Mathematical Logic and Linguistics

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BGSMath Course
Class 8

Linguistic applications of connectives

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Stoups (cf. the linear logic of Girard 2011[3]) (ζ) are stores read as multisets for re-usable (nonlinear) resources which appear at the left of a configuration marked off by a semicolon (when the stoup is empty the semicolon may be omitted).

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A configuration together with a stoup is a *zone* (Ω). The bracket constructor applies not to a configuration alone but to a configuration with a stoup, i.e a zone: reusable resources are specific to their domain.

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\textbf{Stoup} & ::= & \emptyset \mid \textbf{Tp}_0, \textbf{Stoup} \\
\textbf{Config} & ::= & \Lambda \mid \textbf{TreeTerm}, \textbf{Config} \\
\textbf{TreeTerm} & ::= & 1 \mid \textbf{Tp}_0 \mid \textbf{Tp}_{i>0}\{\underbrace{\textbf{Config} : \ldots : \textbf{Config}}_{i\ \textbf{Config}'\mathrm{s}}\} \mid [\textbf{Zone}]
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Where a type A of sort $i > 0$ includes $\alpha_0+1+\alpha_1+\cdots+\alpha_{i-1}+1+\alpha_i$ and $\beta_1 \in \Delta_1, \ldots, \beta_i \in \Delta_i$, $A\{\Delta_1 : \ldots : \Delta_i\}$ contains $\alpha_0+\beta_1+\alpha_1+\cdots+\alpha_{i-1}+\beta_i+\alpha_i$.

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$$\begin{array}{l} sN; 1, 1, (S{\uparrow_1}N){\uparrow_2}N\{N/CN, CN : 1\} = \\ s1, 1, (S{\uparrow_1}N){\uparrow_2}N\{N/CN, CN : 1\} = 3 \end{array}$$

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$$\overrightarrow{A} = \begin{cases} A & \text{if } sA = 0 \\ A\{\underbrace{1 : \ldots : 1}_{sA\ 1\text{'s}}\} & \text{if } sA > 0 \end{cases}$$

Where Γ is a configuration of sort i and $\Delta_1, \ldots, \Delta_i$ are configurations, the *fold* $\Gamma \otimes \langle \Delta_1 : \ldots : \Delta_i \rangle$ is the result of replacing the successive 1's in Γ by $\Delta_1, \ldots, \Delta_i$ respectively;

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i.e. $\Delta\,|_k\,\Gamma$ is the configuration resulting from replacing by Γ the kth separator in Δ.

A semantically labelled sequent is a sequent in which the antecedent types $A_1, \ldots, A_n$ are labelled by distinct variables $x_1, \ldots, x_n$ of types $T(A_1), \ldots, T(A_n)$ respectively, and the succedent type A is labelled by a term of type $T(A)$ with free variables drawn from $x_1, \ldots, x_n$.

The identity rules

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The identity axiom is (a); since we adopt the convention that empty stoups can be omitted, we write (b):

$$a. \quad \frac{}{\emptyset; \overrightarrow{A}: x \Rightarrow A: x}\, id \qquad b. \quad \frac{}{\overrightarrow{A}: x \Rightarrow A: x}\, id$$

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The Cut rule is:

$$\frac{\zeta_1; \Gamma \Rightarrow A \qquad \Omega(\zeta_2; \Delta\langle \overrightarrow{A} \rangle) \Rightarrow B}{\Omega(\zeta_1 \uplus \zeta_2; \Delta\langle \Gamma \rangle) \Rightarrow B}\, Cut$$

Continuous multiplicatives

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1. $$\frac{\zeta_1; \Gamma \Rightarrow B\colon \psi \qquad \zeta_2; \Delta\langle \overrightarrow{C}\colon z\rangle \Rightarrow D\colon \omega}{\zeta_1 \uplus \zeta_2; \Delta\langle \overrightarrow{C/B}\colon x, \Gamma\rangle \Rightarrow D\colon \omega\{(x\ \psi)/z\}}\,/L \qquad \frac{\zeta; \Gamma, \overrightarrow{B}\colon y \Rightarrow C\colon \chi}{\zeta; \Gamma \Rightarrow C/B\colon \lambda y \chi}\,/R$$

