrm{max},N}(?S) - \#^{\bullet}{}_{\mathrm{min},N}(N, N, N\backslash S) = X(\overset{\circ}{\#}_{\mathrm{max},N}(S)) - \overset{\bullet}{\#}_{\mathrm{min},N}(N) - \overset{\bullet}{\#}_{\mathrm{min},N}(N) - \overset{\bullet}{\#}_{\mathrm{min},N}(N\backslash S) = X(0) - 1 - 1 - \overset{\bullet}{\#}_{\mathrm{min},N}(S) + \overset{\bullet}{\#}_{\mathrm{max},N}(N) = 0 - 2 - 0 + 1 = -1 \not\geq 0$ which means that the count-invariance is not satisfied.

5 Illustration

Morrill and Valentín (2016[42]) defines as the *Montague Test* the task of providing a computational grammar of the PTQ fragment of Montague (1973[21]), and shows how CatLog fulfils this task. We are not aware of any other system which has passed the Montague Test. The example sentences of Chapter 7 of Dowty et al. (1981[7]) are given in Figure 14; the lexicon is given in Figure 15.

The CatLog3 LaTeX output for the (ambiguous) last sentence is as follows:

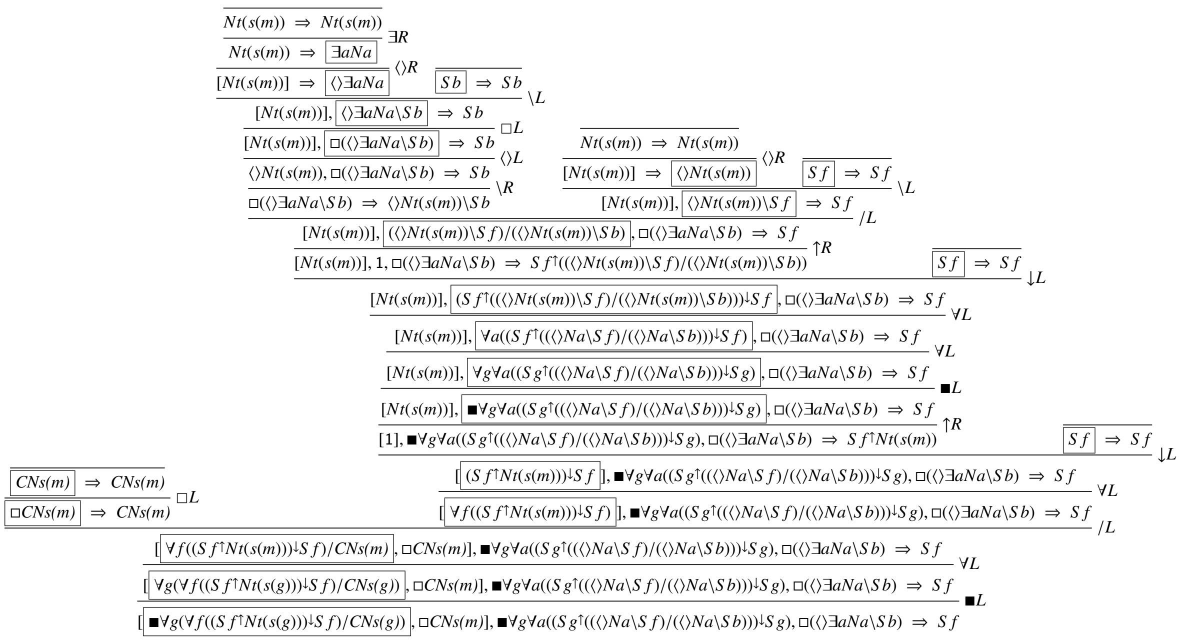
(dwp((7-116, 118))) [**every**+**man**]+**doesnt**+**walk** : Sf

