Displacement Logic for Grammar

Glyn Morrill & Oriol Valentín

Department of Computer Science
Universitat Politècnica de Catalunya
morrill@cs.upc.edu & oriol.valentin@gmail.com

ESSLLI 2016, Bolzano – Bozen

Lecture 3

From Displacement Calculus to Displacement Logic:
Polymorphism

Displacement *calculus* constitutes the ‘multiplicative’ fragment of a much wider non-commutative intuitionistic sublinear displacement *logic* for categorial grammar.

2nd-Order Displacement Logic

	cont. mult.	disc. mult.	secon. disc. mult.	add.	qu.	norm. mod.	brack. mod.	exp.	limited contr. & weak.
primitive	$/$ $\backslash$ $\bullet$ I	$\uparrow$ $\downarrow$ $\odot$ J	$\Uparrow$ $\Downarrow$ $\circledcirc$ K	$\&$ $\oplus$	$\bigwedge$ $\bigvee$	$\Box$ $\Diamond$	$[\,]^{-1}$ $\langle\rangle$	$!$ $?$	$\mid$ W
sem. inactive variants	$\bullet\!\!-$ $-\!\!\circ$ $\circ\!\!-$ $-\!\!\bullet$ ◐ ◑	$\uparrow\!\bullet$ $\downarrow\!\circ$ $\uparrow\!\circ$ $\downarrow\!\bullet$ ◓ ◒	$\Uparrow\!\bullet$ $\Downarrow\!\circ$ $\Uparrow\!\circ$ $\Downarrow\!\bullet$ $\underline{\underline{◓}}$ $\underline{\underline{◒}}$	$\sqcap$ $\sqcup$	$\forall$ $\exists$	$\blacksquare$ $\blacklozenge$			
det. synth.	$\triangleleft^{-1}$ $\triangleright^{-1}$ $\triangleleft$ $\triangleright$	$\check{}$ $\hat{}$	$\check{\check{}}$ $\hat{\hat{}}$						diff.
nondet. synth.	$\div$ $\times$	$\triangleq$ $\Downarrow$ $\diamond$							$-$

(1st-Order) Displacement Logic

	cont. mult.	disc. mult.	add.	qu.	norm. mod.	brack. mod.	exp.	limited contr. & weak.
primitive	$/$ $\backslash$ $\bullet$ I	$\uparrow$ $\downarrow$ $\odot$ J	$\&$ $\oplus$	$\bigwedge$ $\bigvee$	$\Box$ $\Diamond$	$[\,]^{-1}$ $\langle\rangle$	$!$ $?$	$\mid$ W
sem. inactive variants	•– –∘ ∘– –• ◐ ◑	⫯ ⫰ ⫯ ⫰ ◓ ◒	$\sqcap$ $\sqcup$	$\forall$ $\exists$	$\blacksquare$ $\blacklozenge$			
det. synth.	$\triangleleft^{-1}$ $\triangleright^{-1}$ $\triangleleft$ $\triangleright$	$\check{}$ $\hat{}$						diff.
nondet. synth.	$\div$ $\times$	$\triangleq$ $\Downarrow$ $\diamond$						$-$

Multiplicatives

1. $$\frac{\Gamma \Rightarrow B \qquad \Delta\langle\overrightarrow{C}\rangle \Rightarrow D}{\Delta\langle\overrightarrow{C/B},\Gamma\rangle \Rightarrow D}/L \qquad \frac{\Gamma,\overrightarrow{B} \Rightarrow C}{\Gamma \Rightarrow C/B}/R$$

2. $$\frac{\Gamma \Rightarrow A \qquad \Delta\langle\overrightarrow{C}\rangle \Rightarrow D}{\Delta\langle\Gamma,\overrightarrow{A\backslash C}\rangle \Rightarrow D}\backslash L \qquad \frac{\overrightarrow{A},\Gamma \Rightarrow C}{\Gamma \Rightarrow A\backslash C}\backslash R$$

3. $$\frac{\Delta\langle\overrightarrow{A},\overrightarrow{B}\rangle \Rightarrow D}{\Delta\langle\overrightarrow{A\bullet B}\rangle \Rightarrow D}\bullet L \qquad \frac{\Gamma_1 \Rightarrow A \qquad \Gamma_2 \Rightarrow B}{\Gamma_1,\Gamma_2 \Rightarrow A\bullet B}\bullet R$$

4. $$\frac{\Delta\langle\Lambda\rangle \Rightarrow A}{\Delta\langle\overrightarrow{I}\rangle \Rightarrow A}IL \qquad \frac{}{\Lambda \Rightarrow I}IR$$