2. $$\frac{\zeta_1; \Gamma \Rightarrow A\colon \phi \qquad \zeta_2; \Delta\langle \overrightarrow{C}\colon z\rangle \Rightarrow D\colon \omega}{\zeta_1 \uplus \zeta_2; \Delta\langle \Gamma, \overrightarrow{A\backslash C}\colon y\rangle \Rightarrow D\colon \omega\{(y\ \phi)/z\}}\,\backslash L \qquad \frac{\zeta; \overrightarrow{A}\colon x, \Gamma \Rightarrow C\colon \chi}{\zeta; \Gamma \Rightarrow A\backslash C\colon \lambda x \chi}\,\backslash R$$

3. $$\frac{\zeta; \Delta\langle \overrightarrow{A}\colon x, \overrightarrow{B}\colon y\rangle \Rightarrow D\colon \omega}{\zeta; \Delta\langle \overrightarrow{A \bullet B}\colon z\rangle \Rightarrow D\colon \omega\{\pi_1 z/x, \pi_2 z/y\}}\,\bullet L \qquad \frac{\zeta_1; \Gamma_1 \Rightarrow A\colon \phi \qquad \zeta_2; \Gamma_2 \Rightarrow B\colon \psi}{\zeta_1 \uplus \zeta_2; \Gamma_1, \Gamma_2 \Rightarrow A \bullet B\colon (\phi, \psi)}\,\bullet R$$

4. $$\frac{\zeta; \Delta\langle \Lambda\rangle \Rightarrow A\colon \phi}{\zeta; \Delta\langle \overrightarrow{I}\colon x\rangle \Rightarrow A\colon \phi}\,IL \qquad \frac{}{\Lambda \Rightarrow I\colon 0}\,IR$$

The directional divisions over, $/$, and under, $\backslash$, are exemplified by assignments such as *the*: N/CN for *the man*: N and *sings*: $N\backslash S$ for *John sings*: S, and *loves*: $(N\backslash S)/N$ for *John loves Mary*: S.

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And for *John sings* and *John loves Mary*:

$$\frac{N \Rightarrow N \qquad S \Rightarrow S}{N, N\backslash S \Rightarrow S}\,\backslash L \qquad \frac{N \Rightarrow N \qquad \dfrac{N \Rightarrow N \qquad S \Rightarrow S}{N, N\backslash S \Rightarrow S}\,\backslash L}{N, (N\backslash S)/N, N \Rightarrow S}\,/L$$

The continuous product $\bullet$ is exemplified by a 'small clause' assignment such as *considers*: $(N\backslash S)/(N{\bullet}(CN/CN))$ for *John considers Mary socialist*: S.

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$$

Of course this use of product is not essential: we could just as well have used $((N\backslash S)/(CN/CN))/N$ since in general we have both $A/(C\bullet B) \Rightarrow (A/B)/C$ (currying) and $(A/B)/C \Rightarrow A/(C\bullet B)$ (uncurrying).

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6.
$$\frac{\zeta_1; \Gamma \Rightarrow A\colon \phi \qquad \zeta_2; \Delta\langle \overrightarrow{C}\colon z\rangle \Rightarrow D\colon \omega}{\zeta_1 \uplus \zeta_2; \Delta\langle \Gamma\,|_k\,\overrightarrow{A{\downarrow_k}C}\colon y\rangle \Rightarrow D\colon \omega\{(y\ \phi)/z\}}\ {\downarrow_k}L \qquad \frac{\zeta; \overrightarrow{A}\colon x\,|_k\,\Gamma \Rightarrow C\colon \chi}{\zeta; \Gamma \Rightarrow A{\downarrow_k}C\colon \lambda x \chi}\ {\downarrow_k}R$$

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$$\frac{\zeta; \Delta\langle \overrightarrow{A}\colon x\,|_k\,\overrightarrow{B}\colon y\rangle \Rightarrow D\colon \omega}{\zeta; \Delta\langle \overrightarrow{A{\odot_k}B}\colon z\rangle \Rightarrow D\colon \omega\{\pi_1 z/x, \pi_2 z/y\}}\ {\odot_k}L \qquad \frac{\zeta_1; \Gamma_1 \Rightarrow A\colon \phi \qquad \zeta_2; \Gamma_2 \Rightarrow B\colon \psi}{\zeta_1 \uplus \zeta_2; \Gamma_1\,|_k\,\Gamma_2 \Rightarrow A{\odot_k}B}\ {\odot_k}R$$