$[\blacksquare\forall g(\forall f((Sf{\uparrow}Nt(s(g))){\downarrow}Sf)/CNs(g)) : \lambda A\lambda B\forall C[(A\ C) \rightarrow (B\ C)], \square CNs(m) : man],$
$\blacksquare\forall g\forall a((Sg{\uparrow}((\langle\rangle Na\backslash Sf)/(\langle\rangle Na\backslash Sb))){\downarrow}Sg) : \lambda D\neg(D\ \lambda E\lambda F(E\ F)), \square(\langle\rangle(\exists aNa{-}\exists gNt(s(g)))\backslash Sf) :$
$\hat{}\lambda G(Pres\ (\check{}walk\ G)) \ \Rightarrow\ Sf$

```
str(dwp('(7-7)'), [b([john]), walks], s(f)).
str(dwp('(7-16)'), [b([every, man]), talks], s(f)).
str(dwp('(7-19)'), [b([the, fish]), walks], s(f)).
str(dwp('(7-32)'), [b([every, man]), b([b([walks, or, talks])])], s(f)).
str(dwp('(7-34)'), [b([b([b([every, man]), walks, or, b([every, man]), talks])])], s(f)).
str(dwp('(7-39)'), [b([b([b([a, woman]), walks, and, b([she]), talks])])], s(f)).
str(dwp('(7-43, 45)'), [b([john]), believes, that, b([a, fish]), walks], s(f)).
str(dwp('(7-48, 49, 52)'), [b([every, man]), believes, that, b([a, fish]), walks], s(f)).
str(dwp('(7-57)'), [b([every, fish, such, that, b([it]), walks]), talks], s(f)).
str(dwp('(7-60, 62)'), [b([john]), seeks, a, unicorn], s(f)).
str(dwp('(7-73)'), [b([john]), is, bill], s(f)).
str(dwp('(7-76)'), [b([john]), is, a, man], s(f)).
str(dwp('(7-83)'), [necessarily, b([john]), walks], s(f)).
str(dwp('(7-86)'), [b([john]), walks, slowly], s(f)).
str(dwp('(7-91)'), [b([john]), tries, to, walk], s(f)).
str(dwp('(7-94)'), [b([john]), tries, to, b([b([catch, a, fish, and, eat, it])])], s(f)).
str(dwp('(7-98)'), [b([john]), finds, a, unicorn], s(f)).
str(dwp('(7-105)'), [b([every, man, such, that, b([he]), loves, a, woman]), loses, her], s(f)).
str(dwp('(7-110)'), [b([john]), walks, in, a, park], s(f)).
str(dwp('(7-116, 118)'), [b([every, man]), doesnt, walk], s(f)).
```

Figure 14: Montague sentences

$$[\blacksquare\forall g(\forall f((Sf{\uparrow}Nt(s(g))){\downarrow}Sf)/CNs(g)) : \lambda A\lambda B\forall C[(A\ C) \rightarrow (B\ C)], \Box CNs(m) : man],$$
$$\blacksquare\forall g\forall a((Sg{\uparrow}((\langle\rangle Na\backslash Sf)/(\langle\rangle Na\backslash Sb))){\downarrow}Sg) : \lambda D\neg(D\ \lambda E\lambda F(E\ F)), \Box(\langle\rangle\exists aNa\backslash Sb) :$$
$$\hat{}\lambda G(\check{}walk\ G) \ \Rightarrow\ Sf$$



$$\forall C[(\check{}man\ C) \rightarrow \neg(\check{}walk\ C)]$$

a : $\blacksquare\forall g(\forall f((S f\uparrow\blacksquare Nt(s(g)))\downarrow S f)/CNs(g)) : \lambda A\lambda B\exists C[(A\ C) \wedge (B\ C)]$

and : $\blacksquare\forall f((\blacksquare ?S f\backslash[\,]^{-1}[\,]^{-1}S f)/\blacksquare S f) : (\Phi^{n+}\ 0\ and)$

and : $\blacksquare\forall a\forall f((\blacksquare ?(\langle\rangle Na\backslash S f)\backslash[\,]^{-1}[\,]^{-1}(\langle\rangle Na\backslash S f))/\blacksquare(\langle\rangle Na\backslash S f)) : (\Phi^{n+}\ (s\ 0)\ and)$

believes : $\square((\langle\rangle\exists gNt(s(g))\backslash S f)/(CPthat\sqcup\square S f)) : \hat{}\lambda A\lambda B(Pres\ ((\check{}believe\ A)\ B))$

bill : $\blacksquare Nt(s(m)) : b$

catch : $\square((\langle\rangle\exists aNa\backslash S b)/\exists aNa) : \hat{}\lambda A\lambda B((\check{}catch\ A)\ B)$

doesnt : $\blacksquare\forall g\forall a((S g\uparrow((\langle\rangle Na\backslash S f)/(\langle\rangle Na\backslash S b)))\downarrow S g) : \lambda A\neg(A\ \lambda B\lambda C(B\ C))$

