# ENRICHING CATEGORIAL GRAMMAR

Chapter three has introduced categorial grammar as formulated on the foundations of type theory and substructural logic presented in chapter two. As logic formalisms, the Lambek calculi attract mathematical interest in their own right. In terms of application to natural language grammar however, their mathematical elegance can seem to imply unsuitability: surely not many facets of natural language can be construed in such simple structures? Indeed we do not get far at all before it is apparent that extensions to the basic systems are required.

In fact, the tendency in linguistics was to regard the Lambek calculi as being already one of many possible augmentations of **AB** categorial grammar, two other traditions of which are categorial grammar with combinators (see e.g. Steedman 1987 and Szabolcsi 1987) and categorial grammar with unification (see e.g. Uszkoreit 1986; Pollard and Sag 1987, 1993; Zeevat, Klein and Calder 1987 and Bouma 1993). But on the present view, classical categorial grammar is no more than a suggestive fractional notation which discovered its algebraic foundations in residuation and cancellation under product with the Lambek calculi. When we generalise categorial grammar we wish to respect these foundations. This is what is done in what we may call the logical tradition of categorial grammar, where basic Lambek calculus with division and product operators is enriched with additional operators to increase its expressivity.

## 1. MULTIMODAL SYSTEMS

One way of obtaining a richer system than is given by the connectives of either the non-associative calculus **NL** or the associative calculus **L** is by combining two (or more) families of these connectives: of different kinds — non-associative and associative (for attempts see Oehrle and Zhang 1989, Morrill 1990c), or of the same kind — non-associative and non-associative or associative and associative (see Moortgat and Morrill 1991). We refer to such logics as hybrid, or multimodal, including multiple modes of prosodic adjunction.[1] In the case of double non-associativity for example, the categorial formulas are generated by two families of operators, say $\backslash_l, /_l, \cdot_l$ and $\backslash_r, /_r, \cdot_r$ and are (prosodically) interpreted in a 'bimodal groupoid' algebra $(L, +_l, +_r)$ closed under *two* binary operations. Each family is interpreted by residuation with respect to its associated mode of adjunction.

(1)
$$\begin{array}{lcl} D(A\cdot_l B) & = & \{s_1+_l s_2 | s_1 \in D(A) \wedge s_2 \in D(B)\} \\ D(B/_l A) & = & \{s | \forall s' \in D(A), s+_l s' \in D(B)\} \\ D(A\backslash_l B) & = & \{s | \forall s' \in D(A), s'+_l s \in D(B)\} \end{array}$$

(2)
$$\begin{array}{lcl} D(A\cdot_r B) & = & \{s_1+_r s_2 | s_1 \in D(A) \wedge s_2 \in D(B)\} \\ D(B/_r A) & = & \{s | \forall s' \in D(A), s+_r s' \in D(B)\} \\ D(A\backslash_r B) & = & \{s | \forall s' \in D(A), s'+_r s \in D(B)\} \end{array}$$

Thus each family respects the residuation laws:

(3)
$$\begin{array}{lllll} A \Rightarrow C/_l B & =\!\mid\models & A\cdot_l B \Rightarrow C & =\!\mid\models & B \Rightarrow A\backslash_l C \\ A \Rightarrow C/_r B & =\!\mid\models & A\cdot_r B \Rightarrow C & =\!\mid\models & B \Rightarrow A\backslash_r C \end{array}$$

The Gentzen-style sequent calculus is like that for **NL** in that configurations have a binary bracketing (i.e. binary tree) structure in relation to which inference is regulated. But the brackets (mother nodes) are each of one of two 'colours' — $l$ or $r$ — and operators are controlled according to their kind.

(4) a.
$$\frac{}{A \Rightarrow A}\text{id} \qquad \frac{\Gamma \Rightarrow A \qquad \Delta[A] \Rightarrow B}{\Delta[\Gamma] \Rightarrow B}\text{Cut}$$

b.
$$\frac{\Gamma \Rightarrow A \qquad \Delta[B] \Rightarrow C}{\Delta[(_l\Gamma, A\backslash_l B)] \Rightarrow C}\backslash_l\text{L} \qquad \frac{(_l A, \Gamma) \Rightarrow B}{\Gamma \Rightarrow A\backslash_l B}\backslash_l\text{R}$$

c.
$$\frac{\Gamma \Rightarrow A \qquad \Delta[B] \Rightarrow C}{\Delta[(_l B/_l A, \Gamma)] \Rightarrow C}/_l\text{L} \qquad \frac{(_l\Gamma, A) \Rightarrow B}{\Gamma \Rightarrow B/_l A}/_l\text{R}$$

[1] In the sense that algebraic models can be seen as defining particular kinds of ternary accessibility relations, with categorial connectives being diadic modal operators (Dirk Roorda, p.c.), the term modal in multimodal can also be seen as obtaining as in modal logic.

d. $$\frac{\Gamma[({}_l A, B)] \Rightarrow C}{\Gamma[A \cdot_l B] \Rightarrow C}\cdot_l \mathrm{L} \qquad \frac{\Gamma \Rightarrow A \quad \Delta \Rightarrow B}{({}_l \Gamma, \Delta) \Rightarrow A \cdot_l B}\cdot_l \mathrm{R}$$

e. $$\frac{\Gamma \Rightarrow A \quad \Delta[B] \Rightarrow C}{\Delta[({}_r \Gamma, A\backslash_r B)] \Rightarrow C}\backslash_r \mathrm{L} \qquad \frac{({}_r A, \Gamma) \Rightarrow B}{\Gamma \Rightarrow A\backslash_r B}\backslash_r \mathrm{R}$$

f. $$\frac{\Gamma \Rightarrow A \quad \Delta[B] \Rightarrow C}{\Delta[({}_r B/_r A, \Gamma)] \Rightarrow C}/_r \mathrm{L} \qquad \frac{({}_r \Gamma, A) \Rightarrow B}{\Gamma \Rightarrow B/_r A}/_r \mathrm{R}$$

g. $$\frac{\Gamma[({}_r A, B)] \Rightarrow C}{\Gamma[A \cdot_r B] \Rightarrow C}\cdot_r \mathrm{L} \qquad \frac{\Gamma \Rightarrow A \quad \Delta \Rightarrow B}{({}_r \Gamma, \Delta) \Rightarrow A \cdot_r B}\cdot_r \mathrm{R}$$

Such a system characterises, for instance, headed binary constituent structure, each mother node being understood as marked for having either its left ($l$) subconstituent or right ($r$) subconstituent as head. One instance of such structures is provided by metrical trees (Liberman and Prince 1977) and Moortgat and Morrill (1991) exemplify application to prosodic stress assignment in modelling rhythmic patterns of speech. Alternatively the devices could be applied to projection of head-dependent structure, without subjectivity to the traditional correlation of heads with categorial functors, which is questionable in relation to determiners, adjectives, modifiers, and so on.

For a system combining two modes of adjunction each of which is associative, i.e. one based on an algebra $(L, +_l, +_r)$ such that $s_1 +_l (s_2 +_l s_3) = (s_1 +_l s_2) +_l s_3$ and $s_1 +_r (s_2 +_r s_3) = (s_1 +_r s_2) +_r s_3$ (so $(L, +_l)$ and $(L, +_r)$ are semigroups), the logical rules are just the same, but structural rules of association are added:

(5) $$\frac{\Gamma[({}_l \Delta_1, ({}_l \Delta_2, \Delta_3))] \Rightarrow A}{\Gamma[({}_l ({}_l \Delta_1, \Delta_2), \Delta_3)] \Rightarrow A}\mathrm{A}_l \qquad \frac{\Gamma[({}_r \Delta_1, ({}_r \Delta_2, \Delta_3))] \Rightarrow A}{\Gamma[({}_r ({}_r \Delta_1, \Delta_2), \Delta_3)] \Rightarrow A}\mathrm{A}_r$$

Gentzen-style sequent calculus for a doubly associative system can also be given representing equivalence classes of configurations by $n+2$-ary bracketing (Moortgat and Morrill 1991).

In the case of mixing **L** and **NL**, the categorial language generated by associative operators \ ("under"), / ("over"), $\cdot$ and non-associative operators which we shall henceforth notate $>$ ("to"), $<$ ("from"), $\diamond$ is to be interpreted in an algebra $(L, +, (.,.))$ where $+$ and $(.,.)$ are binary operators and $s_1+(s_2+s_3) = (s_1+s_2)+s_3$, i.e. $+$ (but not $(.,.)$) is required to be associative (so $(L, +)$ is a semigroup and $(L, (.,.))$ a groupoid). Interpretation is again by residuation with respect to the associated adjunction.

$$
(6)\quad \begin{array}{lcl} D(A{\cdot}B) & = & \{s_1{+}s_2 | s_1 \in D(A) \wedge s_2 \in D(B)\} \\ D(B/A) & = & \{s | \forall s' \in D(A), s{+}s' \in D(B)\} \\ D(A\backslash B) & = & \{s | \forall s' \in D(A), s'{+}s \in D(B)\} \end{array}
$$

$$
(7)\quad \begin{array}{lcl} D(A{\diamond}B) & = & \{(s_1, s_2) | s_1 \in D(A) \wedge s_2 \in D(B)\} \\ D(B{<}A) & = & \{s | \forall s' \in D(A), (s, s') \in D(B)\} \\ D(A{>}B) & = & \{s | \forall s' \in D(A), (s', s) \in D(B)\} \end{array}
$$

Gentzen sequent calculus for this partially associative system is as follows. The binary tree structured antecedent configurations are bracketed with $\{.,.\}$ indicating the associative adjunction and $(.,.)$ the non-associative.

(8) a.
$$
\frac{}{A \Rightarrow A}\mathrm{id} \qquad \frac{\Gamma \Rightarrow A \quad \Delta[A] \Rightarrow B}{\Delta[\Gamma] \Rightarrow B}\mathrm{Cut}
$$

b.
$$
\frac{\Gamma \Rightarrow A \quad \Delta[B] \Rightarrow C}{\Delta[\{\Gamma, A\backslash B\}] \Rightarrow C}\backslash\mathrm{L} \qquad \frac{\{A, \Gamma\} \Rightarrow B}{\Gamma \Rightarrow A\backslash B}\backslash\mathrm{R}
$$

c.
$$
\frac{\Gamma \Rightarrow A \quad \Delta[B] \Rightarrow C}{\Delta[\{B/A, \Gamma\}] \Rightarrow C}/\mathrm{L} \qquad \frac{\{\Gamma, A\} \Rightarrow B}{\Gamma \Rightarrow B/A}/\mathrm{R}
$$

d.
$$
\frac{\Gamma[\{A, B\}] \Rightarrow C}{\Gamma[A{\cdot}B] \Rightarrow C}{\cdot}\mathrm{L} \qquad \frac{\Gamma \Rightarrow A \quad \Delta \Rightarrow B}{\{\Gamma, \Delta\} \Rightarrow A{\cdot}B}{\cdot}\mathrm{R}
$$

e.
$$
\frac{\Gamma \Rightarrow A \quad \Delta[B] \Rightarrow C}{\Delta[(\Gamma, A{>}B)] \Rightarrow C}{>}\mathrm{L} \qquad \frac{(A, \Gamma) \Rightarrow B}{\Gamma \Rightarrow A{>}B}{>}\mathrm{R}
$$

f.
$$
\frac{\Gamma \Rightarrow A \quad \Delta[B] \Rightarrow C}{\Delta[(B{<}A, \Gamma)] \Rightarrow C}{<}\mathrm{L} \qquad \frac{(\Gamma, A) \Rightarrow B}{\Gamma \Rightarrow B{<}A}{<}\mathrm{R}
$$

g. $$\frac{\Gamma[(A, B)] \Rightarrow C}{\Gamma[A{\circ}B] \Rightarrow C}{\circ}L \qquad \frac{\Gamma \Rightarrow A \quad \Delta \Rightarrow B}{(\Gamma, \Delta) \Rightarrow A{\circ}B}{\circ}R$$

h. $$\frac{\Gamma[\{\Delta_1, \{\Delta_2, \Delta_3\}\}] \Rightarrow A}{\Gamma[\{\{\Delta_1, \Delta_2\}, \Delta_3\}] \Rightarrow A}A_{\{\}}$$

The structural rule (8h) represents associativity for the associative mode of adjunction. We could alternatively represent equivalence classes of bracketings under associativity by flat sequences (cf. Morrill 1990c).

In general we may introduce multimodal logics with any number $n$ of families $\{\cdot_i,\ /_i,\ \backslash_i\}$, $1 \le i \le n$, of multiplicative type-constructors, interpreted by residuation in a multigroupoid $(L, \{+\}_{i\in\{1,\ldots,n\}})$:

(9) $$\begin{array}{lcl} D(A{\cdot_i}B) & = & \{s_1{+}s_2 | s_1 \in D(A) \wedge s_2 \in D(B)\} \\ D(B/_iA) & = & \{s | \forall s' \in D(A), s{+_i}s' \in D(B)\} \\ D(A\backslash_iB) & = & \{s | \forall s' \in D(A), s'{+_i}s \in D(B)\} \end{array}$$

For example phonological theory has identified a number of prosodic units; Nespor and Vogel (1986) asserts a prosodic hierarchy comprising syllable, foot, phonological word, clitic group, phonological phrase, intonational phrase, and phonological utterance. In order to recognise such structure we may introduce a mode of prosodic adjunction, with an associated family of multiplicatives, for each prosodic unit. Each family will respect the residuation laws:

(10) $A \Rightarrow C/_iB \quad =\!\!|\!\!\models \quad A{\cdot_i}B \Rightarrow C \quad =\!\!|\!\!\models \quad B \Rightarrow A\backslash_iC$

Where we identify the mode of prosodic combination in question by subscripting sequent bracketing, the Gentzen sequent logic is as follows for each mode.

(11) a. $$\frac{}{A \Rightarrow A}\text{id} \qquad \frac{\Gamma \Rightarrow A \quad \Delta[A] \Rightarrow B}{\Delta[\Gamma] \Rightarrow B}\text{Cut}$$

b. $$\frac{\Gamma \Rightarrow A \quad \Delta[B] \Rightarrow C}{\Delta[(_i\Gamma, A\backslash_iB)] \Rightarrow C}\backslash_nL \qquad \frac{(_iA, \Gamma) \Rightarrow B}{\Gamma \Rightarrow A\backslash_iB}\backslash_iR$$

c. $$\frac{\Gamma \Rightarrow A \quad \Delta[B] \Rightarrow C}{\Delta[({}_i B/_i A, \Gamma)] \Rightarrow C}/_i\text{L} \qquad \frac{({}_i\Gamma, A) \Rightarrow B}{\Gamma \Rightarrow B/_i A}/_i\text{R}$$

d. $$\frac{\Gamma[({}_i A, B)] \Rightarrow C}{\Gamma[A\cdot_i B] \Rightarrow C}\cdot_i\text{L} \qquad \frac{\Gamma \Rightarrow A \quad \Delta \Rightarrow B}{({}_i\Gamma, \Delta) \Rightarrow A\cdot_i B}\cdot_i\text{R}$$

In addition there are structural rules corresponding to structural axioms such as associativity on the multigroupoid. When such axioms relate different adjunctions, we refer to them as linking rules. We shall see in section 3 how linking rules enable formulation of a multimodal logic of discontinuity; before that we introduce the notion of labelling, which supplies a form of presentation related to the PTQ-style presentation of chapter one.

## 2. LABELLED DEDUCTION

In the sequent calculus for categorial grammar as we have used it so far, prosodic operations are represented by the linear and hierarchical organisation of configurations. Semantic operations, as we have seen, can be represented by labelling antecedent formulas with variables, and labelling succedent formulas with semantic terms over those variables, but prosodic operations were left implicit. In what follows we will present discontinuity operators for which implicit prosodic interpretation by organisation of antecedents into configurations is not so natural, because word order is not always given by the left to right yield of the structures. The situation will be ameliorated by labelling prosodics, like semantics, explicitly. This practice falls within Gabbay's (1991) general discipline of *labelled deduction*, the slogan of which is to "bring semantics back into syntax". What that will mean for grammar is PTQ-style prosodic and semantic labelling of rules of formation. We will also present in section 4 modal operators for which labelling provides a convenient proof format.