5, k.

$$\frac{\Gamma \Rightarrow B \qquad \Delta\langle \overrightarrow{C} \rangle \Rightarrow D}{\Delta\langle \overrightarrow{C\uparrow_k B} \,|_k\, \Gamma \rangle \Rightarrow D}\uparrow_k L \qquad \frac{\Gamma \,|_k\, \overrightarrow{B} \Rightarrow C}{\Gamma \Rightarrow C\uparrow_k B}\uparrow_k R$$

6, k.

$$\frac{\Gamma \Rightarrow A \qquad \Delta\langle \overrightarrow{C} \rangle \Rightarrow D}{\Delta\langle \Gamma \,|_k\, \overrightarrow{A\downarrow_k C} \rangle \Rightarrow D}\downarrow_k L \qquad \frac{\overrightarrow{A} \,|_k\, \Gamma \Rightarrow C}{\Gamma \Rightarrow A\downarrow_k C}\downarrow_k R$$

7, k.

$$\frac{\Delta\langle \overrightarrow{A} \,|_k\, \overrightarrow{B} \rangle \Rightarrow D}{\Delta\langle \overrightarrow{A\odot_k B} \rangle \Rightarrow D}\odot_k L \qquad \frac{\Gamma_1 \Rightarrow A \qquad \Gamma_2 \Rightarrow B}{\Gamma_1 \,|_k\, \Gamma_2 \Rightarrow A\odot_k B}\odot_k R$$

8.

$$\frac{\Delta\langle 1 \rangle \Rightarrow A}{\Delta\langle \overrightarrow{J} \rangle \Rightarrow A} JL \qquad \frac{}{1 \Rightarrow J} JR$$

Additives

$$
\begin{array}{lll}
\mathcal{F}_i ::= \mathcal{F}_i \& \mathcal{F}_i & T(A\&B) = T(A)\&T(B) & \text{additive conjunction} \\
\mathcal{F}_i ::= \mathcal{F}_i \oplus \mathcal{F}_i & T(A\oplus B) = T(A)+T(B) & \text{additive disjunction}
\end{array}
$$

Approximate algebraic interpretation:

$$
\begin{array}{rcl}
[[A\&B]] & = & [[A]] \cap [[B]] \\
[[A\oplus B]] & = & [[A]] \cup [[B]]
\end{array}
$$

9.

$$\frac{\Gamma\langle \overrightarrow{A} \rangle \Rightarrow C}{\Gamma\langle \overrightarrow{A\&B} \rangle \Rightarrow C}\ \&L_1 \qquad \frac{\Gamma\langle \overrightarrow{B} \rangle \Rightarrow C}{\Gamma\langle \overrightarrow{A\&B} \rangle \Rightarrow C}\ \&L_2$$

$$\frac{\Gamma \Rightarrow A \qquad \Gamma \Rightarrow B}{\Gamma \Rightarrow A\&B}\ \&R$$

10.

$$\frac{\Gamma\langle \overrightarrow{A} \rangle \Rightarrow C \qquad \Gamma\langle \overrightarrow{B} \rangle \Rightarrow C}{\Gamma\langle \overrightarrow{A\oplus B} \rangle \Rightarrow C}\ \oplus L$$

$$\frac{\Gamma \Rightarrow A}{\Gamma \Rightarrow A\oplus B}\ \oplus R_1 \qquad \frac{\Gamma \Rightarrow B}{\Gamma \Rightarrow A\oplus B}\ \oplus R_2$$

Quantifiers

$$\mathcal{F}_i ::= \bigwedge x \mathcal{F}_i \quad T(\bigwedge xA) = F \rightarrow T(A) \quad \text{universal quantifier}$$
$$\mathcal{F}_i ::= \bigvee \mathcal{F}_i \quad T(\bigvee xA) = F \& T(A) \quad \text{existential quantifier}$$

Approximate algebraic interpretation:

$$[[\bigwedge A]] = \bigcap_t [[A\{t/x\}]]$$
$$[[\bigvee A]] = \bigcup_t [[A\{t/x\}]]$$

11. $$\frac{\Gamma\langle\overrightarrow{A[t/v]}\rangle \Rightarrow B}{\Gamma\langle\overrightarrow{\bigwedge vA}\rangle \Rightarrow B}\bigwedge L \qquad \frac{\Gamma \Rightarrow A[a/v]}{\Gamma \Rightarrow \bigwedge vA}\bigwedge R^{\dagger}$$

