Mathematical Logic and Linguistics

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BGSMath Course
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Fields

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- Mathematics

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- Logic

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- Logic
- Linguistics

Mathematics

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Einstein also said: “You shouldn’t worry about your difficulties with mathematics, I assure you that mine are much greater!

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- And science is fragmentary and partial
- e.g. quantum theory and relativity theory are mutually contradictory
- And there is no guarantee that there will ever exist a single Grand Unified scientific world view

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- Aristotle, Frege, Gödel, Gentzen, Tarski, . . .

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- The former, *syntax*, is physical
- the latter, *semantics*, is mental
- Hence language is a massive mind/body phenomenon

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- Linear logic (Girard 1987). Multiplicity. $2H_2 + O_2 \Rightarrow 2H_2O$
- Lambek calculus (Lambek 1958). Temporality. $N/CN, CN, N\backslash S \Rightarrow S$

Syllabus

- Syntactic types; grammar as an intuitionistic sublinear logic. Tree-based hypersequent calculus; absorbing structural rules
- Operations on sets; semantic types. Semantic representation language; higher-order logic as a simply typed lambda-calculus with logical constants
- Rules of grammatical inference; linguistic applications of connectives
- Algebraic and frame semantics (Dosen and Schroder-Heister 1993); Galatos et al. 2007)
- Soundness and completeness

Syllabus (cont.): Syntactic and semantic analyses

- Lexicon
- Initial examples
- The PTQ fragment (Montague 1973)
- Discontinuity (Morrill et al. 2011)
- Relativization
- Coordination

Syllabus (cont.): Computation

- Focusing (Andreoli 1992)
- Cut-elimination (Lambek 1958)
- Count-invariance (van Benthem 1991)

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$$\mathcal{F} \quad := \quad \mathcal{P}$$

$$
\begin{array}{llllll}
\mathcal{F} & ::= & \mathcal{F}/\mathcal{F} & T(C/B) & = & T(B)\rightarrow T(C) & \text{over} \\
\mathcal{F} & ::= & \mathcal{F}\backslash\mathcal{F} & T(A\backslash C) & = & T(A)\rightarrow T(C) & \text{under} \\
\mathcal{F} & ::= & \mathcal{F}\bullet\mathcal{F} & T(A\bullet B) & = & T(A)\&T(B) & \text{continuous product} \\
\mathcal{F} & ::= & I & T(I) & = & \top & \text{continuous unit}
\end{array}
$$

Syntactical interpretation

$$
\begin{aligned}
[[C/B]] &= \{s_1 |\ \forall s_2 \in [[B]], s_1 + s_2 \in [[C]]\} \\
[[A\backslash C]] &= \{s_2 |\ \forall s_1 \in [[A]], s_1 + s_2 \in [[C]]\} \\
[[A \bullet B]] &= \{s_1 + s_2 |\ s_1 \in [[A]]\ \&\ s_2 \in [[B]]\} \\
[[I]] &= \{0\}
\end{aligned}
$$

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$$\mathcal{O} \Rightarrow \mathcal{F}$$

Sequent calculus

The logical rules are as follows, where $\Delta(\Gamma)$ signifies context configuration Δ with a distinguished subconfiguration Γ.

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$$\frac{\Gamma \Rightarrow B \qquad \Delta(C) \Rightarrow D}{\Delta(C/B, \Gamma) \Rightarrow D} /L \qquad \frac{\Gamma, B \Rightarrow C}{\Gamma \Rightarrow C/B} /R$$

$$\frac{\Gamma \Rightarrow A \qquad \Delta(C) \Rightarrow D}{\Delta(\Gamma, A\backslash C) \Rightarrow D} \backslash L \qquad \frac{A, \Gamma \Rightarrow C}{\Gamma \Rightarrow A\backslash C} \backslash R$$

$$\frac{\Delta(A, B) \Rightarrow D}{\Delta(A \bullet B) \Rightarrow D} \bullet L \qquad \frac{\Gamma_1 \Rightarrow A \qquad \Gamma_2 \Rightarrow B}{\Gamma_1, \Gamma_2 \Rightarrow A \bullet B} \bullet R$$

$$\frac{\Delta(\Lambda) \Rightarrow A}{\Delta(I) \Rightarrow A} IL \qquad \frac{}{\Lambda \Rightarrow I} IR$$