8.
$$\frac{\zeta; \Delta\langle 1\rangle \Rightarrow A\colon \phi}{\zeta; \Delta\langle \overrightarrow{J}\colon x\rangle \Rightarrow A\colon \phi}\ JL \qquad \frac{}{1 \Rightarrow J\colon 0}\ JR$$

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Infixation, $\downarrow$, and extraction together are exemplified by a quantifier assignment *everyone*: $(S\uparrow N)\downarrow S$ simulating Montague's S14 quantifying in:

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$$\dfrac{\dfrac{\ldots, N, \ldots \Rightarrow S}{\ldots, 1, \ldots \Rightarrow S\uparrow N}\uparrow R \qquad \dfrac{}{S \Rightarrow S}\,id}{\ldots, (S\uparrow N)\downarrow S, \ldots \Rightarrow S}\downarrow L$$

Circumfixation and discontinuous product, $\odot$, are illustrated in an assignment to a relative pronoun *that*: $(CN\backslash CN)/((S{\uparrow}N)\odot I)$ allowing both peripheral and medial extraction, *that John likes*: $CN\backslash CN$ and *that John saw today*: $CN\backslash CN$:

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$$\frac{\Omega\langle \overrightarrow{A}: x\rangle \Rightarrow C: \chi}{\Omega\langle \overrightarrow{A\&B}: z\rangle \Rightarrow C: \chi\{\pi_1 z/x\}}\&L_1 \qquad \frac{\Omega\langle \overrightarrow{B}: y\rangle \Rightarrow C: \chi}{\Omega\langle \overrightarrow{A\&B}: z\rangle \Rightarrow C: \chi\{\pi_2 z/y\}}\&L_2$$

$$\frac{\Omega \Rightarrow A: \phi \qquad \Omega \Rightarrow B: \psi}{\Omega \Rightarrow A\&B: (\phi, \psi)}\&R$$

10.
$$\frac{\Omega\langle \overrightarrow{A}: x\rangle \Rightarrow C: \chi_1 \qquad \Omega\langle \overrightarrow{B}: y\rangle \Rightarrow C: \chi_2}{\Omega\langle \overrightarrow{A\oplus B}: z\rangle \Rightarrow C: z{-}{>}x.\chi_1; y.\chi_2}\oplus L$$

$$\frac{\Omega \Rightarrow A: \phi}{\Omega \Rightarrow A\oplus B: \iota_1\phi}\oplus R_1 \qquad \frac{\Omega \Rightarrow B: \psi}{\Omega \Rightarrow A\oplus B: \iota_2\psi}\oplus R_2$$

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$$\dfrac{\dfrac{N \Rightarrow N}{N\&CN \Rightarrow N}\&L_1 \qquad S \Rightarrow S}{N\&CN, N\backslash S \Rightarrow S}\backslash L \qquad \dfrac{N/CN, CN, N\backslash S \Rightarrow S}{N/CN, N\&CN, N\backslash S \Rightarrow S}\&L_2$$

The additive disjunction $\oplus$ can be used for *is*: $(N\backslash S)/(N\oplus(CN/CN))$ as in *Tully is Cicero*: S and *Tully is humanist*: S:

$$\dfrac{\dfrac{N \Rightarrow N}{N \Rightarrow N\oplus(CN/CN)}\oplus R_1 \qquad N\backslash S \Rightarrow N\backslash S}{(N\backslash S)/(N\oplus(CN/CN)), N \Rightarrow N\backslash S}/L \qquad \dfrac{\dfrac{CN/CN \Rightarrow CN/CN}{CN/CN \Rightarrow N\oplus(CN/CN)}\oplus R_2 \qquad N\backslash S \Rightarrow N\backslash S}{(N\backslash S)/(N\oplus(CN/CN)), CN/CN \Rightarrow N\backslash S}/L$$

Quantifiers

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The first-order quantifiers $\{\bigwedge, \bigvee\}$ of Morrill (1994[16]), have application to features.