eat : $\square((\langle\rangle\exists aNa\backslash S b)/\exists aNa) : \hat{}\lambda A\lambda B((\check{}eat\ A)\ B)$

every : $\blacksquare\forall g(\forall f((S f\uparrow Nt(s(g)))\downarrow S f)/CNs(g)) : \lambda A\lambda B\forall C[(A\ C) \rightarrow (B\ C)]$

finds : $\square((\langle\rangle\exists gNt(s(g))\backslash S f)/\exists aNa) : \hat{}\lambda A\lambda B(Pres\ ((\check{}find\ A)\ B))$

fish : $\square CNs(n) : fish$

he : $\blacksquare[\,]^{-1}\forall g((\blacksquare S g|\blacksquare Nt(s(m)))/(\langle\rangle Nt(s(m))\backslash S g)) : \lambda AA$

her : $\blacksquare\forall g\forall a(((\langle\rangle Na\backslash S g)\uparrow\blacksquare Nt(s(f)))\downarrow(\blacksquare(\langle\rangle Na\backslash S g)|\blacksquare Nt(s(f)))) : \lambda AA$

in : $\square(\forall a\forall f((\langle\rangle Na\backslash S f)\backslash(\langle\rangle Na\backslash S f))/\exists aNa) : \hat{}\lambda A\lambda B\lambda C((\check{}in\ A)\ (B\ C))$

is : $\blacksquare((\langle\rangle\exists gNt(s(g))\backslash S f)/(\exists aNa\oplus(\exists g((CNg/CNg)\sqcup(CNg\backslash CNg))-I))) : \lambda A\lambda B(Pres\ (A \rightarrow C.[B = C]; D.((D\ \lambda E[E = B])\ B)))$

it : $\blacksquare\forall f\forall a(((\langle\rangle Na\backslash S f)\uparrow\blacksquare Nt(s(n)))\downarrow(\blacksquare(\langle\rangle Na\backslash S f)|\blacksquare Nt(s(n)))) : \lambda AA$

it : $\blacksquare[\,]^{-1}\forall f((\blacksquare S f|\blacksquare Nt(s(n)))/(\langle\rangle Nt(s(n))\backslash S f)) : \lambda AA$

john : $\blacksquare Nt(s(m)) : j$

loses : $\square((\langle\rangle\exists gNt(s(g))\backslash S f)/\exists aNa) : \hat{}\lambda A\lambda B(Pres\ ((\check{}lose\ A)\ B))$

loves : $\square((\langle\rangle\exists gNt(s(g))\backslash S f)/\exists aNa) : \hat{}\lambda A\lambda B(Pres\ ((\check{}love\ A)\ B))$

man : $\square CNs(m) : man$

necessarily : $\blacksquare(S A/\square S A) : Nec$

or : $\blacksquare\forall f((\blacksquare ?S f\backslash[\,]^{-1}[\,]^{-1}S f)/\blacksquare S f) : (\Phi^{n+}\ 0\ or)$

or : $\blacksquare\forall a\forall f((\blacksquare ?(\langle\rangle Na\backslash S f)\backslash[\,]^{-1}[\,]^{-1}(\langle\rangle Na\backslash S f))/\blacksquare(\langle\rangle Na\backslash S f)) : (\Phi^{n+}\ (s\ 0)\ or)$

or : $\blacksquare\forall f((\blacksquare ?(S f/(\langle\rangle\exists gNt(s(g))\backslash S f))\backslash[\,]^{-1}[\,]^{-1}(S f/(\langle\rangle\exists gNt(s(g))\backslash S f)))/\blacksquare(S f/(\langle\rangle\exists gNt(s(g))\backslash S f))) : (\Phi^{n+}\ (s\ 0)\ or)$

park : $\square CNs(n) : park$

seeks : $\square((\langle\rangle\exists gNt(s(g))\backslash S f)/\square\forall a\forall f(((Na\backslash S f)/\exists bNb)\backslash(Na\backslash S f))) : \hat{}\lambda A\lambda B((\check{}tries\ \hat{}((\check{}A\ \check{}find)\ B))\ B)$

she : $\blacksquare[\,]^{-1}\forall g((\blacksquare S g|\blacksquare Nt(s(f)))/(\langle\rangle Nt(s(f))\backslash S g)) : \lambda AA$