A prosodically labelled sequent has the form:

(12) $a_1\colon A_1, ..., a_n\colon A_n \Rightarrow \alpha\colon A$

No prosodic variable $a_i$ may occur more than once in the antecedent which is thus a functional assignment of categorial types to prosodic variables, and $\alpha$ is a prosodic term over the variables $a_1, ..., a_n$. Such a sequent states that applying the prosodic operation represented by $\alpha$ to any objects in $A_1, ..., A_n$ (according to the labelling) yields an object in $A$. When semantic labelling is also included, we have statements of formation as in chapter one:

(13) $a_1 - x_1 \colon A_1, ..., a_n - x_n \colon A_n \Rightarrow \alpha - \phi \colon A$

Order of antecedents is now unimportant: as noted in chapter one, validity is preserved under permutation (and contraction and weakening). In our labelling, we maintain the convention that antecedent formulas are labelled with prosodic and semantic *variables*. Other versions of labelling involve labelling antecedent formulas with prosodic and semantic *terms* in general. We keep to the stricter discipline here in order that the antecedents in the theorems $a_1 - x_1 \colon A_1, \ldots, a_n - x_n \colon A_n \Rightarrow \alpha - \phi \colon A$ just show to what categories the prosodic and semantic operations $\alpha$ and $\phi$ apply in a Montagovian rule of formation with input categories $A_1, \ldots, A_n$ and output category $A$. Pattern-matching against antecedent semantic terms would constitute essential use of logical form in the way which removes the guarantee of convertibility of a PTQ-style grammar into an EFL-style one. In the logical setting, the analogue is that the 'bringing of semantics into syntax' would not be a transparent reflection of the model theory (though of course it might be valid). We advocate the same transparency with respect to the prosodic dimension as for the semantic dimension, so that antecedent prosodic terms are also restricted to variables.

For a given model theory each labelled sequent either is or is not valid, and we would like a labelled proof theory generating exactly the validities. For the Lambek calculi we can obtain a labelled deductive system (LDS) fairly directly from the usual Gentzen-style sequent formulation. The process is really just one of compiling configuration structure into the succedent prosodic term. Henceforth we allow the distinguished occurrence notation [.] to apply to terms as well as configurations. Working for the moment with just prosodic labelling, we obtain the following for **NL**:

(14) a. $\dfrac{}{a \colon A \Rightarrow a \colon A}\text{id} \qquad \dfrac{\Gamma \Rightarrow \alpha \colon A \qquad a \colon A, \Delta \Rightarrow \beta[a] \colon B}{\Gamma, \Delta \Rightarrow \beta[\alpha] \colon B}\text{Cut}$

b. $$\frac{\Gamma \Rightarrow \alpha\colon A \qquad b\colon B, \Delta \Rightarrow \gamma[b]\colon C}{\Gamma, d\colon A\backslash B, \Delta \Rightarrow \gamma[(\alpha{+}d)]\colon C}\backslash L \qquad \frac{\Gamma, a\colon A \Rightarrow (a{+}\gamma)\colon B}{\Gamma \Rightarrow \gamma\colon A\backslash B}\backslash R$$

c. $$\frac{\Gamma \Rightarrow \alpha\colon A \qquad b\colon B, \Delta \Rightarrow \gamma[b]\colon C}{\Gamma, d\colon B/A, \Delta \Rightarrow \gamma[(d{+}\alpha)]\colon C}/L \qquad \frac{\Gamma, a\colon A \Rightarrow (\gamma{+}a)\colon B}{\Gamma \Rightarrow \gamma\colon B/A}/R$$

d. $$\frac{a\colon A, b\colon B, \Delta \Rightarrow \gamma[(a{+}b)]\colon C}{d\colon A{\cdot}B, \Delta \Rightarrow \gamma[d]\colon C}{\cdot}L \qquad \frac{\Gamma \Rightarrow \alpha\colon A \qquad \Delta \Rightarrow \beta\colon B}{\Gamma, \Delta \Rightarrow (\alpha{+}\beta)\colon A{\cdot}B}{\cdot}R$$

A derivation of lifting, for example, is as follows.

(15) $$\cfrac{\cfrac{a\colon A \Rightarrow a\colon A \qquad b\colon B \Rightarrow b\colon B}{a\colon A, d\colon A\backslash B \Rightarrow (a{+}d)\colon B}\backslash L}{a\colon A \Rightarrow a\colon B/(A\backslash B)}/R$$

The difference between associative and non-associative labelling lies in an understanding that + in the labels of the former, but not the latter, is associative. Thus in an LDS for **L** there is the following structural rule.

(16) $$\frac{\Gamma \Rightarrow \alpha[(\alpha_1{+}(\alpha_2{+}\alpha_3))]\colon A}{\Gamma \Rightarrow \alpha[((\alpha_1{+}\alpha_2){+}\alpha_3)]\colon A}A \quad \text{(double line)}$$

Then for example composition in **L** is derived thus:

(17) $$\cfrac{\cfrac{\cfrac{c\colon C \Rightarrow c\colon C \qquad \cfrac{a\colon A \Rightarrow a\colon A \qquad b\colon B \Rightarrow b\colon B}{a\colon A, d\colon A\backslash B \Rightarrow (a{+}d)\colon B}\backslash L}{c\colon C, e\colon C\backslash A, d\colon A\backslash B \Rightarrow ((c{+}e){+}d)\colon B}\backslash L}{c\colon C, e\colon C\backslash A, d\colon A\backslash B \Rightarrow (c{+}(e{+}d))\colon B}A}{e\colon C\backslash A, d\colon A\backslash B \Rightarrow (e{+}d)\colon C\backslash B}\backslash R$$

When we in addition label for semantics, a sequent has the form $a_1 - x_1\colon A_1, \ldots, a_n - x_n\colon A_n \Rightarrow\ \alpha - \phi\colon A$, no prosodic or semantic variable is associated with more than one category, $\alpha$ is a prosodic term over variables $a_1, \ldots, a_n$ and $\phi$ is a semantic term over (free) variables $x_1, \ldots, x_n$. The prosodically and semantically labelled calculus is as follows; in ·L the [.] distinguished occurrence notation is extended to [., .] indicating two distinguished occurrences.

(18) a.
$$\frac{}{a - x\colon A \Rightarrow a - x\colon A}\text{id}$$

b.
$$\frac{\Gamma \Rightarrow \alpha - \phi\colon A \qquad a - x\colon A, \Delta \Rightarrow \beta[a] - \psi[x]\colon B}{\Gamma, \Delta \Rightarrow \beta[\alpha] - \psi[\phi]\colon B}\text{Cut}$$

c.
$$\frac{\Gamma \Rightarrow \alpha - \phi\colon A \qquad b - y\colon B, \Delta \Rightarrow \gamma[b] - \chi[y]\colon C}{\Gamma, d - w\colon A\backslash B, \Delta \Rightarrow \gamma[(\alpha{+}d)] - \chi[(w\ \phi)]\colon C}\backslash\text{L}$$

d.
$$\frac{\Gamma, a - x\colon A \Rightarrow (a{+}\gamma) - \psi\colon B}{\Gamma \Rightarrow \gamma - \lambda x\psi\colon A\backslash B}\backslash\text{R}$$

e.
$$\frac{\Gamma \Rightarrow \alpha - \phi\colon A \qquad b - y\colon B, \Delta \Rightarrow \gamma[b] - \psi[y]\colon C}{\Gamma, d - w\colon B/A, \Delta \Rightarrow \gamma[(d{+}\alpha)] - \psi[(w\ \phi)]\colon C}/\text{L}$$

f.
$$\frac{\Gamma, a - x\colon A \Rightarrow (\gamma{+}a) - \psi\colon B}{\Gamma \Rightarrow \gamma - \lambda x\psi\colon B/A}/\text{R}$$

g.
$$\frac{a - x\colon A, b - y\colon B, \Delta \Rightarrow \gamma[(a{+}b)] - \chi[x, y]\colon C}{d - w\colon A{\cdot}B, \Delta \Rightarrow \gamma[d] - \chi[\pi_1 w, \pi_2 w]\colon C}{\cdot}\text{L}$$

h.
$$\frac{\Gamma \Rightarrow \alpha - \phi\colon A \qquad \Delta \Rightarrow \beta - \psi\colon B}{\Gamma, \Delta \Rightarrow (\alpha{+}\beta) - (\phi, \psi)\colon A{\cdot}B}{\cdot}\text{R}$$

With semantics a subject lifting derivation becomes the following.

(19)
$$\frac{\dfrac{a - x\colon \text{N} \Rightarrow a - x\colon \text{N} \qquad c - z\colon \text{S} \Rightarrow c - z\colon \text{S}}{a - x\colon \text{N},\ b - y\colon \text{N}\backslash\text{S} \Rightarrow (a{+}b) - (y\ x)\colon \text{S}}\backslash\text{L}}{a - x\colon \text{N} \Rightarrow a - \lambda y(y\ x)\colon \text{S}/(\text{N}\backslash\text{S})}/\text{R}$$

We shall now use labelling to present a Fitch-style natural deduction format for categorial derivations (see Morrill 1993) which is typographically friendly in that derivations of reasonable complexity can be represented in full down the page, whereas sequent proofs and labelled Prawitz-style natural deduction quickly exhaust space across the page. Fitch-style natural deduction is serial, i.e. linearly structured, whereas Prawitz-style is parallel, i.e. tree structured. The former represents conditional reasoning by 'smart' block structure which indicates the scope of hypothetical subderivations.

For labelled Fitch-style categorial derivation, there are lexical assignment, subderivation hypothesis, and term label equation rules thus:

(20) a. $n.$ $\alpha - \phi: A$ for any lexical entry

b.
$$\begin{array}{ll|ll} & n. & a_1 - x_1: A_1 & \text{H} \\ & & \vdots & \vdots \\ & n+m. & a_m - x_m: A_n & \text{H} \end{array}$$

c.
$$n. \quad \frac{\alpha - \phi: A}{\alpha' - \phi': A} \quad = n, \text{if } \alpha = \alpha' \ \& \ \phi = \phi'$$

The lexical assignment rule allows introduction of a lexical declaration at any stage in a derivation. The subderivation rule allows commencement of a subderivation with one or more hypotheses, at one level of embedding down, and the label equation rule allows rewriting of labels under equality. As usual there are two rules for each operator: a rule of elimination (corresponding to the Gentzen-style left rule) showing how to use a formula with that operator as principal connective, and a rule of introduction (corresponding to the Gentzen-style right rule) showing how to prove a formula with that operator as principal connective. Logical rules for the categorial connectives are as follows.

(21) a.
$$\begin{array}{ll} n. & \alpha - \phi: A \\ m. & \gamma - \chi: A\backslash B \\ \hline & (\alpha{+}\gamma) - (\chi\ \phi): B \quad \text{E}\backslash\ n, m \end{array}$$

b.
$$\begin{array}{l|ll} n. & a - x: A & \text{H} \\ m. & (a{+}\gamma) - \psi: B & \text{unique } a \text{ as indicated} \\ & \gamma - \lambda x \psi: A\backslash B & \text{I}\backslash\ n, m \end{array}$$

(22) a.
$$\begin{array}{ll} n. & \alpha - \phi: A \\ m. & \gamma - \chi: B/A \\ \hline & (\gamma{+}\alpha) - (\chi\ \phi): B \quad \text{E}/\ n, m \end{array}$$

$$\begin{array}{lll}
1. & a - x{:}\,\mathrm{N} & \\
2. & \quad b - y{:}\,\mathrm{N}\backslash\mathrm{S} & \mathrm{H} \\
3. & \quad (a{+}b) - (y\ x){:}\,\mathrm{S} & \mathrm{E}\backslash\ 1,2 \\
4. & a - \lambda y(y\ x){:}\,\mathrm{S}/(\mathrm{N}\backslash\mathrm{S}) & \mathrm{I}/\ 2,3
\end{array}$$

FIGURE 4.1. Derivation of lifting in labelled non-associative calculus

b.
$$\begin{array}{lll}
n. & \quad a - x{:}\,A & \mathrm{H} \\
m. & \quad (\gamma{+}a) - \psi{:}\,B & \text{unique } a \text{ as indicated} \\
 & \gamma - \lambda x\psi{:}\,B/A & \mathrm{I}/\ n, m
\end{array}$$

(23) a.
$$\begin{array}{lll}
n. & \gamma - \chi{:}\,A{\cdot}B & \\
m. & \quad a - x{:}\,A & \mathrm{H} \\
m+1. & \quad b - y{:}\,B & \mathrm{H} \\
p. & \quad \delta[(a{+}b)] - \omega[x,y]{:}\,D & \text{unique } a \text{ and } b \text{ as indicated} \\
 & \delta[\gamma] - \omega[\pi_1\chi, \pi_2\chi]{:}\,D & \mathrm{E}{\cdot}\ n, m, m+1, p
\end{array}$$

b.
$$\begin{array}{lll}
n. & \alpha - \phi{:}\,A & \\
m. & \beta - \psi{:}\,B & \\
\hline
 & (\alpha{+}\beta) - (\phi,\psi){:}\,A{\cdot}B & \mathrm{I}{\cdot}\ n, m
\end{array}$$

The previous lifting theorem is now derived as in Figure 4.1

A Fitch-style labelled calculus for the associative Lambek calculus **L** can be obtained from that for the non-associative calculus by adding a prosodic label equation applying to arbitrary subterms in a derivation:

(24) $((\alpha_1{+}\alpha_2){+}\alpha_3) = (\alpha_1{+}(\alpha_2{+}\alpha_3))$

Alternatively, the associative Lambek calculus can be given by dropping parentheses in prosodic labels (using in effect $n$-ary +). Fitch-style this gives the following:

(25) a.
$$\begin{array}{lll}
n. & \alpha - \phi{:}\,A & \\
m. & \gamma - \chi{:}\,A\backslash B & \\
\hline
 & \alpha{+}\gamma - (\chi\ \phi){:}\,B & \mathrm{E}\backslash\ n, m
\end{array}$$

b.
$$\begin{array}{lll}
n. & \quad a - x{:}\,A & \mathrm{H} \\
m. & \quad a{+}\gamma - \psi{:}\,B & \text{unique } a \text{ as indicated} \\
 & \gamma - \lambda x\psi{:}\,A\backslash B & \mathrm{I}\backslash\ n, m
\end{array}$$

| | | | |
|---|---|---|---|
| 1. | $d - w\colon \mathrm{VP/PP}$ | | |
| 2. | $e - u\colon \mathrm{PP/N}$ | | |
| 3. | $\mid c - z\colon \mathrm{N}$ | H | |
| 4. | $\mid e{+}c - (u\ z)\colon \mathrm{PP}$ | E/ 2, 3 | |
| 5. | $\mid d{+}e{+}c - (w\ (u\ z))\colon \mathrm{VP}$ | E/ 1, 4 | |
| 6. | $d{+}e - \lambda z(w\ (u\ z))\colon \mathrm{VP/N}$ | I/ 3, 5 | |

FIGURE 4.2. Derivation of composition in labelled associative calculus

(26) c.
$$\frac{n.\ \ \alpha - \phi\colon A \qquad m.\ \ \gamma - \chi\colon B/A}{\gamma{+}\alpha - (\chi\ \phi)\colon B}\ \mathrm{E}/\ n, m$$

d.
$n.\ \ \mid a - x\colon A$ H

$m.\ \ \mid \gamma{+}a - \psi\colon B$ unique $a$ as indicated

$\gamma - \lambda x\psi\colon B/A$ I/ $n, m$

(27) e.
$n.\ \ \gamma - \chi\colon A{\cdot}B$

$m.\ \ \mid a - x\colon A$ H

$m+1.\ \ \mid b - y\colon B$ H

$p.\ \ \mid \delta[a{+}b] - \omega[x, y]\colon D$ unique $a$ and $b$ as indicated

$\delta[\gamma] - \omega[\pi_1\chi, \pi_2\chi]\colon D$ E$\cdot$ $n, m, m+1, p$

f.
$$\frac{n.\ \ \alpha - \phi\colon A \qquad m.\ \ \beta - \psi\colon B}{\alpha{+}\beta - (\phi, \psi)\colon A{\cdot}B}\ \mathrm{I}{\cdot}\ n, m$$

This allows derivation of e.g. composition theorems not valid in the non-associative case; see Figure 4.2.

We have already seen relativisation examples such as (28).

(28) which John talked about

The relativisation can be derived as shown in Figure 4.3 in Fitch-style natural deduction **L** without parentheses.