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11. $$\frac{\Omega\langle \overrightarrow{A[t/v]}: x\rangle \Rightarrow B: \psi}{\Omega\langle \overrightarrow{\bigwedge vA}: z\rangle \Rightarrow B: \psi\{(z\ t)/x\}} \bigwedge L \qquad \frac{\Omega \Rightarrow A[a/v]: \phi}{\Omega \Rightarrow \bigwedge vA: \lambda v\phi} \bigwedge R^{\dagger}$$

12. $$\frac{\Omega\langle \overrightarrow{A[a/v]}: x\rangle \Rightarrow B: \psi}{\Omega\langle \overrightarrow{\bigvee vA}: z\rangle \Rightarrow B: \psi\{\pi_2 z/x\}} \bigvee L^{\dagger} \qquad \frac{\Omega \Rightarrow A[t/v]: \phi}{\Omega \Rightarrow \bigvee vA: (t, \phi)} \bigvee R$$

where † indicates that there is no a in the conclusion

For example, we can generalise over singular and plural number in *sheep*: $\bigwedge nCNn$ for *the sheep grazes*: S and *the sheep graze*: S:

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$$\frac{CNsg \Rightarrow CNsg}{\bigwedge nCNn \Rightarrow CNsg} \wedge L \qquad \frac{CNpl \Rightarrow CNpl}{\bigwedge nCNn \Rightarrow CNpl} \wedge L$$

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And we can express a past, present or future tense finite sentence complement: *said*: $(N\backslash S)/\bigvee tSf(t)$ in *John said Mary walked*: S, *John said Mary walks*: S and *John said Mary will walk*: S:

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And we can express a past, present or future tense finite sentence complement: *said*: $(N\backslash S)/\bigvee tSf(t)$ in *John said Mary walked*: S, *John said Mary walks*: S and *John said Mary will walk*: S:

$$\frac{Sf(past) \Rightarrow Sf(past)}{Sf(past) \Rightarrow \bigvee tSf(t)} \bigvee R \qquad \frac{Sf(pres) \Rightarrow Sf(pres)}{Sf(pres) \Rightarrow \bigvee tSf(t)} \bigvee R \qquad \frac{Sf(fut) \Rightarrow Sf(fut)}{Sf(fut) \Rightarrow \bigvee tSf(t)} \bigvee R$$

Normal modalities

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With respect to the normal modalities $\{\Box, \Diamond\}$ of Morrill (1990[11]) and Moortgat (1997[9]), the universal has application to intensionality.

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13. $$\frac{\Omega\langle\overrightarrow{A}: x\rangle \Rightarrow B: \psi}{\Omega\langle\overrightarrow{\Box A}: z\rangle \Rightarrow B: \psi\{^{\vee}z/x\}}\Box L \qquad \frac{\boxtimes\Omega \Rightarrow A: \phi}{\boxtimes\Omega \Rightarrow \Box A: {}^{\wedge}\phi}\Box R$$

14. $$\frac{\boxtimes\Omega\langle\overrightarrow{A}: x\rangle \Rightarrow \diamondplus B: \psi}{\boxtimes\Omega\langle\overrightarrow{\Diamond A}: z\rangle \Rightarrow \diamondplus B: \psi\{^{\cup}z/x\}}\Diamond L \qquad \frac{\Omega \Rightarrow A: \phi}{\Omega \Rightarrow \Diamond A: {}^{\cap}\phi}\Diamond R$$

where $\boxtimes/\diamondplus$ marks a structure all the types of which have principal connective a box/diamond

For example, for a propositional attitude verb we can have an assignment such as *believes*: $\square((N\backslash S)/\square S)$ with a modality outermost since the word has a sense, and its sentential complement is an intensional domain, but its subject is not.

Bracket modalities

Bracket modalities

The bracket modalities $\{[\,]^{-1}, \langle\rangle\}$ of Morrill (1992[12]) and Moortgat (1995[8]), have application to syntactical domains such as islands.