12. $$\frac{\Gamma\langle\overrightarrow{A[a/v]}\rangle \Rightarrow B}{\Gamma\langle\overrightarrow{\bigvee vA}\rangle \Rightarrow B}\bigvee L^{\dagger} \qquad \frac{\Gamma \Rightarrow A[t/v]}{\Gamma \Rightarrow \bigvee vA}\bigvee R$$

† indicates that there is no *a* in the conclusion

Semantically labelled continuous multiplicatives

1. $$\frac{\Gamma \Rightarrow B: \psi \qquad \Delta\langle \overrightarrow{C}: z\rangle \Rightarrow D: \omega}{\Delta\langle \overrightarrow{C/B}: x, \Gamma\rangle \Rightarrow D: \omega\{(x\ \psi)/z\}}\,/L \qquad \frac{\Gamma, \overrightarrow{B}: y \Rightarrow C: \chi}{\Gamma \Rightarrow C/B: \lambda y \chi}\,/R$$

2. $$\frac{\Gamma \Rightarrow A: \phi \qquad \Delta\langle \overrightarrow{C}: z\rangle \Rightarrow D: \omega}{\Delta\langle \Gamma, \overrightarrow{A\backslash C}: y\rangle \Rightarrow D: \omega\{(y\ \phi)/z\}}\,\backslash L \qquad \frac{\overrightarrow{A}: x, \Gamma \Rightarrow C: \chi}{\Gamma \Rightarrow A\backslash C: \lambda x \chi}\,\backslash R$$

3. $$\frac{\Delta\langle \overrightarrow{A}: x, \overrightarrow{B}: y\rangle \Rightarrow D: \omega}{\Delta\langle \overrightarrow{A \bullet B}: z\rangle \Rightarrow D: \omega\{\pi_1 z/x, \pi_2 z/y\}}\,\bullet L \qquad \frac{\Gamma_1 \Rightarrow A: \phi \qquad \Gamma_2 \Rightarrow B: \psi}{\Gamma_1, \Gamma_2 \Rightarrow A \bullet B: (\phi, \psi)}\,\bullet R$$

4. $$\frac{\Delta\langle \Lambda\rangle \Rightarrow A: \phi}{\Delta\langle \overrightarrow{I}: x\rangle \Rightarrow A: \phi}\,IL \qquad \frac{}{\Lambda \Rightarrow I: 0}\,IR$$

Semantically labelled discontinuous multiplicatives

5.
$$\frac{\Gamma \Rightarrow B: \psi \qquad \Delta\langle \overrightarrow{C}: z\rangle \Rightarrow D: \omega}{\Delta\langle \overrightarrow{C{\uparrow_k}B}: x \,|_k\, \Gamma\rangle \Rightarrow D: \omega\{(x\ \psi)/z\}} \uparrow_k L \qquad \frac{\Gamma \,|_k\, \overrightarrow{B}: y \Rightarrow C: \chi}{\Gamma \Rightarrow C{\uparrow_k}B: \lambda y \chi} \uparrow_k R$$

6.
$$\frac{\Gamma \Rightarrow A: \phi \qquad \Delta\langle \overrightarrow{C}: z\rangle \Rightarrow D: \omega}{\Delta\langle \Gamma \,|_k\, \overrightarrow{A{\downarrow_k}C}: y\rangle \Rightarrow D: \omega\{(y\ \phi)/z\}} \downarrow_k L \qquad \frac{\overrightarrow{A}: x \,|_k\, \Gamma \Rightarrow C: \chi}{\Gamma \Rightarrow A{\downarrow_k}C: \lambda x \chi} \downarrow_k R$$

7.
$$\frac{\Delta\langle \overrightarrow{A}: x \,|_k\, \overrightarrow{B}: y\rangle \Rightarrow D: \omega}{\Delta\langle \overrightarrow{A{\odot_k}B}: z\rangle \Rightarrow D: \omega\{\pi_1 z/x, \pi_2 z/y\}} \odot_k L \qquad \frac{\Gamma_1 \Rightarrow A: \phi \qquad \Gamma_2 \Rightarrow B: \psi}{\Gamma_1 \,|_k\, \Gamma_2 \Rightarrow A{\odot_k}B: (\phi, \psi)} \odot_k R$$

8.
$$\frac{\Delta\langle 1\rangle \Rightarrow A: \phi}{\Delta\langle \overrightarrow{J}: x\rangle \Rightarrow A: \phi} JL \qquad \frac{}{1 \Rightarrow J: 0} JR$$