Any multimodal system can be given a Fitch-style presentation as follows with each connective correlated with its adjunction constructor in the labels. First there are always the lexical assignment, subderivation

| | | |
|---|---|---|
| 1. | *which* – $\lambda x\lambda y\lambda z[(y\ z) \wedge (x\ z)]$: (CN\CN)/(S/N) | |
| 2. | *John* – $\mathbf{j}$: N | |
| 3. | *talked* – **talk**: (N\S)/PP | |
| 4. | *about* – **about**: PP/N | |
| 5. | \| $a$ – $x$: N | H |
| 6. | \| *about*+$a$ – (**about** $x$): PP | 4, 5 E/ |
| 7. | \| *talked*+*about*+$a$ – (**talk** (**about** $x$)): N\S | 3, 6 E/ |
| 8. | \| *John*+*talked*+*about*+$a$ – ((**talk** (**about** $x$)) $\mathbf{j}$): S | 2, 7 E\ |
| 9. | *John*+*talked*+*about* – $\lambda x$((**talk** (**about** $x$)) $\mathbf{j}$): S/N | 5, 8 I/ |
| 10. | *which*+*John*+*talked*+*about* – ($\lambda x\lambda y\lambda z[(y\ z) \wedge (x\ z)]$ $\lambda x$((**talk** (**about** $x$)) $\mathbf{j}$)): CN\CN | 1, 9 E/ |
| 11. | *which*+*John*+*talked*+*about* – $\lambda y\lambda z[(y\ z) \wedge$ ((**talk** (**about** $z$)) $\mathbf{j}$)]: CN\CN | = 10 |

FIGURE 4.3. Derivation of 'which John talked about'

hypothesis, and term label equations:

(29) a. $n. \quad \alpha - \phi\colon A \quad$ for any lexical entry

b.
$$\begin{array}{ll|ll} & n. & a_1 - x_1\colon A_1 & \mathrm{H} \\ & & \vdots & \vdots \\ & n+m. & a_m - x_m\colon A_n & \mathrm{H} \\ \hline \end{array}$$

c.
$$n. \quad \frac{\alpha - \phi\colon A}{\alpha' - \phi'\colon A} \quad = n, \text{if } \alpha = \alpha' \ \&\ \phi = \phi'$$

Then there are logical rules with each connective associated with its adjunction in the labels:

(30) a.
$$\begin{array}{ll} n. & \alpha - \phi\colon A \\ m. & \gamma - \chi\colon A\backslash_i B \\ \hline & (\alpha +_i \gamma) - (\chi\ \phi)\colon B \quad \mathrm{E}\backslash_i\ n, m \end{array}$$

b.
$$\begin{array}{ll|ll} n. & & a - x\colon A & \mathrm{H} \\ \hline m. & & (a +_i \gamma) - \psi\colon B & \text{unique } a \text{ as indicated} \\ & \gamma - \lambda x\psi\colon A\backslash_i B & & \mathrm{I}\backslash_i\ n, m \end{array}$$

(31) a.
$$\begin{array}{ll} n. & \alpha - \phi\colon A \\ m. & \gamma - \chi\colon B/_i A \\ \hline & (\gamma +_i \alpha) - (\chi\ \phi)\colon B \quad \mathrm{E}/_i\ n, m \end{array}$$

b. $n.$ $a - x{:}\, A$ H
$m.$ $(\gamma{+_i}a) - \psi{:}\, B$ unique $a$ as indicated
$\gamma - \lambda x\psi{:}\, B/_i A$ $\mathrm{I}/_i$ $n, m$

(32) a. $n.$ $\gamma - \chi{:}\, A{\cdot_i}B$
$m.$ $a - x{:}\, A$ H
$m+1.$ $b - y{:}\, B$ H
$p.$ $\delta[(a{+_i}b)] - \omega[x, y]{:}\, D$ unique $a$ and $b$ as indicated
$\delta[\gamma] - \omega[\pi_1\chi, \pi_2\chi]{:}\, D$ $\mathrm{E}{\cdot_i}$ $n, m, m+1, p$

b. $n.$ $\alpha - \phi{:}\, A$
$m.$ $\beta - \psi{:}\, B$
$(\alpha{+_i}\beta) - (\phi, \psi){:}\, A{\cdot_i}B$ $\mathrm{I}{\cdot_i}$ $n, m$

Label equations are to be added according to the algebras of interpretation.

Furthermore any multimodal system has a prosodically and semantically labelled Gentzen-style LDS presentation as follows, together with suitable label structural rules.

(33) a.
$$\frac{}{a - x{:}\, A \Rightarrow a - x{:}\, A}\mathrm{id}$$

b.
$$\frac{\Gamma \Rightarrow \alpha - \phi{:}\, A \qquad a - x{:}\, A, \Delta \Rightarrow \beta[a] - \psi[x]{:}\, B}{\Gamma, \Delta \Rightarrow \beta[\alpha] - \psi[\phi]{:}\, B}\mathrm{Cut}$$

c.
$$\frac{\Gamma \Rightarrow \alpha - \phi{:}\, A \qquad b - y{:}\, B, \Delta \Rightarrow \gamma[b] - \chi[y]{:}\, C}{\Gamma, d - w{:}\, A\backslash_i B, \Delta \Rightarrow \gamma[(\alpha{+_i}d)] - \chi[(w\ \phi)]{:}\, C}\backslash\mathrm{L}_i$$

d.
$$\frac{\Gamma, a - x{:}\, A \Rightarrow (a{+_i}\gamma) - \psi{:}\, B}{\Gamma \Rightarrow \gamma - \lambda x\psi{:}\, A\backslash_i B}\backslash_i\mathrm{R}$$

e.
$$\frac{\Gamma \Rightarrow \alpha - \phi{:}\, A \qquad b - y{:}\, B, \Delta \Rightarrow \gamma[b] - \psi[y]{:}\, C}{\Gamma, d - w{:}\, B/_i A, \Delta \Rightarrow \gamma[(d{+_i}\alpha)] - \psi[(w\ \phi)]{:}\, C}/_i\mathrm{L}$$

f. $$\frac{\Gamma, a - x{:}A \Rightarrow (\gamma{+_i}a) - \psi{:}B}{\Gamma \Rightarrow \gamma - \lambda x\psi{:}B/_iA}/_i\mathrm{R}$$

g. $$\frac{a - x{:}A, b - y{:}B, \Delta \Rightarrow \gamma[(a{+_i}b)] - \chi[x,y]{:}C}{d - w{:}A{\cdot_i}B, \Delta \Rightarrow \gamma[d] - \chi[\pi_1 w, \pi_2 w]{:}C}\cdot_i\mathrm{L}$$

h. $$\frac{\Gamma \Rightarrow \alpha - \phi{:}A \qquad \Delta \Rightarrow \beta - \psi{:}B}{\Gamma, \Delta \Rightarrow (\alpha{+_i}\beta) - (\phi,\psi){:}A{\cdot_i}B}\cdot_i\mathrm{R}$$

It is primarily in terms of such Fitch-style and Gentzen-style labelling that the subsequent sections present discontinuity and modal operators.

## 3. DISCONTINUITY OPERATORS

Consider the following paradigm of 'quantifier floating' (see Smith and Wilson 1979, p.61).

(34) a. *All* the children might have been shouting at once.
b. The children *all* might have been shouting at once.
c. The children might *all* have been shouting at once.
d. The children might have *all* been shouting at once.
e. The children might have been *all* shouting at once

One characterisation might treat 'all' as both a prenominal determiner (which may also occur with object nominals, etc.), and a verbal modifier which can occur within the auxiliary group. But for the purpose of exemplification let us take this paradigm at face value and consider how the (floating) quantifier 'all' may precede the subject, or appear anywhere within the auxiliary verb sequence. In order to capture such a generalisation we may define a 'non-directional' division such that $\frac{B}{A}$ represents a functor into $B$ capable of combining with its argument $A$ to both the left and the right:[2]

(35) $$D(\tfrac{B}{A}) = \{s|\forall s' \in D(A), s'{+}s \in D(B) \wedge s{+}s' \in D(B)\} = D(A\backslash B) \cap D(B/A)$$

[2] The possibility of such a type-constructor has been part of the categorial folklore for a long time. Its type mapping would naturally be $T(\frac{B}{A}) = T(B)^{T(A)}$.

We assume an associative context. Ordered natural deduction rules are as follows.

(36)
$$\frac{\begin{matrix}\vdots & \vdots\\ \frac{B}{A} & A\end{matrix}}{B}\text{-E}_a \qquad\qquad \frac{\begin{matrix}\vdots & \vdots\\ A & \frac{B}{A}\end{matrix}}{B}\text{-E}_b$$

(37)
$$\frac{\begin{matrix}\overline{A}^{\,i} & & \Gamma\\ & \vdots & \\ & B & \end{matrix} \qquad \begin{matrix}\overline{\Gamma}^{\,i} & & \overline{A}^{\,i}\\ & \vdots & \\ & B & \end{matrix}}{\frac{B}{A}}\text{-I}^i$$

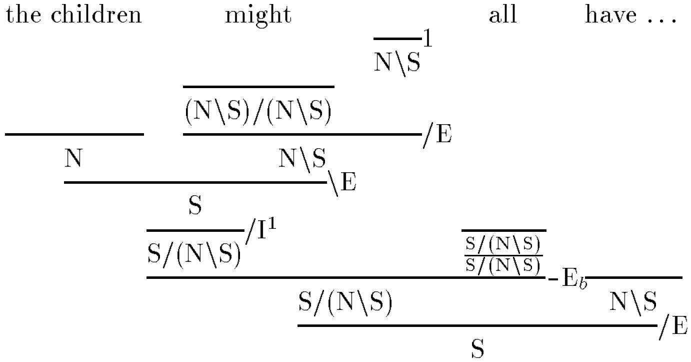
The elimination rules are straightforward. Indexed overline over Γ signifies discharge of the sequence of assumptions Γ so that the introduction rule states that where there is a proof of $B$ from Γ plus $A$ at the left periphery, and such a proof from Γ plus $A$ at the right periphery, then $\frac{B}{A}$ is proved from Γ alone.

Where we evade some details of the semantics of plurals and definites, the distributional facts in (34) are captured by assignment of 'all' to $\frac{S/(N\backslash S)}{S/(N\backslash S)}$. Thus (34a) is obtained as follows by direct forward application to the lifted subject, and (34b) is obtained similarly, by direct backward application of 'all' to 'the boys'.

(38) all the children might have ...
$$\frac{\dfrac{\dfrac{}{\frac{S/(N\backslash S)}{S/(N\backslash S)}} \qquad \dfrac{\dfrac{\dfrac{\text{the children}}{N} \quad \dfrac{}{N\backslash S}^{1}}{S}\backslash E}{S/(N\backslash S)}/I^1}{S/(N\backslash S)}\text{-E}_a \qquad \dfrac{\dfrac{\text{might}}{(N\backslash S)/(N\backslash S)} \quad \dfrac{\text{have} \ldots}{N\backslash S}}{N\backslash S}/E}{S}/E$$

The derivation for (34c) is as follows, and that for (34d-e) is similar. The associativity assumed means that the subject together with some auxiliary verbs has type S/(N\S).

(39) the children might all have ...




This case of quantifier floating shows how an apparently puzzling distribution can receive a quite simple characterisation in the right technical setting. In what follows we shall consider a range of discontinuous constructions: particle verbs, discontinuous idioms and other discontinuous functors, quantifier-scoping, pied-piping, gapping, and object-antecedent reflexivisation. Discontinuity refers to phenomena wherein signs differ markedly in their prosodic and semantic structures. There is no precise pre-theoretic definition, and it is not our pretence to provide a comprehensive account of the instances cited. However, we do intend to show how each is rendered amenable in its basic form.

The directional divisions of standard categorial grammar signify functors that adjoin to the left or right of their arguments. Moortgat (1988b), following Bach (1981, 1984), considers the idea of operators $B{\uparrow}A$ for functors that wrap themselves around an argument of category $A$ to form a $B$, and (in our notation) $A{\downarrow}B$ for functors that infix themselves in an argument of category $A$ to form a $B$. Assuming an associative context, Moortgat observes that for each operator two variants can be conceived: existential and universal. Leaving the semantic dimension aside, we interpret now in a monoid $(L^*, +, \epsilon)$, i.e. a semigroup $(L^*, +)$ together with an element $\epsilon \in L^*$ satisfying $s{+}\epsilon = \epsilon{+}s = s$ (an identity element). Formulas will be interpreted as subsets of $L = L^* - \{\epsilon\}$, but the identity element will be appropriate in order that peripherality is accommodated as an instance of discontinuity. Interpretation in $L^*$ would commit us to $\epsilon \in D(A/A), D(A\backslash A)$ since $\epsilon$ would always satisfy the residuation condition that it adjoins to any element in $D(A)$

on the right/left to give an element in $D(A)$. Then assignment of e.g. 'extremely' to (CN/CN)/(CN/CN) for such examples as 'the extremely deaf man' would also permit *'the extremely man'; see Morrill (1990b). Defining interpretation in $L = L^* - \{\epsilon\}$ circumvents this problem.

(40) Existential

$$D(B\uparrow_{\exists}A) = \{s|\exists s_1, s_2 \in L^*, \; [s = s_1{+}s_2 \wedge \forall s' \in D(A), s_1{+}s'{+}s_2 \in D(B)]\}$$

Universal

$$D(B\uparrow_{\forall}A) = \{s|\forall s_1, s_2 \in L^*, \; [s = s_1{+}s_2 \rightarrow \forall s' \in D(A), s_1{+}s'{+}s_2 \in D(B)]\}$$

For example $(N\backslash S)\uparrow_{\exists}N$ would be the lexical category of discontinuous functors such as particle verbs ('rings . up') and discontinuous idioms like ' gives . the cold shoulder', which wrap around their objects to form verb phrases and which convey a meaning as a whole not attributable to the component word meanings. Note that there is a specific point at which interpolation is required. By way of further example, $S\uparrow_{\exists}N$ would be the category of a sentence containing at some point a nominal gap, e.g. 'Mary met . yesterday', 'Mary met .', and '. walks', so that a relative pronoun category $(CN\backslash CN)/(S\uparrow_{\exists}N)$ for 'that' would generate each of 'that Mary met yesterday', 'that Mary met', and 'that walks'. It is less apparent what application there might be for universal wrap, but such a functor would circumscribe its argument admitting all interpolation points. Evidently use of $\uparrow_{\forall}$ instead of $\uparrow_{\exists}$ for discontinuous idioms and so on would permit incorrect word order such as *'Mary gave the John cold shoulder'

For infixation the two possibilities in a unimodal associative setting are:

(41) Existential

$$D(A\downarrow_{\exists}B) = \{s|\forall s' \in D(A), \exists s_1, s_2 \in L^*, \; [s' = s_1{+}s_2 \wedge s_1{+}s'{+}s_2 \in D(B)]\}$$

Universal

$$D(A\downarrow_{\forall}B) = \{s|\forall s' \in D(A), \forall s_1, s_2 \in L^*, \; [s' = s_1{+}s_2 \rightarrow s_1{+}s'{+}s_2 \in D(B)]\}$$

By way of example here, $Sneg\downarrow_{\forall}Spos$ would be the category of a freely floating negation particle, if there really were such an element. Existential infixation is reminiscent of quantifying-in if we think of a quantifier phrase as wanting to infix itself in a sentence lacking a nominal, at the position of the missing nominal. Thus a quantifier phrase like 'every man' might have type $(\mathrm{S}{\uparrow_\exists}\mathrm{N}){\downarrow_\exists}\mathrm{S}$, (cf. Moortgat 1991) but we would need to ensure that the two existentials were effectively referring to the same point of interpolation, an issue we consider shortly.

Inspecting the possibilities of ordered sequent presentation, of the eight possible rules of inference (use and proof for each of four division operators), only $\uparrow_\exists$R and $\downarrow_\forall$L are expressible:

(42) a.
$$\frac{\Gamma_1, A, \Gamma_2 \Rightarrow B}{\Gamma_1, \Gamma_2 \Rightarrow B{\uparrow_\exists}A}{\uparrow_\exists}\mathrm{R}$$

b.
$$\frac{\Gamma_1, \Gamma_2 \Rightarrow A \qquad \Delta_1, B, \Delta_2 \Rightarrow C}{\Delta_1, \Gamma_1, A{\downarrow_\forall}B, \Gamma_2, \Delta_2 \Rightarrow C}{\downarrow_\forall}\mathrm{L}$$

This is the partial logic of Moortgat (1988b). A left ordered sequent rule for $\uparrow_\exists$ cannot be formulated: if we ask how the sequent (43) might be proved, it is apparent that we lack a handle on the prosodic object in $B{\uparrow_\exists}A$ and the point around which it wraps.

(43) $\Gamma_1, B{\uparrow_\exists}A, \Gamma_2 \Rightarrow C$

Such a rule is needed however for a complete logic. In relation to this Moortgat (1991) observes that labelled deduction seems promising, and possibilities are considered in Versmissen (1991) which involve marking of insertion points. Yet the interpretations in (40) and (41) in a unimodal monoid algebra $(L^*, +, \epsilon)$ do not make reference to prosodic objects marked with insertion points.