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15. $$\frac{\Omega\langle\overrightarrow{A}: x\rangle \Rightarrow B: \psi}{\Omega\langle[\overrightarrow{[\,]^{-1}A}: x]\rangle \Rightarrow B: \psi}[\,]^{-1}L \qquad \frac{[\Omega] \Rightarrow A: \phi}{\Omega \Rightarrow [\,]^{-1}A: \phi}[\,]^{-1}R$$

16. $$\frac{\Omega\langle[\overrightarrow{A}: x]\rangle \Rightarrow B: \psi}{\Omega\langle\overrightarrow{\langle\rangle A}: x\rangle \Rightarrow B: \psi}\langle\rangle L \qquad \frac{\Omega \Rightarrow A: \phi}{[\Omega] \Rightarrow \langle\rangle A: \phi}\langle\rangle R$$

For example, *walks*: $\langle\rangle N\backslash S$ for the subject condition, and *before*: $[\,]^{-1}(VP\backslash VP)/VP$ for the adverbial island constraint, which are weak islands, and can contain parasitic gaps, see the next section;

For example, *walks*: $\langle\rangle N\backslash S$ for the subject condition, and *before*: $[\,]^{-1}(VP\backslash VP)/VP$ for the adverbial island constraint, which are weak islands, and can contain parasitic gaps, see the next section; for a strong island such as a coordinate structure, which cannot contain a parasitic gap, we define doubly bracketed strong islands — *and*: $(S\backslash[\,]^{-1}[\,]^{-1}S)/S$.

$$\dfrac{\dfrac{N \Rightarrow N}{[N] \Rightarrow \langle\rangle N}\langle\rangle R \qquad S \Rightarrow S}{[N], \langle\rangle N\backslash S \Rightarrow S}\backslash L$$

$$\dfrac{S \Rightarrow S \qquad \dfrac{S \Rightarrow S \qquad \dfrac{\dfrac{S \Rightarrow S}{[[\,]^{-1}S] \Rightarrow S}[\,]^{-1}L}{[[[\,]^{-1}[\,]^{-1}S]] \Rightarrow S}[\,]^{-1}L}{[[S, S\backslash[\,]^{-1}[\,]^{-1}S]] \Rightarrow S}\backslash S}{[[S, (S\backslash[\,]^{-1}[\,]^{-1}S)/S, S]] \Rightarrow S}/S$$

Exponentials

Exponentials

The exponentials $\{!, ?\}$ of Girard (1987[2]), Barry et al. (1991[1]) and Morrill (1994[16]), have application to sharing.

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17.

$$\frac{\Omega(\zeta \uplus \{A\colon x\}; \Gamma_1, \Gamma_2) \Rightarrow B\colon \psi}{\Omega(\zeta; \Gamma_1, !A\colon x, \Gamma_2) \Rightarrow B\colon \psi}\ !L \qquad \frac{\zeta; \Lambda \Rightarrow A\colon \phi}{\zeta; \Lambda \Rightarrow\ !A\colon \phi}\ !R$$

$$\frac{\Omega(\zeta; \Gamma_1, A\colon x, \Gamma_2) \Rightarrow B\colon \psi}{\Omega(\zeta \uplus \{A\colon x\}; \Gamma_1, \Gamma_2) \Rightarrow B\colon \psi}\ !P$$

$$\frac{\Omega(\zeta \uplus \{A\colon x\}; \Gamma_1, [\{A\colon y\}; \Gamma_2], \Gamma_3) \Rightarrow B\colon \psi}{\Omega(\zeta \uplus \{A\colon x\}; \Gamma_1, \Gamma_2, \Gamma_3) \Rightarrow B\colon \psi\{x/y\}}\ !C$$

18.

$$\frac{\Omega \Rightarrow A\colon \phi}{\Omega \Rightarrow\ ?A\colon [\phi]}\ ?R \qquad \frac{\zeta; \Gamma \Rightarrow A\colon \phi \qquad \zeta'; \Delta \Rightarrow\ ?A\colon \psi}{\zeta \uplus \zeta'; \Gamma, \Delta \Rightarrow\ ?A\colon [\phi|\psi]}\ ?M$$

Using the universal exponential, !, we can assign a relative pronoun type *that*: $(CN\backslash CN)/(S/!N)$ allowing parasitic extraction, Morrill (2011[17]), Morrill and Valentín (2015[14]), such as *paper that John filed without reading*: CN, where parasitic gaps can appear only in (weak) islands, but can be iterated in (weak) subislands.

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Using the existential exponential, ?, we can assign a coordinator type *and*: $(?N\backslash N)/N$ allowing iterated coordination as in *John, Bill, Mary and Suzy*: N.

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