Semantically labelled additives

9.
$$\frac{\Delta\langle\overrightarrow{A}: x\rangle \Rightarrow C: \chi}{\Delta\langle\overrightarrow{A\&B}: z\rangle \Rightarrow C: \chi\{\pi_1 z/x\}}\&L_1 \qquad \frac{\Delta\langle\overrightarrow{B}: y\rangle \Rightarrow C: \chi}{\Delta\langle\overrightarrow{A\&B}: z\rangle \Rightarrow C: \chi\{\pi_2 z/y\}}\&L_2$$

$$\frac{\Gamma \Rightarrow A: \phi \qquad \Gamma \Rightarrow B: \psi}{\Gamma \Rightarrow A\&B: (\phi, \psi)}\&R$$

10.
$$\frac{\Delta\langle\overrightarrow{A}: x\rangle \Rightarrow C: \chi_1 \qquad \Delta\langle\overrightarrow{B}: y\rangle \Rightarrow C: \chi_2}{\Delta\langle\overrightarrow{A\oplus B}: z\rangle \Rightarrow C: z \rightarrow x.\chi_1; y.\chi_2}\oplus L$$

$$\frac{\Delta \Rightarrow A: \phi}{\Gamma \Rightarrow A\oplus B: \iota_1\phi}\oplus R_1 \qquad \frac{\Gamma \Rightarrow B: \psi}{\Delta \Rightarrow A\oplus B: \iota_2\psi}\oplus R_2$$

The additives of Lambek (1961[2]), Morrill (1990[3]), Kanazawa (1992[1]), have application to polymorphism. For example the additive conjunction & can be used for **rice**: $N\&CN$ as in **rice grows**: S and **the rice grows**: S.

Note the computational advantage of this approach over assuming an empty determiner: if empty operators were allowed they could occur any number of times in any positions.

The additive disjunction $\oplus$ can be used for **is**: $(N\backslash S)/(N\oplus(CN/CN))$ as in **Tully is Cicero**: S and **Tully is humanist**: S.

The additive disjunction can be used together with the continuous unit to express the optionality of a complement as in **eats**: $(N\backslash S)/(N\oplus I)$ for **John eats fish**: S and **John eats**: S.

Note the advantage of this over simply listing intransitive and transitive lexical entries: empirically the latter does not capture the generalisation that in both cases the verb *eats* combines with a subject to the left, and computationally every lexical ambiguity doubles the lexical insertion search space. Appeal to lexical ambiguity is at best a promissory solution, unless there is true ambiguity.

Semantically labelled quantifiers

11. $$\frac{\Delta\langle\overrightarrow{A[t/v]}: x\rangle \Rightarrow B: \psi}{\Delta\langle\overrightarrow{\bigwedge vA}: z\rangle \Rightarrow B: \psi\{(z\ t)/x\}} \bigwedge L \qquad \frac{\Gamma \Rightarrow A[a/v]: \phi}{\Gamma \Rightarrow \bigwedge vA: \lambda v\phi} \bigwedge R^{\dagger}$$

12. $$\frac{\Delta\langle\overrightarrow{A[a/v]}: x\rangle \Rightarrow B: \psi}{\Delta\langle\overrightarrow{\bigvee vA}: z\rangle \Rightarrow B: \psi\{\pi_2 z/x\}} \bigvee L^{\dagger} \qquad \frac{\Gamma \Rightarrow A[t/v]: \phi}{\Gamma \Rightarrow \bigvee vA: (t, \phi)} \bigvee R$$

The quantifiers of Morrill (1994[4]) have application to features. For example, singular and plural number in **sheep**: $\bigwedge nCNn$ for **the sheep grazes**: S and **the sheep graze**: S. And for a past, present or future tense finite sentence complement we can have **said**: $(N\backslash S)/\bigvee tSf(t)$ in **John said Mary walked**: S, **John said Mary walks**: S and **John said Mary will walk**: S.

References

M. Kanazawa.
The Lambek calculus enriched with additional connectives.
Journal of Logic, Language and Information, 1:141–171, 1992.

J. Lambek.
On the Calculus of Syntactic Types.
In Roman Jakobson, editor, *Structure of Language and its Mathematical Aspects, Proceedings of the Symposia in Applied Mathematics XII*, pages 166–178. American Mathematical Society, Providence, Rhode Island, 1961.

Glyn Morrill.
Grammar and Logical Types.
In Martin Stockhof and Leen Torenvliet, editors, *Proceedings of the Seventh Amsterdam Colloquium*, pages 429–450, Amsterdam, 1990. Universiteit van Amsterdam.

Glyn V. Morrill.
Type Logical Grammar: Categorial Logic of Signs.
Kluwer Academic Publishers, Dordrecht, 1994.