In Moortgat (1991) a discontinuity 'substring' product is proposed, again implicitly assuming just a semigroup algebra for interpretation:[3]

(44) $D(A{\odot}B) = \{s_1{+}s_2{+}s_1' | s_1{+}s_1' \in D(A) \wedge s_2 \in D(B)\}$

As for the discontinuity divisions, ordered sequent presentation cannot express rules of both use and proof: only $\odot$R can be represented:

(45)
$$\frac{\Gamma_1, \Gamma_2 \Rightarrow A \qquad \Delta \Rightarrow B}{\Gamma_1, \Delta, \Gamma_2 \Rightarrow A{\odot}B}{\odot}\mathrm{R}$$

[3] The version given is actually just the existential case of two possibilities, existential and universal, as before. No rules for the universal version can be expressed in ordered sequent calculus.

Even using labelling, the problem for $\odot$L remains and is the same as that before: there is no proper management of separation points.[4]

Tension between finding a proof theory with rules of both use and proof, and a model theory for linguistically useful discontinuity operators to which it corresponds, is addressed in Morrill and Solias (1993) by using a bimodal prosodic algebra $(L^*, +, \langle ., . \rangle, \epsilon)$ where $(L^*, +, \epsilon)$ is a monoid and $(L^*, \langle ., . \rangle)$ is a free groupoid, involving a 'tupling' operation $\langle ., . \rangle$ introduced in Solias (1992). The significance of such an operation is that its parts are recoverable (by projection functions 1 and 2), enabling a definition of wrapping and infixation in terms of + and $\langle ., . \rangle$ in which $\langle ., . \rangle$ remembers separation points, in contrast to the attempts in terms of associative + which do not. Use of the tuple operation collapses the former distinction between existential and universal in (40) and (41): both cases become 'there exists exactly one'. This is because tuples express a unique insertion point: tupling guarantees the unicity of decomposition. Existential and universal wrappers collapse into a single wrapper and existential and universal infixers collapse into a single infixer. The system is a three family multimodal one, with interpretation by residuation with respect to +, $\langle ., . \rangle$ and a wrapping adjunction $W$ defined by $s_1 W s_2 = 1s_1 + s_2 + 2s_1$.

Note that the tuple prosodic operation is not simply that of a groupoid, but that of a free groupoid, since its components must be recoverable for the wrapping adjunction to be defined. But this raises a problem, because while the non-associative calculus with product is complete for interpretation by residuation in groupoids, it is not complete in free groupoids (see Venema 1993b). So the multimodal calculus based on tupling would be incomplete.[5]

The solution we propose here is one which departs from the unimodal proposals of Moortgat (1988b, 1991) and the bimodal proposals of Solias (1992) and Morrill and Solias (1993) in respect of the status of the wrapping adjunction. Instead of being defined, it is introduced from the start as a primitive operation $W$ in a trimodal algebra of interpretation $(L^*, +, (., .), W, \epsilon)$. $(L^*, +, \epsilon)$ is a monoid and $(L^*, (., .))$ and $(L^*, W)$ groupoids, and the significant properties of the wrapping adjunction are

[4] See Hepple (1993) for an attempt to give full logic for Moortgat interpretations via a complex system of labelling.

[5] There are also certain questions as to the definability of $s_1 W s_2$ when $s_1$ is not a tuple element, and to discrimination according to 'prosodic sort' (tuple or non-tuple), rather than soley according to categorial type.

specified by the linking rule $(s_1, s_3)Ws_2 = s_1{+}s_2{+}s_3$.[6]

To formulate discontinuity we have a community comprising three families of multiplicatives: the usual associative 'surface' operators, 'split-point' non-associative operators, and discontinuity operators. The category formulas $\mathcal{F}$ are defined in terms of a set $\mathcal{A}$ of atomic category formulas thus:

(46) $\mathcal{F} = \mathcal{A} \mid \mathcal{F}{\cdot}\mathcal{F} \mid \mathcal{F}\backslash\mathcal{F} \mid \mathcal{F}/\mathcal{F} \mid \mathcal{F}{\circ}\mathcal{F} \mid \mathcal{F}{>}\mathcal{F} \mid \mathcal{F}{<}\mathcal{F} \mid \mathcal{F}\odot\mathcal{F} \mid \mathcal{F}{\downarrow}\mathcal{F} \mid \mathcal{F}{\uparrow}\mathcal{F}$

Spelt out in full the prosodic interpretation by residuation with respect to each adjunction is as follows:

(47)
$$\begin{array}{lcl} D(A{\cdot}B) & = & \{s_1{+}s_2 | s_1 \in D(A) \wedge s_2 \in D(B)\} \\ D(A\backslash B) & = & \{s | \forall s' \in D(A), s'{+}s \in D(B)\} \\ D(B/A) & = & \{s | \forall s' \in D(A), s{+}s' \in D(B)\} \end{array}$$

(48)
$$\begin{array}{lcl} D(A{\circ}B) & = & \{(s_1, s_2) | s_1 \in D(A) \wedge s_2 \in D(B)\} \\ D(A{>}B) & = & \{s | \forall s' \in D(A), (s', s) \in D(B)\} \\ D(B{<}A) & = & \{s | \forall s' \in D(A), (s, s') \in D(B)\} \end{array}$$

(49)
$$\begin{array}{lcl} D(A\odot B) & = & \{s_1 W s_2 | s_1 \in D(A) \wedge s_2 \in D(B)\} \\ D(A{\downarrow}B) & = & \{s | \forall s' \in D(A), s' W s \in D(B)\} \\ D(B{\uparrow}A) & = & \{s | \forall s' \in D(A), s W s' \in D(B)\} \end{array}$$

Since this is interpretation by residuation in a multigroupoid, proof rules can be provided in the standard formats. In particular it now becomes possible to present rules of both use and proof in the prosodically labelled sequent formats (see Morrill 1993 for the ordered sequent format). The full set of labelled Gentzen-style sequent rules are as follows.

(50) a.
$$\frac{}{a - x\colon A \Rightarrow a - x\colon A}\,\mathrm{id}$$

b.
$$\frac{\Gamma \Rightarrow \alpha - \phi\colon A \qquad a - x\colon A, \Delta \Rightarrow \beta[a] - \psi[x]\colon B}{\Gamma, \Delta \Rightarrow \beta[\alpha] - \psi[\phi]\colon B}\,\mathrm{Cut}$$

[6] This both removes questions about the partiality of $W$, which is defined as a total function, and replaces the problematic free groupoid under $\langle .,. \rangle$ by an unproblematic groupoid under $(.,.)$. On the other hand the prosodic algebra becomes more abstract, with prosodic forms no longer all corresponding to just strings or split strings.

(51) a.
$$\frac{\Gamma \Rightarrow \alpha - \phi{:}\, A \qquad b - y{:}\, B, \Delta \Rightarrow \gamma[b] - \chi[y]{:}\, C}{\Gamma, d - w{:}\, A\backslash B, \Delta \Rightarrow \gamma[(\alpha{+}d)] - \chi[(w\ \phi)]{:}\, C}\backslash\mathrm{L}$$

b.
$$\frac{\Gamma, a - x{:}\, A \Rightarrow (a{+}\gamma) - \psi{:}\, B}{\Gamma \Rightarrow \gamma - \lambda x\psi{:}\, A\backslash B}\backslash\mathrm{R}$$

c.
$$\frac{\Gamma \Rightarrow \alpha - \phi{:}\, A \qquad b - y{:}\, B, \Delta \Rightarrow \gamma[b] - \psi[y]{:}\, C}{\Gamma, d - w{:}\, B/A, \Delta \Rightarrow \gamma[(d{+}\alpha)] - \psi[(w\ \phi)]{:}\, C}/\mathrm{L}$$

d.
$$\frac{\Gamma, a - x{:}\, A \Rightarrow (\gamma{+}a) - \psi{:}\, B}{\Gamma \Rightarrow \gamma - \lambda x\psi{:}\, B/A}/\mathrm{R}$$

e.
$$\frac{a - x{:}\, A, b - y{:}\, B, \Delta \Rightarrow \gamma[(a{+}b)] - \chi[x, y]{:}\, C}{d - w{:}\, A{\cdot}B, \Delta \Rightarrow \gamma[d] - \chi[\pi_1 w, \pi_2 w]{:}\, C}\cdot\mathrm{L}$$

f.
$$\frac{\Gamma \Rightarrow \alpha - \phi{:}\, A \qquad \Delta \Rightarrow \beta - \psi{:}\, B}{\Gamma, \Delta \Rightarrow (\alpha{+}\beta) - (\phi, \psi){:}\, A{\cdot}B}\cdot\mathrm{R}$$

(52) a.
$$\frac{\Gamma \Rightarrow \alpha - \phi{:}\, A \qquad b - y{:}\, B, \Delta \Rightarrow \gamma[b] - \chi[y]{:}\, C}{\Gamma, d - w{:}\, A{>}B, \Delta \Rightarrow \gamma[(\alpha, d)] - \chi[(w\ \phi)]{:}\, C}{>}\mathrm{L}$$

b.
$$\frac{\Gamma, a - x{:}\, A \Rightarrow (a, \gamma) - \psi{:}\, B}{\Gamma \Rightarrow \gamma - \lambda x\psi{:}\, A{>}B}{>}\mathrm{R}$$

c.
$$\frac{\Gamma \Rightarrow \alpha - \phi{:}\, A \qquad b - y{:}\, B, \Delta \Rightarrow \gamma[b] - \psi[y]{:}\, C}{\Gamma, d - w{:}\, B{<}A, \Delta \Rightarrow \gamma[(d, \alpha)] - \psi[(w\ \phi)]{:}\, C}{<}\mathrm{L}$$

d.
$$\frac{\Gamma, a - x{:}\, A \Rightarrow (\gamma, a) - \psi{:}\, B}{\Gamma \Rightarrow \gamma - \lambda x\psi{:}\, B{<}A}{<}\mathrm{R}$$

e.
$$\frac{a - x{:}\, A, b - y{:}\, B, \Delta \Rightarrow \gamma[(a, b)] - \chi[x, y]{:}\, C}{c - z{:}\, A{\diamond}B, \Delta \Rightarrow \gamma[c] - \chi[\pi_1 z, \pi_2 z]{:}\, C}\diamond\mathrm{L}$$

f. $$\frac{\Gamma \Rightarrow \alpha - \phi\colon A \qquad \Delta \Rightarrow \beta - \psi\colon B}{\Gamma, \Delta \Rightarrow (\alpha, \beta) - (\phi, \psi)\colon A{\diamond}B}{\diamond}\mathrm{R}$$

(53) a. $$\frac{\Gamma \Rightarrow \alpha - \phi\colon A \qquad b - y\colon B, \Delta \Rightarrow \gamma[b] - \chi[y]\colon C}{\Gamma, d - w\colon A{\downarrow}B, \Delta \Rightarrow \gamma[(\alpha W d)] - \chi[(w\ \phi)]\colon C}{\downarrow}\mathrm{L}$$

b. $$\frac{\Gamma, a - x\colon A \Rightarrow (aW\gamma) - \psi\colon B}{\Gamma \Rightarrow \gamma - \lambda x\psi\colon A{\downarrow}B}{\downarrow}\mathrm{R}$$

c. $$\frac{\Gamma \Rightarrow \alpha - \phi\colon A \qquad b - y\colon B, \Delta \Rightarrow \gamma[b] - \psi[y]\colon C}{\Gamma, d - w\colon B{\uparrow}A, \Delta \Rightarrow \gamma[(dW\alpha)] - \psi[(w\ \phi)]\colon C}{\uparrow}\mathrm{L}$$

d. $$\frac{\Gamma, a - x\colon A \Rightarrow (\gamma W a) - \psi\colon B}{\Gamma \Rightarrow \gamma - \lambda x\psi\colon B{\uparrow}A}{\uparrow}\mathrm{R}$$

e. $$\frac{a - x\colon A, b - y\colon B, \Delta \Rightarrow \gamma[(aWb)] - \chi[x, y]\colon C}{d - w\colon A{\odot}B, \Delta \Rightarrow \gamma[d] - \chi[\pi_1 w, \pi_2 w]\colon C}{\odot}\mathrm{L}$$

f. $$\frac{\Gamma \Rightarrow \alpha - \phi\colon A \qquad \Delta \Rightarrow \beta - \psi\colon B}{\Gamma, \Delta \Rightarrow (\alpha W \beta) - (\phi, \psi)\colon A{\odot}B}{\odot}\mathrm{R}$$

There are label structural rules for associativity, adjunction identity, and the split-wrap linking rule:

(54) a. $$\frac{\Gamma \Rightarrow \alpha[((\alpha_1{+}\alpha_2){+}\alpha_3)] - \phi\colon A}{\Gamma \Rightarrow \alpha[(\alpha_1{+}(\alpha_2{+}\alpha_3))] - \phi\colon A}\mathrm{A}$$

b. $$\frac{\Gamma \Rightarrow \alpha[(\epsilon{+}\beta)] - \phi\colon A}{\Gamma \Rightarrow \alpha[\beta] - \phi\colon A}\mathrm{I} \qquad \frac{\Gamma \Rightarrow \alpha[(\beta{+}\epsilon)] - \phi\colon A}{\Gamma \Rightarrow \alpha[\beta] - \phi\colon A}\mathrm{I}$$

c. $$\frac{\Gamma \Rightarrow \alpha[((\alpha_1, \alpha_2)W\alpha_3)] - \phi\colon A}{\Gamma \Rightarrow \alpha[((\alpha_1{+}\alpha_3){+}\alpha_2)] - \phi\colon A}\mathrm{SW}$$

Alternatively, since there is only one associative adjunction, its associativity can be represented by omitting parentheses for the relevant term constructor.

If the relative pronoun 'that' is assigned a category (CN\CN)/(S↑N), both peripheral and medial relativisation is generated; thus 'that Mary met yesterday' can be derived via the following (leaving semantics aside):

(55)
$$\cfrac{\cfrac{\cfrac{a\colon \mathrm{N} \Rightarrow a\colon \mathrm{N} \qquad b\colon \mathrm{N},\ e\colon \mathrm{N}\backslash \mathrm{S},\ d\colon (\mathrm{N}\backslash \mathrm{S})\backslash(\mathrm{N}\backslash \mathrm{S}) \ \Rightarrow b{+}e{+}d\colon \mathrm{S}}{b\colon \mathrm{N},\ c\colon (\mathrm{N}\backslash \mathrm{S})/\mathrm{N},\ d\colon (\mathrm{N}\backslash \mathrm{S})\backslash(\mathrm{N}\backslash \mathrm{S}),\ a\colon \mathrm{N} \ \Rightarrow b{+}c{+}a{+}d\colon \mathrm{S}}/L}{b\colon \mathrm{N},\ c\colon (\mathrm{N}\backslash \mathrm{S})/\mathrm{N},\ d\colon (\mathrm{N}\backslash \mathrm{S})\backslash(\mathrm{N}\backslash \mathrm{S}),\ a\colon \mathrm{N} \ \Rightarrow (b{+}c, d)Wa\colon \mathrm{S}}SW}{b\colon \mathrm{N},\ c\colon (\mathrm{N}\backslash \mathrm{S})/\mathrm{N},\ d\colon (\mathrm{N}\backslash \mathrm{S})\backslash(\mathrm{N}\backslash \mathrm{S}) \ \Rightarrow (b{+}c, d)\colon \mathrm{S}{\uparrow}\mathrm{N}}{\uparrow}R$$

This gives the prosodic form *that*+(*Mary*+*met*, *yesterday*), where the extraction domain is marked off, and partitioned at the extraction site. We shall later see a similar domain effect relating to prosodic phrasing and islandhood of relative clauses and other domains. The partitioning at the extraction site is also interesting, in relation to "wanna' contraction' phenomena showing that phonological processes are interrupted at extraction sites. Nevertheless later chapters pursue an alternative line on relativisation, one offering greater sensitivity with respect to island phenomena.

We now give the Fitch-style proof theory. The operation for + has the only associative constructor, and we represent it omitting parentheses. There are the following prosodic term label equations:

(56)
$$((\alpha, \gamma)W\beta) = \alpha{+}\beta{+}\gamma$$
$$\alpha{+}\epsilon = \epsilon{+}\alpha = \alpha$$

The lexical assignment, subderivation hypotheses, and term rewriting rules are as usual:

(57) a. $n.\quad \alpha - \phi\colon A$ for any lexical entry

b.
$$\begin{array}{ll|ll} n. & & a_1 - x_1\colon A_1 & \mathrm{H} \\ & & \vdots & \vdots \\ n+m. & & a_m - x_m\colon A_n & \mathrm{H} \\ \end{array}$$

c.
$$n.\ \ \cfrac{\alpha - \phi\colon A}{\alpha' - \phi'\colon A} \ = n, \text{if } \alpha = \alpha' \ \&\ \phi = \phi'$$

The logical rules are as follows.