slowly : $\square\forall a\forall f(\square(\langle\rangle Na\backslash S f)\backslash(\langle\rangle\square Na\backslash S f)) : \hat{}\lambda A\lambda B(\check{}slowly\ \hat{}(\check{}A\ \check{}B))$

such+that : $\blacksquare\forall n((CNn\backslash CNn)/(S f|\blacksquare Nt(n))) : \lambda A\lambda B\lambda C[(B\ C) \wedge (A\ C)]$

talks : $\square(\langle\rangle\exists gNt(s(g))\backslash S f) : \hat{}\lambda A(Pres\ (\check{}talk\ A))$

that : $\blacksquare(CPthat/\square S f) : \lambda AA$

the : $\blacksquare\forall n(Nt(n)/CNn) : \iota$

to : $\blacksquare((PPto/\exists aNa)\sqcap\forall n((\langle\rangle Nn\backslash S i)/(\langle\rangle Nn\backslash S b))) : \lambda AA$

tries : $\square((\langle\rangle\exists gNt(s(g))\backslash S f)/\square(\langle\rangle\exists gNt(s(g))\backslash S i)) : \hat{}\lambda A\lambda B((\check{}tries\ \hat{}(\check{}A\ B))\ B)$

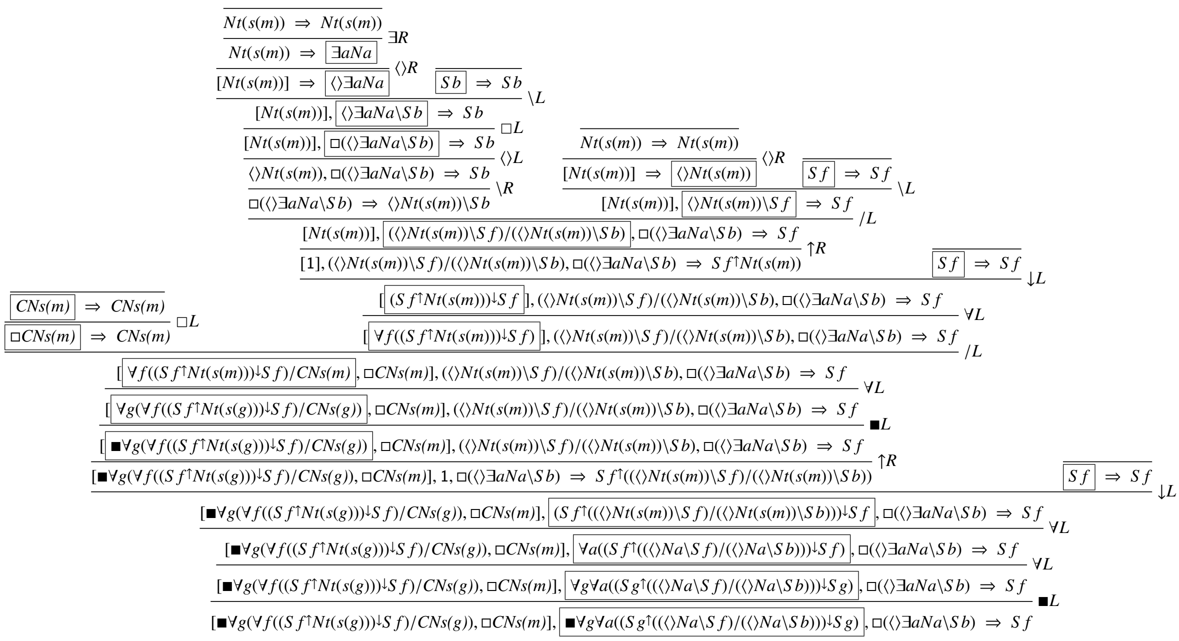
unicorn : $\square CNs(n) : unicorn$

walk : $\square(\langle\rangle\exists aNa\backslash S b) : \hat{}\lambda A(\check{}walk\ A)$

walks : $\square(\langle\rangle\exists gNt(s(g))\backslash S f) : \hat{}\lambda A(Pres\ (\check{}walk\ A))$

woman : $\square CNs(f) : woman$

Figure 15: Montague lexicon



$\neg\forall G[(\check{}man\ G) \to (\check{}walk\ G)]$

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