(58) a.
$$\begin{array}{ll} n. & \alpha - \phi{:}\, A \\ m. & \underline{\gamma - \chi{:}\, A\backslash B} \\ & \alpha{+}\gamma - (\chi\ \phi){:}\, B \quad \mathrm{E}\backslash\ n, m \end{array}$$

b.
$$\begin{array}{lll} n. & \left| \underline{a - x{:}\, A} \right. & \mathrm{H} \\ m. & \left| a{+}\gamma - \psi{:}\, B \right. & \text{unique } a \text{ as indicated} \\ & \gamma - \lambda x\psi{:}\, A\backslash B & \mathrm{I}\backslash\ n, m \end{array}$$

(59) a.
$$\begin{array}{ll} n. & \alpha - \phi{:}\, A \\ m. & \underline{\gamma - \chi{:}\, B/A} \\ & \gamma{+}\alpha - (\chi\ \phi){:}\, B \quad \mathrm{E}/\ n, m \end{array}$$

b.
$$\begin{array}{lll} n. & \left| \underline{a - x{:}\, A} \right. & \mathrm{H} \\ m. & \left| \gamma{+}a - \psi{:}\, B \right. & \text{unique } a \text{ as indicated} \\ & \gamma - \lambda x\psi{:}\, B/A & \mathrm{I}/\ n, m \end{array}$$

(60) a.
$$\begin{array}{lll} n. & \gamma - \chi{:}\, A{\cdot}B & \\ m. & \left| a - x{:}\, A \right. & \mathrm{H} \\ m+1. & \left| \underline{b - y{:}\, B} \right. & \mathrm{H} \\ p. & \left| \delta[a{+}b] - \omega[x, y]{:}\, D \right. & \text{unique } a \text{ and } b \text{ as indicated} \\ & \delta[\gamma] - \omega[\pi_1\chi, \pi_2\chi]{:}\, D & \mathrm{E}{\cdot}\ n, m, m+1, p \end{array}$$

b.
$$\begin{array}{ll} n. & \alpha - \phi{:}\, A \\ m. & \underline{\beta - \psi{:}\, B} \\ & \alpha{+}\beta - (\phi, \psi){:}\, A{\cdot}B \quad \mathrm{I}{\cdot}\ n, m \end{array}$$

(61) a.
$$\begin{array}{ll} n. & \alpha - \phi{:}\, A \\ m. & \underline{\gamma - \chi{:}\, A{>}B} \\ & (\alpha, \gamma) - (\chi\ \phi){:}\, B \quad \mathrm{E}{>}\ n, m \end{array}$$

b.
$$\begin{array}{lll} n. & \left| \underline{a - x{:}\, A} \right. & \mathrm{H} \\ m. & \left| (a, \gamma) - \psi{:}\, B \right. & \text{unique } a \text{ as indicated} \\ & \gamma - \lambda x\psi{:}\, A{>}B & \mathrm{I}{>}\ n, m \end{array}$$

(62) a.
$$\begin{array}{ll} n. & \alpha - \phi{:}\, A \\ m. & \underline{\gamma - \chi{:}\, B{<}A} \\ & (\gamma, \alpha) - (\chi\ \phi){:}\, B \quad \mathrm{E}{<}\ n, m \end{array}$$

b.
$$\begin{array}{lll} n. & \left| \underline{a - x{:}\, A} \right. & \mathrm{H} \\ m. & \left| (\gamma, a) - \psi{:}\, B \right. & \text{unique } a \text{ as indicated} \\ & \gamma - \lambda x\psi{:}\, B{<}A & \mathrm{I}{<}\ n, m \end{array}$$

(63) a.

$$\begin{array}{lll}
n. & \gamma - \chi\colon A{\diamond}B & \\
m. & \quad| \; a - x\colon A & \text{H} \\
m+1. & \quad| \; \underline{b - y\colon B} & \text{H} \\
p. & \quad| \; \delta[(a,b)] - \omega[x,y]\colon D & \text{unique } a \text{ and } b \text{ as indicated} \\
 & \delta[\gamma] - \omega[\pi_1\chi, \pi_2\chi]\colon D & \text{E}{\diamond}\; n, m, m+1, p
\end{array}$$

b.

$$\begin{array}{ll}
n. & \alpha - \phi\colon A \\
m. & \underline{\beta - \psi\colon B} \\
 & (\alpha,\beta) - (\phi,\psi)\colon A{\diamond}B \quad \text{I}{\diamond}\; n, m
\end{array}$$

(64) a.

$$\begin{array}{ll}
n. & \alpha - \phi\colon A \\
m. & \underline{\gamma - \chi\colon A{\downarrow}B} \\
 & (\alpha W \gamma) - (\chi\ \phi)\colon B \quad \text{E}{\downarrow}\; n, m
\end{array}$$

b.

$$\begin{array}{lll}
n. & | \; \underline{a - x\colon A} & \text{H} \\
m. & | \; (aW\gamma) - \psi\colon B & \text{unique } a \text{ as indicated} \\
 & \gamma - \lambda x\psi\colon A{\downarrow}B & \text{I}{\downarrow}\; n, m
\end{array}$$

(65) a.

$$\begin{array}{ll}
n. & \alpha - \phi\colon A \\
m. & \underline{\gamma - \chi\colon B{\uparrow}A} \\
 & (\gamma W \alpha) - (\chi\ \phi)\colon B \quad \text{E}{\uparrow}\; n, m
\end{array}$$

b.

$$\begin{array}{lll}
n. & | \; \underline{a - x\colon A} & \text{H} \\
m. & | \; (\gamma W a) - \psi\colon B & \text{unique } a \text{ as indicated} \\
 & \gamma - \lambda x\psi\colon B{\uparrow}A & \text{I}{\uparrow}\; n, m
\end{array}$$

(66) a.

$$\begin{array}{lll}
n. & \gamma - \chi\colon A{\odot}B & \\
m. & \quad| \; a - x\colon A & \text{H} \\
m+1. & \quad| \; \underline{b - y\colon B} & \text{H} \\
p. & \quad| \; \delta[(aWb)] - \omega[x,y]\colon D & \text{unique } a \text{ and } b \text{ as indicated} \\
 & \delta[\gamma] - \omega[\pi_1\chi, \pi_2\chi]\colon D & \text{E}{\odot}\; n, m, m+1, p
\end{array}$$

b.

$$\begin{array}{ll}
n. & \alpha - \phi\colon A \\
m. & \underline{\beta - \psi\colon B} \\
 & (\alpha W \beta) - (\phi,\psi)\colon A{\odot}B \quad \text{I}{\odot}\; n, m
\end{array}$$

The examples in the next subsections are derived using this format.

| | | |
|---|---|---|
| 1. | $(rang, up)$ – **phone**: $(N\backslash S)\uparrow N$ | |
| 2. | *John* – **j**: N | |
| 3. | *Mary* – **m**: N | |
| 4. | $((rang, up)W John)$ – (**phone j**): $N\backslash S$ | 1, 2 E↑ |
| 5. | *rang+John+up* – (**phone j**): $N\backslash S$ | = 4 |
| 6. | *Mary+rang+John+up* – ((**phone j**) **m**): S | 3, 5 E\ |

FIGURE 4.4. Derivation of 'Mary rang John up'

| | | |
|---|---|---|
| 1. | $(gave, the+cold+shoulder)$ – **give-tcs**: $(N\backslash S)\uparrow N$ | |
| 2. | *John* – **j**: N | |
| 3. | *Mary* – **m**: N | |
| 4. | $((gave, the+cold+shoulder)W John)$ – (**give-tcs j**): $N\backslash S$ | 1, 2 E↑ |
| 5. | *gave+John+the+cold+shoulder* – (**give-tcs j**): $N\backslash S$ | = 4 |
| 6. | *Mary+gave+John+the+cold+shoulder* – ((**give-tcs j**) **m**): S | 3, 5 E\ |

FIGURE 4.5. Derivation of 'Mary gave John the cold shoulder'

### 3.1. *Discontinuous Functors*

Consider the following.

(67) a. Mary *rang* John *up*.
b. Mary *gave* John *the cold shoulder*.
c. Mary *both/either/neither* walks *and/or/nor* talks.

Each case involves a discontinuous functor shown in italics. The example 'Mary rang John up' is derived as shown in Figure 4.4. The particle verb has a complex lexical form constructed out of the splitting adjunction, and its lexical type is that of a wrapping functor. After combination by wrapping application with the object at line 3, prosodic evaluation gives the discontinuous word order.

A discontinuous idiom construction such as 'Mary gave John the cold shoulder' is treated in exactly the same way; see Figure 4.5.

1. *Mary* – $\mathbf{m}$: N
2. $(neither, nor)$ – $\lambda x \lambda y \lambda z \neg[(x\ z) \vee (y\ z)]$
   : $((\mathrm{N}\backslash\mathrm{S})/(\mathrm{N}\backslash\mathrm{S}))\uparrow(\mathrm{N}\backslash\mathrm{S})$
3. *walks* – **walk**: N\S
4. *talks* – **talk**: N\S
5. *neither+walks+nor* – 2, 3 E↑
   $\lambda y \lambda z \neg[(\mathbf{walk}\ z) \vee (y\ z)]$: $(\mathrm{N}\backslash\mathrm{S})/(\mathrm{N}\backslash\mathrm{S})$
6. *neither+walks+nor+talks* – 4, 5 E/
   $\lambda z \neg[(\mathbf{walk}\ z) \vee (\mathbf{talk}\ z)]$: N\S
7. *Mary+neither+walks+nor+talks* – 1, 6 E\
   $\neg[(\mathbf{walk}\ \mathbf{m}) \vee (\mathbf{talk}\ \mathbf{m})]$: S

FIGURE 4.6. Derivation of 'Mary neither walks nor talks'

Discontinuous coordination particles can be treated as functors which combine with their left conjunct by wrap, and then their right by regular division, as shown in Figure 4.6. Here and henceforth we sometimes allow ourselves the liberty of performing label manipulations implicitly within derivation steps.

### 3.2. *Quantifier Raising*

In Moortgat (1990a) a binary "binder" operator which we write here as $\Uparrow$ is defined for which the rule of use is essentially quantifying-in, so that a Montagovian treatment of quantifier-scoping is achieved by assignment of a quantifier phrase like 'something' to $\mathrm{N}\Uparrow\mathrm{S}$, and assignment of determiners like 'every' to $(\mathrm{N}\Uparrow\mathrm{S})/\mathrm{CN}$. As we already noted, in Moortgat (1991) it is suggested that a category such as $A\Uparrow B$ might be definable (in our notation) as $(B\uparrow A)\downarrow B$, but Moortgat observed that this definability does not hold for the unimodal interpretation given, for which furthermore, the logic is problematic in ways we have already considered. Moortgat's intuitions, however, are fully realised in the present trimodal formulation. The category $(\mathrm{S}\uparrow\mathrm{N})\downarrow\mathrm{S}$ is a suitable category for a quantifier phrase such as 'everything' or 'some man', characterising sentential quantifier scope, and quantificational ambiguity. Consider first 'Every man walks' as in Figure 4.7. The generation up to line 5 of 'every man' with the standard semantics, and type $(\mathrm{S}\uparrow\mathrm{N})\downarrow\mathrm{S}$ is straightforward. In

| | | |
|---|---|---|
| 1. | $every$ – $\lambda x\lambda y\forall z[(x\ z) \to (y\ z)]$: $((S{\uparrow}N){\downarrow}S)/CN$ | |
| 2. | $man$ – **man**: CN | |
| 3. | $walks$ – **walk**: N\S | |
| 4. | $every{+}man$ – $(\lambda x\lambda y\forall z[(x\ z) \to (y\ z)]\ \mathbf{man})$: $(S{\uparrow}N){\downarrow}S$ | E/ 1, 2 |
| 5. | $every{+}man$ – $\lambda y\forall z[(\mathbf{man}\ z) \to (y\ z)]$: $(S{\uparrow}N){\downarrow}S$ | = 4 |
| 6. | $a$ – $x$: N | H |
| 7. | $a{+}walks$ – $(\mathbf{walk}\ x)$: S | E\ 3, 6 |
| 8. | $\epsilon{+}a{+}walks$ – $(\mathbf{walk}\ x)$: S | = 7 |
| 9. | $((\epsilon, walks)Wa)$ – $(\mathbf{walk}\ x)$: S | = 8 |
| 10. | $(\epsilon, walks)$ – $\lambda x(\mathbf{walk}\ x)$: $S{\uparrow}N$ | I↑ 6, 9 |
| 11. | $((\epsilon, walks)W\,every{+}man)$ – | |
| | $(\lambda y\forall z[(\mathbf{man}\ z) \to (y\ z)]\ \lambda x(\mathbf{walk}\ x))$: S | E↓ 5, 10 |
| 12. | $\epsilon{+}every{+}man{+}walks$ – $\forall z[(\mathbf{man}\ z) \to (\mathbf{walk}\ z)]$: S | = 11 |
| 13. | $every{+}man{+}walks$ – $\forall z[(\mathbf{man}\ z) \to (\mathbf{walk}\ z)]$: S | = 12 |

FIGURE 4.7. Derivation of 'Every man walks'

lines 7 to 9 a sentence is constructed on the basis of the nominal $a$ – $x$ hypothesised at line 6. Prosodic equations are used to show that the prosodics can be expressed in a form in which $W$ is the main constructor, and in which $a$ is its right hand operand. The left hand operand is a split string term in which $a$ is to be interpolated. Now because the wrap connective is the divisional residuation with respect to the right hand operand of $W$, this split string term is derivable at line 10 as of the wrap type S↑N, by I↑. Since 'every man' is an infix functor over S↑N, it can apply by E↓ (line 11), and on prosodic evaluation interpolates itself at the position in which the hypothesised nominal was used in the subderivational sentence. Thus the quantifier phrase binds semantically a semantic variable for the position in which it occurs prosodically.

There can be no deviance from this pattern, that is, a quantifier phrase cannot bind the wrong position, for there can be no way that the last line of the relevant subderivation can have the form required for I↑, i.e. $(\alpha Wa)$ – $\phi$ where $a$ – $x$ is the hypothesis, without $\alpha$ being a split string marking the interpolation position for the prosodics that corresponds to semantics $\phi$ in terms of $x$: the equations do not allow anything else. So when a quantifier phrase infixes itself, it will semantically bind the position it occupies prosodically.

| | | |
|---|---|---|
| 1. | $John$ – $\mathbf{j}$: N | |
| 2. | $likes$ – $\mathbf{like}$: $(N\backslash S)/N$ | |
| 3. | $everything$ – $\lambda x \forall y(x\ y)$: $(S{\uparrow}N){\downarrow}S$ | |
| 4. | $a$ – $x$: N | H |
| 5. | $likes{+}a$ – $(\mathbf{like}\ x)$: $N\backslash S$ | 2, 4 E/ |
| 6. | $John{+}likes{+}a$ – $((\mathbf{like}\ x)\ \mathbf{j})$: S | 1, 5 E\ |
| 7. | $John{+}likes{+}a{+}\epsilon$ – $((\mathbf{like}\ x)\ \mathbf{j})$: S | = 6 |
| 8. | $((John{+}likes, \epsilon)Wa)$ – $((\mathbf{like}\ x)\ \mathbf{j})$: S | = 7 |
| 9. | $(John{+}likes, \epsilon)$ – $\lambda x((\mathbf{like}\ x)\ \mathbf{j})$: $S{\uparrow}N$ | 4, 8 I↑ |
| 10. | $((John{+}likes, \epsilon)W\,everything)$ – $(\lambda x \forall y(x\ y)\ \lambda x((\mathbf{like}\ x)\ \mathbf{j}))$: S | 3, 9 E↓ |
| 11. | $John{+}likes{+}everything$ – $\forall y((\mathbf{like}\ y)\ \mathbf{j})$: S | = 10 |

FIGURE 4.8. Derivation of 'John likes everything'

The derivation in Figure 4.8 shows the object position binding of 'John likes everything'.

The next two derivations we consider will deliver the subject wide scope and object wide scope readings of 'Everyone loves something'. The first is these is given in Figure 4.9. A nominal hypothesis for the subject is made at line 4, and another subderivation hypothesis for the object at line 5. Since subderivations are last in first out, the subject position is bound last, that is the subject wide scope reading is obtained. The sentence already with the object quantifier phrase is obtained at line 11 just like 'John likes everything' in the previous example, but the subject is a hypothetical variable not a lexical form, and we have worked nested one level down.

In Figure 4.10 the hypothesis of the wider scope subderivation is used in object position, so that the object wide scope reading is obtained.

In the examples so far the quantifier is peripheral in the sentence and (in associative calculus) a category $(S/N)\backslash S$ could have been used for a quantifier phrase to appear in object position and $S/(N\backslash S)$ for the quantifier phrase to appear in subject position. But further assignments still would be required for a quantifier phrase to appear in sentence-medial positions. Some generality with respect to the latter can be achieved by assuming second-order polymorphic categories (see Emms 1990), but two assignments, one forward-looking and another backward

| | | |
|---|---|---|
| 1. | *everyone* – $\lambda x \forall z[(\mathbf{person}\ z) \rightarrow (x\ z)]$: $(S{\uparrow}N){\downarrow}S$ | |
| 2. | *loves* – $\mathbf{love}$: $(N\backslash S)/N$ | |
| 3. | *something* – $\lambda x \exists w[(\mathbf{thing}\ w) \wedge (x\ w)]$: $(S{\uparrow}N){\downarrow}S$ | |
| 4. | *b* – $y$: N | H |
| 5. | *a* – $x$: N | H |
| 6. | *loves+a* – $(\mathbf{love}\ x)$: $N\backslash S$ | E/ 2, 5 |
| 7. | *b+loves+a* – $((\mathbf{love}\ x)\ y)$: S | E\ 4, 6 |
| 8. | $((b{+}loves, \epsilon)Wa)$ – $((\mathbf{love}\ x)\ y)$: S | = 7 |
| 9. | $(b{+}loves, \epsilon)$ – $\lambda x((\mathbf{love}\ x)\ y)$: $S{\uparrow}N$ | I↑ 5, 8 |
| 10. | $((b{+}loves, \epsilon)W something)$ – $(\lambda x \exists w[(\mathbf{thing}\ w) \wedge (x\ w)]\ \lambda x((\mathbf{love}\ x)\ y))$: S | E↓ 3, 9 |
| 11. | *b+loves+something* – $\exists w[(\mathbf{thing}\ w) \wedge ((\mathbf{love}\ w)\ y)]$: S | = 10 |
| 12. | $((\epsilon, loves{+}something)Wb)$ – $\exists w[(\mathbf{thing}\ w) \wedge ((\mathbf{love}\ w)\ y)]$: S | = 11 |
| 13. | $(\epsilon, loves{+}something)$ – $\lambda y \exists w[(\mathbf{thing}\ w) \wedge ((\mathbf{love}\ w)\ y)]$: $S{\uparrow}N$ | I↑ 4, 12 |
| 14. | *everyone+loves+something* – $\forall z[(\mathbf{person}\ z) \rightarrow \exists w[(\mathbf{thing}\ w) \wedge ((\mathbf{love}\ w)\ z)]]$: S | E↓ 3, 13 |

FIGURE 4.9. Subject wide scope derivation of 'Everyone loves something'

looking are nevertheless uniformly required by all quantifiers. The single assignment we have given allows appearance in all N positions without further ado, and allows all the relative quantifier scopings at S nodes. Thus for the example 'John believes someone walks', the derivation in Figure 4.11 gives the narrow scope, non-specific, quantifier reading, but that in Figure 4.12, the wide scope, specific reading, which involves the quantifier raising to the superordinate sentence, in which it is medial.

### 3.3. *Pied-Piping*

Pied-piping refers to relativisation in which a fronted relative pronoun draws along with it additional material from its extraction site. Compare (68a) and (68b), which are paraphrases.

(68) a. (a girl) John knows the brother of
b. (a girl) the brother of whom John knows

| | | |
|---|---|---|
| 1. | *everyone* – $\lambda x \forall z[(\mathbf{person}\ z) \rightarrow (x\ z)]$: $(S\uparrow N)\downarrow S$ | |
| 2. | *loves* – **love**: $(N\backslash S)/N$ | |
| 3. | *something* – $\lambda x \exists w[(\mathbf{thing}\ w) \wedge (x\ w)]$: $(S\uparrow N)\downarrow S$ | |
| 4. | *a* – $x$: N | H |
| 5. | *b* – $y$: N | H |
| 6. | *loves+a* – $(\mathbf{love}\ x)$: $N\backslash S$ | E/ 2, 4 |
| 7. | *b+loves+a* – $((\mathbf{love}\ x)\ y)$: S | E\ 5, 6 |
| 8. | $((\epsilon, loves{+}a)Wb)$ – $((\mathbf{love}\ x)\ y)$: S | = 7 |
| 9. | $(\epsilon, loves{+}a)$ – $\lambda y((\mathbf{love}\ x)\ y)$: $S\uparrow N$ | I↑ 5, 8 |
| 10. | $((\epsilon, loves{+}a)W\,everyone)$ – $(\lambda x \forall z[(\mathbf{person}\ z) \rightarrow (x\ z)]\ \lambda y((\mathbf{love}\ x)\ y))$: S | E↓ 1, 9 |
| 11. | *everyone+loves+a* – $\forall z[(\mathbf{person}\ z) \rightarrow ((\mathbf{love}\ x)\ z)]$: S | = 10 |
| 12. | $((everyone{+}loves, \epsilon)Wa)$ – $\forall z[(\mathbf{person}\ z) \rightarrow ((\mathbf{love}\ x)\ z)]$: S | = 11 |
| 13. | $(everyone{+}loves, \epsilon)$ – $\lambda x \forall z[(\mathbf{person}\ z) \rightarrow ((\mathbf{love}\ x)\ z)]$: $S\uparrow N$ | I↑ 4, 12 |
| 14. | *everyone+loves+something* – $\exists w[(\mathbf{thing}\ w) \wedge \forall z[(\mathbf{person}\ z) \rightarrow ((\mathbf{love}\ w)\ z)]]$: S | E↓ 3, 13 |

FIGURE 4.10. Object wide scope derivation for 'Everyone loves something'

| | | |
|---|---|---|
| 1. | *John* – $\mathbf{j}$: N | |
| 2. | *believes* – **believe**: $(N\backslash S)/S$ | |
| 3. | *someone* – $\lambda x \exists y(x\ y)$: $(S\uparrow N)\downarrow S$ | |
| 4. | *walks* – **walk**: $N\backslash S$ | |
| 5. | *a* – $x$: N | H |
| 6. | *a+walks* – $(\mathbf{walk}\ x)$: S | E\ 4, 5 |
| 7. | $((\epsilon, walks)Wa)$ – $(\mathbf{walk}\ x)$: S | = 6 |
| 8. | $(\epsilon, walks)$ – $\lambda x(\mathbf{walk}\ x)$: $S\uparrow N$ | I↑ 5, 7 |
| 9. | *someone+walks* – $\exists y(\mathbf{walk}\ y)$: S | E↓ 3, 8 |
| 10. | *believes+someone+walks* – $(\mathbf{believe}\ \exists y(\mathbf{walk}\ y))$: $N\backslash S$ | E/ 2, 9 |
| 11. | *John+believes+someone+walks* – $((\mathbf{believe}\ \exists y(\mathbf{walk}\ y))\ \mathbf{j})$: S | E\ 1, 10 |

FIGURE 4.11. Derivation of non-specific 'John believes someone walks'

| | | |
|---|---|---|
| 1. | *John* – $\mathbf{j}$: N | |
| 2. | *believes* – **believe**: (N\S)/S | |
| 3. | *someone* – $\lambda x \exists y(x\ y)$: (S↑N)↓S | |
| 4. | *walks* – **walk**: N\S | |
| 5. | *a* – $x$: N | H |
| 6. | *a+walks* – (**walk** $x$): S | E\ 4, 5 |
| 7. | *believes+a+walks* – (**believe** (**walk** $x$)): N\S | E/ 2, 6 |
| 8. | *John+believes+a+walks* – ((**believe** (**walk** $x$)) $\mathbf{j}$): S | E\ 7, 1 |
| 9. | ((*John+believes*, *walks*)*Wa*) – ((**believe** (**walk** $x$)) $\mathbf{j}$): S | = 8 |
| 10. | (*John+believes*, *walks*) – $\lambda x$((**believe** (**walk** $x$)) $\mathbf{j}$): S↑N | I↑ 5, 9 |
| 11. | *John+believes+someone+walks* – $\exists y$((**believe** (**walk** $y$)) $\mathbf{j}$): S | E↓ 3, 10 |

FIGURE 4.12. Derivation for specific 'John believes someone walks'

Historically, pied-piping has played a crucial rôle in the promotion of feature percolation and phrase structural approaches (Gazdar, Klein, Pullum and Sag 1985; Pollard and Sag 1987, 1993). Pollard (1988, p.412) for example regards it as exposing a critical inadequacy of categorial grammar:

(69) "Evidently, there is no principled analysis of pied piping in an extended categorial framework like Steedman's without the addition of a feature-passing mechanism for unbounded dependencies."

On the phrase structural view, a relative pronoun introduces information which may percolate up normal constituent structure to endow larger phrases with the relativisation property of occurring fronted and binding a gap of the same category as the entire fronted constituent. Instances in which there is no pied-piping are, convincingly, obtained as the special case where the fronted constituent comprises only the relative pronoun. That is, a single categorisation covers both pied-piping cases such as (70a) and non-pied-piping cases such as (70b).

(70) a. (the contract) the loss of which after so much wrangling John would finally have to pay for
b. (the contract) which John talked about

| | | |
|---|---|---|
| 1. | $\textit{about} - \mathbf{about}\colon \mathrm{PP/N}$ | |
| 2. | $\textit{which} - \lambda x\lambda y\lambda z\lambda w[(z\ w) \wedge (y\ (x\ w))]\colon$ $(\mathrm{PP}{\uparrow}\mathrm{N}){\downarrow}(\mathrm{R/(S/PP)})$ | |
| 3. | $\textit{John} - \mathbf{j}\colon \mathrm{N}$ | |
| 4. | $\textit{talked} - \mathbf{talk}\colon (\mathrm{N}\backslash\mathrm{S})/\mathrm{PP}$ | |
| 5. | $a - x\colon \mathrm{N}$ | H |
| 6. | $\textit{about}{+}a - (\mathbf{about}\ x)\colon \mathrm{PP}$ | 1, 5 E/ |
| 7. | $\textit{about}{+}a{+}\epsilon - (\mathbf{about}\ x)\colon \mathrm{PP}$ | = 6 |
| 8. | $((\textit{about}, \epsilon)Wa) - (\mathbf{about}\ x)\colon \mathrm{PP}$ | = 7 |
| 9. | $(\textit{about}, \epsilon) - \lambda x(\mathbf{about}\ x)\colon \mathrm{PP}{\uparrow}\mathrm{N}$ | 5, 8 I↑ |
| 10. | $((\textit{about}, \epsilon)W\textit{which}) -$ $(\lambda x\lambda y\lambda z\lambda w[(z\ w) \wedge (y\ (x\ w))]\ \lambda x(\mathbf{about}\ x))]\colon$ $\mathrm{R/(S/PP)}$ | 2, 9 E↓ |
| 11. | $\textit{about}{+}\textit{which} -$ $\lambda y\lambda z\lambda w[(z\ w) \wedge (y\ (\mathbf{about}\ w))\colon \mathrm{R/(S/PP)}$ | = 10 |
| 12. | $a - x\colon \mathrm{PP}$ | H |
| 13. | $\textit{talked}{+}a - (\mathbf{talk}\ x)\colon \mathrm{N}\backslash\mathrm{S}$ | 4, 12 E/ |
| 14. | $\textit{John}{+}\textit{talked}{+}a - ((\mathbf{talk}\ x)\ \mathbf{j})\colon \mathrm{S}$ | 3, 13 E\ |
| 15. | $\textit{John}{+}\textit{talked} - \lambda x((\mathbf{talk}\ x)\ \mathbf{j})\colon \mathrm{S/PP}$ | 12, 14 I/ |
| 16. | $\textit{about}{+}\textit{which}{+}\textit{John}{+}\textit{talked} -$ $(\lambda y\lambda z\lambda w[(z\ w) \wedge (y\ (\mathbf{about}\ w))\ \lambda x((\mathbf{talk}\ x)\ \mathbf{j}))\colon \mathrm{R}$ | 11, 15 E/ |
| 17. | $\textit{about}{+}\textit{which}{+}\textit{John}{+}\textit{talked} -$ $\lambda z\lambda w[(z\ w) \wedge ((\mathbf{talk}\ (\mathbf{about}\ w))\ \mathbf{j})]\colon \mathrm{R}$ | = 16 |

FIGURE 4.13. Derivation of 'about which John talked'

In Moortgat (1991) a three-place operator is considered which is like $A{\Uparrow}B$, except that quantifying-in changes the category of the context expression. Morrill (1992b) shows that this enables capture of pied-piping. It follows from the nature of the present proposals that $(B{\uparrow}C){\downarrow}A$ represents the context-changing complicity desired between the discontinuity operators. As a result, the treatment of Morrill (1992b) can be presented in these terms.

As a first example, note how in Figure 4.13 the pied-piping assignment generates 'about which John talked' with the same semantics as 'which John talked about', considered earlier. We make use of an abbreviation R for CN\CN. This example is potentially manageable in any categorial grammar with composition, by assignment of type (PP/N)\(R/(S/PP)) to the relative pronoun. Such assignments are an obvious possibility in the light of Szabolsci (1987) for example, who discusses pied-piping of reflexives, such as to render them direct functors over verbs. Such an assignment must be additional to the regular one, a situation to be improved upon if possible. Nevertheless, a naïve account is reasonably obtained by assuming types R/(S/N), and additional restricted second-order quantified types such as $\forall^2 X \in \{\mathrm{N}, \mathrm{PP}\}((X/\mathrm{N})\backslash(\mathrm{R}/(\mathrm{S}/X)))$ for relative pronouns which pied-pipe (e.g. 'which' but not 'that'). In view of these possibilities then it is unclear why Pollard's objection is voiced so strongly.

In fact the respect in which a naïve Szabolsci-style treatment is truly unsatisfactory, while phrase structural percolation copes naturally, appears not to have been identified by Pollard. The crucial cases are those like (70a) where the relative pronoun is medial in the pied-piped material. Given only basic categorial tools it would need to be arranged by a lexical assignment additional to those above that 'after so much wrangling' modifies 'loss'. For unclear reasons, it is not easy to find highly acceptable examples of the crucially problematic medial pied-piping cases, but see e.g. (71).

(71) (a statue) for the transport of which by rail John would have to pay $10,000

In other cases the pied-piped constituent occupies subject position:

(72) a. (a supermarket) the opening of which by the queen/in June was heralded a moving and historical occasion
b. (a woman) the painting of whom by Matisse fetched a fortune
c. (a boy) the yelling of whom outside could be heard throughout the sermon

If in reality there were no such cases, which would be to say that pied-piping noun phrases always occur right-peripherally in the fronted constituent, a rudimentary treatment like that deriving from Szabolcsi would suffice for categorial grammar. Furthermore all existing phrase structure accounts would be erroneous in that none predict such right-peripherality. Thus for phrase structural approaches there would be "no principled analysis of pied piping" possible without the addition of directional constraints on feature inheritance. However, since we judge the examples in the text to be acceptable, we do not regard this implication as going through.

Our treatment of medial and peripheral pied-piping, in terms of infixing and wrapping, reduces the latter to the former in just the same way as phrase structure grammar feature percolation. There is the derivation in Figure 4.14 for 'the loss of which after so much wrangling John would finally have to pay for', given the relative pronoun assignment at line 4. In addition, this same assignment generates non pied-piping cases, such as 'which John talked about'. Lines 5 to 9 in Figure 4.15 show that the regular relative pronoun category is derivable from the nominal pied-piping one because $(\epsilon, \epsilon) \in D(\mathrm{N}\uparrow\mathrm{N})$. Thus prepositional pied-piping, nominal pied-piping, and no-pied-piping examples are all obtained by assignment to just the following two types:

(73) $(\mathrm{N}\uparrow\mathrm{N})\downarrow(\mathrm{R}/(\mathrm{S}/\mathrm{N}))$
$(\mathrm{PP}\uparrow\mathrm{N})\downarrow(\mathrm{R}/(\mathrm{S}/\mathrm{PP}))$

If $T(\mathrm{PP})$ can be assumed equal to $T(\mathrm{N})$ the typed semantic terms are the same in each case, so all the examples considered are obtained by a single restricted second-order quantification assignment as in (74).

(74) *which* – $\lambda x \lambda y \lambda z \lambda w[(z\ w) \wedge (y\ (x\ w))]$
: $\forall^2 X \in \{\mathrm{N},\ \mathrm{PP}\}((X\uparrow\mathrm{N})\downarrow(\mathrm{R}/(\mathrm{S}/X)))$

For pied-piping of constituents of different semantic types, a more sophisticated polymorphism and lambda calculus typing is required, an issue we shall not go into. Our final observation here is that since the relative pronoun 'that' cannot pied-pipe it should be assigned the regular type:

(75) *that* – $\lambda x \lambda y \lambda z[(y\ z) \wedge (x\ z)]$ : $(\mathrm{CN}\backslash\mathrm{CN})/(\mathrm{S}/\mathrm{N})$

The matter of characterising relativisation, in particular non-peripherality, is resumed and extended in chapters seven and eight.

### 3.4. *Gapping*

We have seen in chapter three how categorial grammar provides possibilities for 'non-constituent' coordination. These constructions are less amenable to the phrase structure and feature percolation approach because of their inconsistency with constituent structure. We consider next a coordination construction which is highly problematic from all perspectives, gapping. It is entirely unclear how feature percolation could engage such a construction; but as we shall see the discontinuity apparatus already presented succeeds in doing so.

| | | |
|---|---|---|
| 1. | *the* – $\lambda x \iota y(x\ y)$: N/CN | |
| 2. | *loss* – **loss**: CN | |
| 3. | *of* – **of**: (CN\CN)/N | |
| 4. | *which* – $\lambda x \lambda y \lambda z \lambda w[(z\ w) \wedge (y\ (x\ w))]$: (N↑N)↓(R/(S/N)) | |
| 5. | *asmw* – **asmw**: CN\CN | |
| 6. | *John* – **j**: N | |
| 7. | *wfhtpf* – **wfhtpf**: (N\S)/N | |
| 8. | │ *a* – $x$: N | H |
| 9. | │ *of*+*a* – (**of** $x$): CN\CN | E/ 3, 8 |
| 10. | │ *loss*+*of*+*a* – ((**of** $x$) **loss**): CN | E\ 2, 9 |
| 11. | │ *loss*+*of*+*a*+*asmw* – (**asmw** ((**of** $x$) **loss**)): CN | E\ 5, 10 |
| 12. | │ *the*+*loss*+*of*+*a*+*asmw* – $\iota y$((**asmw** ((**of** $x$) **loss**)) $y$): N | E/ 1, 11 |
| 13. | │ (*the*+*loss*+*of*, *asmw*)*W a* – $\iota y$((**asmw** ((**of** $x$) **loss**)) $y$): N | = 12 |
| 14. | (*the*+*loss*+*of*, *asmw*) – $\lambda x \iota y$((**asmw** ((**of** $x$) **loss**)) $y$): N↑N | I↑ 8, 13 |
| 15. | *the*+*loss*+*of*+*which*+*asmw* – $\lambda y \lambda z \lambda w[(z\ w) \wedge (y\ \iota u$((**asmw** ((**of** $w$) **loss**)) $u$))]: R/(S/N) | E↓4, 14 |
| 16. | │ *a* – $x$: N | H |
| 17. | │ *wfhtpf*+*a* – (**wfhtpf** $x$): N\S | E/ 7, 16 |
| 18. | │ *John*+*wfhtpf*+*a* – ((**wfhtpf** $x$) **j**): S | E\ 6, 17 |
| 19. | *John*+*wfhtpf* – $\lambda x$((**wfhtpf** $x$) **j**): S/N | I/ 16, 18 |
| 20. | *the*+*loss*+*of*+*which*+*asmw*+*John*+*wfhtpf* – $\lambda z \lambda w[(z\ w) \wedge$ ((**wfhtpf** $\iota u$((**asmw** ((**of** $w$) **loss**)) $u$)) **j**)]: R | E/ 15, 19 |

FIGURE 4.14. Derivation of 'the loss of which after so much wrangling (asmw) John would finally have to pay for (wfhtpf)'

The proposal to be presented here is that of Morrill and Solias (1993). The kind of example considered is:

(76) John studies logic and Charles, phonetics.

Discussion is presented by reference to such a minimal example gapping a transitive verb TV. The construction is characterised by the absence

| | | |
|---|---|---|
| 1. | *which* – $\lambda x \lambda y \lambda z \lambda w[(z\ w) \wedge (y\ (x\ w))]$: $(N{\uparrow}N){\downarrow}(R/(S/N))$ | |
| 2. | *John* – $\mathbf{j}$: N | |
| 3. | *talked* – **talk**: (N\S)/PP | |
| 4. | *about* – **about**: PP/N | |
| 5. | $a$ – $x$: N | H |
| 6. | $(\epsilon,\epsilon)Wa$ – $x$: N | = 5 |
| 7. | $(\epsilon,\epsilon)$ – $\lambda x x$: $N{\uparrow}N$ | I↑ 5, 6 |
| 8. | $((\epsilon,\epsilon)W\,which)$ – $(\lambda x \lambda y \lambda z \lambda w[(z\ w) \wedge (y\ (x\ w))]\ \lambda x x)$: R/(S/N) | E↓ 1, 7 |
| 9. | *which* – $\lambda y \lambda z \lambda w[(z\ w) \wedge (y\ w)]$: R/(S/N) | |
| 10. | $a$ – $x$: N | H |
| 11. | *about+a* – (**about** $x$): PP | E/ 4, 10 |
| 12. | *talked+about+a* – (**talk** (**about** $x$)): N\S | E/ 3, 11 |
| 13. | *John+talked+about+a* – ((**talk** (**about** $x$)) **j**): S | E\ 2, 12 |
| 14. | *John+talked+about* – $\lambda x$((**talk** (**about** $x$)) **j**): S/N | I/ 10, 13 |
| 15. | *which+John+talked+about* – $\lambda z \lambda w[(z\ w) \wedge$ ((**talk** (**about** $w$)) **j**)]: R | E/ 9, 14 |

FIGURE 4.15. Derivation of 'which John talked about' from pied-piping relative pronoun assignment

in the right hand conjunct of a verbal element, the understood semantics of which is provided by a corresponding verbal element in the left hand conjunct. Clearly, instanciations of a coordinator category schema $(X\backslash X)/X$ will not generate gapping. However, the prosodic character of gapping, with a verbal element missing medially after the coordinator, is marked with respect to that of left node raising coordination reduction with an elided verbal element left-peripheral after the coordinator ('John saw Bill today and Mary yesterday'). Accordingly, gapping will be triggered by a distinct coordinator assignment.

The phenomenon receives categorial attention in Steedman (1990). The approach of Steedman aims to reduce gapping to constituent coordination; furthermore it aims to do this using just (a version of) the standard division operators of categorial grammar. This involves special treatment of both the right and the left conjunct.

With respect to the right hand conjunct, the initial problem is to give a categorisation at all. Steedman does this by reference to a constituent formed by the subject and object with the coordinator. This constituent is essentially TV\S but with a feature both blocking ordinary application, and licensing coordination with a left hand conjunct of the same category. The blocking is necessary because 'and Charles, phonetics' is clearly not of category TV\S: 'Studies and Charles, phonetics' is not a sentence. Now, with respect to the left hand conjunct, Steedman invokes a special syntactic and semantic decomposition of 'John studies logic' analysed as S, into TV and TV\S. There is then constituent coordination between TV\S and TV\S. Finally the coordinate structure of category TV\S combines with the TV to give S.

Although this treatment addresses the two problems that any account of gapping must solve, categorisation of the right hand conjunct and access to the verbal semantics in the left hand conjunct, it attempts to do so within a narrow conception of categorial grammar (only division operators) that necessitates invocation of distinctly contrived mechanisms. The radical reconstruals of grammar implicated by this analysis are not necessary given the general framework including discontinuity operators we have set out.

Within the context of categorial grammar we have established, the right hand conjunct is characterisable as S↑TV.[7] It remains to access the understood verbal semantics from the sentence that is the left hand conjunct. In order to recover from the left hand side the information we miss on the right hand side, we would like to say that this information, the category and semantics of the verb, is made available to the coordinator when it combines with the left conjunct. In accordance with the spirit of Steedman's proposal, we can observe that the left hand conjunct contains a part with the category S↑TV of the right hand constituent, but it is discontinuous, being interpolated by TV. But this is precisely what is expressed by the discontinuous product category (S↑TV)⊙TV. Furthermore, an element of such a category has as its semantics a pair the second projection of which is the semantics of the TV, making the verb semantics accessible. Consequently gapping is generated by assignment of 'and' to the category (((S↑TV)⊙TV)\S)/(S↑TV) with semantics $\lambda x \lambda y[(\pi_1 y\ \pi_2 y) \wedge (x\ \pi_2 y)]$; see Figure 4.16.

A slightly different treatment is proposed in Solias (1992, 1993), where

[7] This is not the only possibility; a structural modality (see chapter seven) could be used: S/△TV.

| | | |
|---|---|---|
| 1. | *John* – $\mathbf{j}$: N | |
| 2. | *studies* – $\mathbf{study}$: TV | |
| 3. | *logic* – $\mathbf{logic}$: N | |
| 4. | *and* – $\lambda x \lambda y[(\pi_1 y\ \pi_2 y) \wedge (x\ \pi_2 y)]$: $(((S{\uparrow}TV) \odot TV)\backslash S)/(S{\uparrow}TV)$ | |
| 5. | *Charles* – $\mathbf{c}$: N | |
| 6. | *phonetics* – $\mathbf{phonetics}$: N | |
| 7. | *a* – $x$: TV | H |
| 8. | *a+phonetics* – $(x\ \mathbf{phonetics})$: $N\backslash S$ | 6, 7 E/ |
| 9. | *Charles+a+phonetics* – $((x\ \mathbf{phonetics})\ \mathbf{c})$: S | 5, 8 E\ |
| 10. | $((Charles, phonetics)W a)$ – $((x\ \mathbf{phonetics})\ \mathbf{c})$: S | = 9 |
| 11. | $(Charles, phonetics)$ – $\lambda x((x\ \mathbf{phonetics})\ \mathbf{c})$: $S{\uparrow}TV$ | 7, 10 I↑ |
| 12. | *a* – $x$: TV | H |
| 13. | *a+logic* – $(x\ \mathbf{logic})$: $N\backslash S$ | 3, 12 E/ |
| 14. | *John+a+logic* – $((x\ \mathbf{logic})\ \mathbf{j})$: S | 1, 13 E\ |
| 15. | $((John, logic)W a)$ – $((x\ \mathbf{logic})\ \mathbf{j})$: S | = 14 |
| 16. | $(John, logic)$ – $\lambda x((x\ \mathbf{logic})\ \mathbf{j})$: $S{\uparrow}TV$ | 12, 15 I↑ |
| 17. | $((John, logic)W\, studies)$ – $(\lambda x((x\ \mathbf{logic})\ \mathbf{j}), \mathbf{study})$: $(S{\uparrow}TV) \odot TV$ | 2, 16 I⊙ |
| 18. | *John+studies+logic* – $(\lambda x((x\ \mathbf{logic})\ \mathbf{j}), \mathbf{study})$: $(S{\uparrow}TV) \odot TV$ | = 17 |
| 19. | *and*+(*Charles*, *phonetics*) – $\lambda y[(\pi_1 y\ \pi_2 y) \wedge ((\pi_2 y\ \mathbf{phonetics})\ \mathbf{c})]$: $((S{\uparrow}TV) \odot TV)\backslash S$ | 4, 11 E/ |
| 20. | *John+studies+logic+and*+(*Charles*, *phonetics*) – $(\lambda y[(\pi_1 y\ \pi_2 y) \wedge ((\pi_2 y\ \mathbf{phonetics})\ \mathbf{c})]$ $(\lambda x((x\ \mathbf{logic})\ \mathbf{j}), \mathbf{study}))$: S | 18, 19 E\ |
| 21. | *John+studies+logic+and*+(*Charles*, *phonetics*) – $[((\mathbf{study}\ \mathbf{logic})\ \mathbf{j}) \wedge ((\mathbf{study}\ \mathbf{phonetics})\ \mathbf{c})]$: S | = 20 |

FIGURE 4.16. Derivation of 'John studies logic and Charles, phonetics'

the coordinator is assigned gapping type (77) (but with a tupling i.e. free groupoid, rather than groupoid, understanding of the splitting product).

(77) $(((N{\diamond}N) \odot TV)\backslash S)/(N{\diamond}N)$

The Morrill and Solias type is 'more reactive' than the Solias type in that the former entails the latter, but not vice-versa, because $N{\diamond}N \Rightarrow S{\uparrow}TV$

but not the reverse (the reader is invited to check such results). In fact these are not the only possibilities; the following two types, also entailed by the Morrill and Solias type (but not by the Solias type) are also suitable; they do not stand in an entailment relation to each other.

(78) a. $(((N \circ N) \odot TV) \backslash S)/(S \uparrow TV)$
b. $(((S \uparrow TV) \odot TV) \backslash S)/(N \circ N)$

Thus a range of options are made available.

For generalisation of discontinuity to multiple cases, with a view to multiple gapping data ('John put beer in the freezer and Fred, wine') see Solias (1993), and see Morrill (1993) for generalisation of the discontinuity proposal here.

### 3.5. *Object-Antecedent Reflexivisation*

We turn finally to object-oriented reflexives (the less problematic subject-oriented reflexives are considered in the next chapter). Consider the following paradigm:

(79) a. John shows Mary the book.
b. John shows Mary herself.
c. *John shows herself Mary.

Although perhaps a little strange (79b) is acceptable (we could think of John showing Mary pictures or photos of various people including herself), whereas (79c) is not. In order for reflexivisation to occur in the semantics, it is necessary for a reflexive to combine with, and reflexivise, a predicate, before the predicate applies to the antecedent: the other way round, the antecedent semantics is not accessible for duplication. The facts in (79) are thus precisely the opposite of those expected if surface form is generated by concatenation of the ditransitive verb first with its adjacent reflexive-antecedent complement, and then with its remote reflexive complement. This observation has been taken to motivate a 'head-wrapping' analysis of such verbs (see e.g. Dowty 1979) in which they combine with the surface-form remote complement first, and then 'head-wrap' around the other complement. Here however we are able to avoid such a global reconsideration of ditransitive verbs in response to the specific demands of object-antecedent reflexivisation, and instead instigate the desired combinatorics through the reflexive assignment. Let us assume assignment of a ditransitive such as 'show' as shown in (80b).

| | | |
|---|---|---|
| 1. | *John* – $\mathbf{j}$: N | |
| 2. | *shows* – $\lambda x((\mathbf{show}\ \pi_1 x)\ \pi_2 x)$: $(N\backslash S)/(N{\cdot}N)$ | |
| 3. | *herself* – $\lambda x \lambda y(x\ (y, y))$: $((N\backslash S)/(N{\cdot}N)) > ((N\backslash S)\uparrow N)$ | |
| 4. | *Mary* – $\mathbf{m}$: N | |
| 5. | $(shows, herself)$ – $(\lambda x \lambda y(x\ (y, y))\ \lambda x((\mathbf{show}\ \pi_1 x)\ \pi_2 x))$: $(N\backslash S)\uparrow N$ | 2, 3 E> |
| 6. | $(shows, herself)$ – $\lambda y((\mathbf{show}\ y)\ y)$: $(N\backslash S)\uparrow N$ | = 5 |
| 7. | $((shows, herself) W Mary)$ – $(\lambda y((\mathbf{show}\ y)\ y)\ \mathbf{m})$: $N\backslash S$ | 4, 6 E↑ |
| 8. | *shows+Mary+herself* – $((\mathbf{show}\ \mathbf{m})\ \mathbf{m})$: $N\backslash S$ | = 7 |
| 9. | *John+shows+Mary+herself* – $(((\mathbf{show}\ \mathbf{m})\ \mathbf{m})\ \mathbf{j})$: S | 1, 8 E\ |

FIGURE 4.17. Derivation of 'John showed Mary herself'

This may be considered the lexical assignment, or a consequence of a lexical assignment (80a) with which it is mutually derivable.

| | | | | |
|---|---|---|---|---|
| (80) | a. | *shows* | – | **show** |
| | | | : | $((N\backslash S)/N/N)$ |
| | b. | *shows* | – | $\lambda x((\mathbf{show}\ \pi_1 x)\ \pi_2 x)$ |
| | | | : | $(N\backslash S)/(N{\cdot}N)$ |

Now an object-oriented reflexive can be assigned a category as shown in line 3 of Figure 4.17: i.e. a splitting functor mapping ditransitives into wrapping transitives. This triggers the order of combination required for a compositional analysis, while the acceptable word order rather than the unacceptable *'John shows herself Mary' is generated because the result of applying the reflexive is a wrapping functor.

This instance of object-antecedent reflexivisation is a 'non-pied-piping' example in that the reflexive is an immediate complement of the verb. The examples (81) are more problematic, especially (81c) which involves medial 'pied-piping', for exactly the same reasons as the relativisation pied-piping examples.

(81) a. Bill shows Mary to herself.
b. Bill shows Mary a picture of herself.
c. Bill shows Mary a picture of herself taken in Paris.

However just as for the relativisation, nominal and prepositional pied-piping, with no pied-piping obtained as a special case of the former, can all be obtained by just the assignment (82).

(82) *herself* – $\lambda x \lambda y \lambda z(y\ (z, (x\ z)))$
: $\forall^2 X \in \{\mathrm{N},\ \mathrm{PP}\}(X{\uparrow}\mathrm{N}){\downarrow}(((\mathrm{N}\backslash\mathrm{S})/(\mathrm{N}{\cdot}\mathrm{X})){>}((\mathrm{N}\backslash\mathrm{S}){\uparrow}\mathrm{N}))$

A further assignment for subject-antecedent reflexivisation will be seen in the next chapter. Note that distinct treatments of the subject-antecedent and object-antecedent cases is motivated by the different prosodic realisations in such languages as Dutch, Icelandic, and Norwegian.

## 4. DOMAIN MODALITIES

The proposals of the previous sections address one particular limitation in the expressivity of basic categorial grammar: that with respect to discontinuity. This section turns to another such limitation: that with respect to domains. Various grammatical phenomena characterise domains with certain properties. The most conspicuous hypothesis of domains is that implicated by constituent structure in traditional grammatical treatments. Associative Lambek calculus induces no such structure; non-associative does. However, constituent structure is not the only notion of domain conducive to characterisation, and nor is it always conducive: in relation to certain 'bracketing paradoxes' the notion is quite counterproductive. In this respect the 'constituent-free' associative Lambek calculus seems to offer an interesting alternative to 'constituent-based' grammar, but it remains to reintroduce domains as and when they *are* required. The move towards this is the concern of this section; there is further consideration in chapters five, seven and eight.

In Morrill (1989a, 1990b) it is proposed to extend categorial grammar by adding modal category formulas. Such modality provides one way of formalising domains in grammar; in particular, these works have been concerned with semantic domains of intensionality, i.e. domains of elements which share the point of reference (possible world) at which they are semantically evaluated. This application is propounded in the next chapter. Here we consider technicalities of modal categorial grammar in general.

The categorial language is modalised by including modal operators: if $A$ is a category formula, $A$ and $\Diamond A$ are category formulas. Interpretation is relativised Kripke-style to points in a set $I$ on which an accessibility relation $R$ is defined. Excluding the semantic dimension for the time being, a category formula $A$ now has an interpretation as a set $D(A)^i$ of prosodic objects relative to each point $i$ in $I$. (Alternatively viewed, each category formula has a single absolute interpretation as a set of pairings of points and prosodic objects.) Where an interpretation function maps atomic category formulas to sets of prosodic objects for each $i$, modal formulas are interpreted as follows, where $iRj$ signifies that $i$ is accessible from $j$.

$$(83)\quad \begin{array}{lcl} D(A)^i & = & \{s|\forall j, iRj \rightarrow s \in D(A)^j\} \\ D(\Diamond A)^i & = & \{s|\exists j, iRi \wedge s \in D(A)^j\} \end{array}$$

The interpretation of formulas obtained by other connectives is fixed point-wise:

$$(84)\quad \begin{array}{lcl} D(A{\cdot}B)^i & = & \{s_1{+}s_2|s_1 \in D(A)^i \wedge s_2 \in D(B)^i\} \\ D(B/A)^i & = & \{s|\forall s' \in D(A)^i, s{+}s' \in D(B)^i\} \\ D(A\backslash B)^i & = & \{s|\forall s' \in D(A)^i, s'{+}s \in D(B)^i\} \end{array}$$

Various modal logics are obtained by setting conditions on the accessibility relation $R$, such as $iRi$ (reflexivity), $iRj{\wedge}jRk{\rightarrow}iRk$ (transitivity), and $iRj{\rightarrow}jRi$ (symmetry). In the modal logic **K** there are no conditions; in **T** reflexivity is added; in **S4** reflexivity and transitivity are imposed; and in **S5** symmetry is also required. Sequent logic for the **K** universal modality is given by the following rule where (as for ! in chapter two) $\Gamma$ denotes sequences $A_1, \ldots, A_n$ of -ed formulas:

$$(85)\quad \frac{\Gamma \Rightarrow A}{\Gamma \Rightarrow A}\mathbf{K}$$

A sequent in modal logic is read as stating that at every index, if the antecedent holds, then the succedent also holds. A sequent for which this is indeed the case is valid. A rule such as (85) is read as stating that if the premise is valid, then the conclusion is valid, i.e. that if at every point the succedent of the premise follows from its antecedent, then at every point the succedent of the conclusion follows from its antecedent. Gentzen-style sequent calculus rules for the universal and existential modalities of **S4** are as follows.

(86) a. $$\frac{\Gamma_1, A, \Gamma_2 \Rightarrow B}{\Gamma_1, A, \Gamma_2 \Rightarrow B}\text{L} \qquad \frac{\Gamma \Rightarrow A}{\Gamma \Rightarrow A}\text{R}$$

b. $$\frac{\Gamma_1, A, \Gamma_2 \Rightarrow \Diamond B}{\Gamma_1, \Diamond A, \Gamma_2 \Rightarrow \Diamond B}\Diamond\text{L} \qquad \frac{\Gamma \Rightarrow A}{\Gamma \Rightarrow \Diamond A}\Diamond\text{R}$$

Giving Gentzen-style sequent logic for **S5** is problematic, but we will see shortly how a certain kind of labelling provides for a formulation.

To gain a feel for modal logic in sequent calculus, observe that in **S4** (and **S5**), $\Diamond A$ ('$A$ is possible') is a consequence of $A$ ('$A$ is the case'), which is a consequence of $A$ ('$A$ is necessary'), but that in general $A$ is not a consequence of $\Diamond A$, and $A$ is not a consequence of $A$. The rule of use for  simply involves removing the box, for example:

(87) $$\cfrac{\cfrac{\text{N} \Rightarrow \text{N} \qquad \cfrac{\cfrac{\text{N} \Rightarrow \text{N}}{\text{N} \Rightarrow \text{N}}\text{L} \quad \text{S} \Rightarrow \text{S}}{\text{N, N}\backslash\text{S} \Rightarrow \text{S}}\backslash\text{L}}{\cfrac{\text{N, (N}\backslash\text{S)/N, N} \Rightarrow \text{S}}{\text{N, (N}\backslash\text{S)/N, N} \Rightarrow \text{S}}\text{L}}/\text{L}}{\text{N, ((N}\backslash\text{S)/N), N} \Rightarrow \text{S}}\text{L}$$

The rule of proof is more restricted. In essence, inference to $A$ requires universally modalised ('necessitated') assumptions. Suppose that some functor takes as argument S. For elements to occur within such an argumental domain their categories must yield universally modalised assumptions; this may be assured by adding an outermost box to each original lexical category.

(88)
$$\dfrac{\dfrac{\dfrac{S \Rightarrow S \qquad N \Rightarrow N}{N, N\backslash S \Rightarrow S}\,\backslash L}{N, N\backslash S \Rightarrow S}\,L \qquad \dfrac{N, ((N\backslash S)/N), N \Rightarrow S}{N, ((N\backslash S)/N), N \Rightarrow S}\,R}{\dfrac{N, (N\backslash S)/S, N, ((N\backslash S)/N), N \Rightarrow S}{N, ((N\backslash S)/S), N, ((N\backslash S)/N), N \Rightarrow S}\,L}\,/L$$

Consider now a relative pronoun category which is a functor over S/N. Analysis of a subject and transitive verb as this argument type is thus:

(89)
$$\dfrac{\dfrac{\dfrac{N \Rightarrow N \qquad N, N\backslash S \Rightarrow S}{N, (N\backslash S)/N, N \Rightarrow S}\,/L}{N, ((N\backslash S)/N), N \Rightarrow S}\,L}{N, ((N\backslash S)/N) \Rightarrow S/N}\,/R$$

However, if the pronoun is meant to bind a position in an embedded modal domain, the derivation fails:

(90)
$$\dfrac{\dfrac{\dfrac{N, N\backslash S \Rightarrow S \qquad \dfrac{}{N, ((N\backslash S)/N), N \Rightarrow S}\,{*R}}{N, (N\backslash S)/S, N, ((N\backslash S)/N), N \Rightarrow S}\,/L}{N, ((N\backslash S)/S), N, ((N\backslash S)/N), N \Rightarrow S}\,L}{N, ((N\backslash S)/S), N, ((N\backslash S)/N) \Rightarrow S/N}\,/R$$

The problem is that the conditionalised N fails to allow inference to S. For this to be allowed, we need N, i.e. the relative pronoun should be a functor over S/N if it is to allow relativisation from the S domain. In this way, binding elements are sensitised to modal domains.

By way of example with respect to existential modality, suppose that a relative pronoun is a functor over S/◇N. Then it will not be able to bind the argument position of a functor over N:

(91)
$$\dfrac{\dfrac{\overline{\Diamond\mathrm{N} \Rightarrow \mathrm{N}}^{\,*} \qquad \mathrm{N},\ \mathrm{N}\backslash\mathrm{S} \Rightarrow \mathrm{S}}{\mathrm{N},\ (\mathrm{N}\backslash\mathrm{S})/\mathrm{N},\ \Diamond\mathrm{N} \Rightarrow \mathrm{S}}/\mathrm{L}}{\mathrm{N},\ (\mathrm{N}\backslash\mathrm{S})/\mathrm{N} \Rightarrow \mathrm{S}/\Diamond\mathrm{N}}/\mathrm{R}$$

To be eligible for binding, such argument positions must be governed by functors over ◇N:

(92)
$$\dfrac{\dfrac{\Diamond\mathrm{N} \Rightarrow \Diamond\mathrm{N} \qquad \dfrac{\mathrm{N} \Rightarrow \mathrm{N} \qquad \mathrm{S} \Rightarrow \mathrm{S}}{\mathrm{N},\ \mathrm{N}\backslash\mathrm{S} \Rightarrow \mathrm{S}}\backslash\mathrm{L}}{\mathrm{N},\ (\mathrm{N}\backslash\mathrm{S})/\Diamond\mathrm{N},\ \Diamond\mathrm{N} \Rightarrow \mathrm{S}}/\mathrm{L}}{\mathrm{N},\ (\mathrm{N}\backslash\mathrm{S})/\Diamond\mathrm{N} \Rightarrow \mathrm{S}/\Diamond\mathrm{N}}/\mathrm{R}$$

(Note that ordinary lexical arguments can occupy diamond argument positions since $A$ yields $\Diamond A$.) Then with a binder a functor over a diamond type, in a language like English where prepositions can be stranded we may categorise them PP/◇N, while in languages without preposition stranding they would be PP/N.

So far we have considered just the prosodic dimension of interpretation of modal category formulas. We shall give two kinds of overall interpretation, these differing with respect to the semantic dimension. The essential step in giving a semantic dimension is to define a type map. In the first, semantically inactive, kind of modality, this is $T(A) = T(\Diamond A) = T(A)$. Then:

(93)
$$\begin{array}{lcl} D(A)^i & = & \{\langle s,m\rangle \mid \forall j, iRj \rightarrow \langle s,m\rangle \in D(A)^j\} \\ D(\Diamond A)^i & = & \{\langle s,m\rangle \mid \exists j, iRj \wedge \langle s,m\rangle \in D(A)^j\} \end{array}$$

In the semantically active version, $T(A) = I{\rightarrow}T(A)$ and $T(\Diamond A) = I{\times}T(A)$, i.e. the semantic value of an object in $A$ is a function from indices into semantic values for $A$, and the semantic value of an object in $\Diamond A$ is a pairing of an index and a semantic value for $A$.

(94) $D(A)^i = \{\langle s, m\rangle | \forall j, iRj \rightarrow \langle s, m(j)\rangle \in D(A)^j\}$
$D(\Diamond A)^i = \{\langle s, \langle j, m\rangle\rangle | iRj \wedge \langle s, m\rangle \in D(A)^j\}$

For the modal logic **S5**, interpretation is particularly simple: because the accessibility relation is universal we can ignore it and just quantify over the set of points. For the semantically active case:

(95) $D(A)^i = \{\langle s, m\rangle | \forall j, \langle s, m(j)\rangle \in D(A)^j\}$
$D(\Diamond A)^i = \{\langle s, \langle j, m\rangle\rangle | \langle s, m\rangle \in D(A)^j\}$

And for the semantically inactive case:

(96) $D(A)^i = \{\langle s, m\rangle | \forall j, \langle s, m\rangle \in D(A)^j\}$
$D(\Diamond A)^i = \{\langle s, m\rangle | \exists j, \langle s, m\rangle \in D(A)^j\}$

The **S5** formulation is particularly attractive in its simplicity, yet as mentioned above presenting sequent logic for **S5** is difficult. We therefore consider here how schematic possible world annotation can be used in the presentation of modal logic (see e.g. Wallen 1990, Mints 1992); this can be seen as a kind of labelled deduction. Hollenberg (1992) shows this formulation of modal categorial grammar for **S4**, in a format simply adaptable to **K** and other model logics, as described by Wallen (1990). For **S5** the annotation is very simple. Each formula in a sequent is labelled with an index variable $(p, q, \ldots)$: $A^p$ refers to the category formula $A$ at index $p$. Theorems are those derivable sequents with all formulas coindexed. Where we include now explicit prosodic labelling, the axiom scheme becomes:

(97) $$\frac{}{a\colon A^p \Rightarrow a\colon A^p}\text{id}$$

The **S5** labelled Gentzen-style rules are:

(98) $$\frac{\Gamma, a\colon A^p \Rightarrow \beta\colon B^q}{\Gamma, a\colon A^r \Rightarrow \beta\colon B^q}\text{L} \qquad \frac{\Gamma \Rightarrow \alpha\colon A^p}{\Gamma \Rightarrow \alpha\colon A^q}\text{R, no } p \text{ in } \Gamma$$

By way of example we have the following derivation of the modal axiom T corresponding to reflexivity of accessibility:

(99) $$\frac{a\colon A^p \Rightarrow a\colon A^p}{a\colon A^p \Rightarrow a\colon A^p}\text{L}$$

But the condition on R blocks the converse:

(100) $$\frac{a\colon A^p \Rightarrow a\colon A^p}{a\colon A^p \Rightarrow a\colon A^p}{*}\mathrm{R}$$

There is also the derivation (101) of the modal axiom 4 corresponding to transitivity of the accessibility relation.

(101) $$\cfrac{\cfrac{\cfrac{a\colon A^p \Rightarrow a\colon A^p}{a\colon A^r \Rightarrow a\colon A^p}\mathrm{L}}{a\colon A^r \Rightarrow a\colon A^q}\mathrm{R}}{a\colon A^r \Rightarrow a\colon A^r}\mathrm{R}$$

Other rules are labelled with indices as follows:

(102) a. $$\frac{\Gamma \Rightarrow \alpha\colon A^p \qquad b\colon B^p, \Delta \Rightarrow \gamma[b]\colon C^q}{\Gamma, d\colon A\backslash B^p, \Delta \Rightarrow \gamma[(\alpha{+}d)]\colon C^q}\backslash\mathrm{L}$$

b. $$\frac{\Gamma, a\colon A^p \Rightarrow (a{+}\gamma)\colon B^p}{\Gamma \Rightarrow \gamma\colon A\backslash B^p}\backslash\mathrm{R}$$

(103) a. $$\frac{\Gamma \Rightarrow \alpha\colon A^p \qquad b\colon B^p, \Delta \Rightarrow \gamma[b]\colon C^q}{\Gamma, d\colon B/A^p, \Delta \Rightarrow \gamma[(d{+}\alpha)]\colon C^q}/\mathrm{L}$$

b. $$\frac{\Gamma, a\colon A^p \Rightarrow (\gamma{+}a)\colon B^p}{\Gamma \Rightarrow \gamma\colon B/A^p}/\mathrm{R}$$

Thus we have:

(104) $$\cfrac{\cfrac{\cfrac{\cfrac{B^q \Rightarrow B^q \qquad A^q \Rightarrow A^q}{A/B^q, B^q \Rightarrow A^q}/\mathrm{L}}{A/B^q, B^r \Rightarrow A^q}\mathrm{L}}{(A/B)^r, B^r \Rightarrow A^q}\mathrm{L}}{(A/B)^r, B^r \Rightarrow A^r}\mathrm{R}$$

But, for instance, (105) is invalid in virtue of the condition on R.

(105)
$$\dfrac{\dfrac{\dfrac{B^r \Rightarrow B^r \qquad A^r \Rightarrow A^r}{A/B^r, B^r \Rightarrow A^r}/\mathrm{L}}{(A/B)^r, B^r \Rightarrow A^r}\mathrm{L}}{(A/B)^r, B^r \Rightarrow A^r}{*}\mathrm{R}$$

Modal calculus is applied to extraction constraints in chapter eight, and to intensionality in the next chapter, where its interaction with quantifier and reflexive binding is addressed. With this introduction to enriched categorial grammar we conclude our initial technical considerations, and move on to the refinement of Montague grammar for which the way has been paved.