Reinforcement Learning

Policy Search: Actor-Critic and Gradient Policy search

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Goal of this lecture

- So far we approximated the value or action-value function using parameters θ (e.g. neural networks)

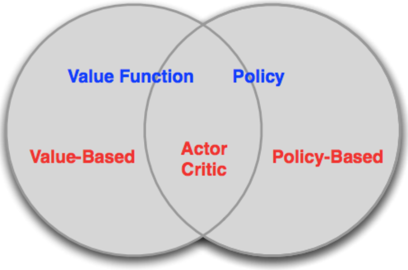
$$\begin{aligned} V_\theta &\approx V^\pi \\ Q_\theta(s,a) &\approx V^\pi(s) \end{aligned}$$

- A policy was generated directly from the value function e.g. using ϵ-greedy
- In this lecture we will directly parameterize the policy in a stochastic setting

$$\pi_\theta(a|s) = P_\theta(a|s)$$

- and do a direct **Policy search**
- Again on model-free setting

Three approaches to RL



Value based learning: **Implicit policy**

- Learn value function $Q_\theta(s, a)$ and from there infer policy $\pi(s) = \arg\max_a Q(s, a)$

Policy based learning: **No value function**

- Explicitly learn policy $\pi_\theta(a|s)$ that implicitly maximize reward over all policies

Actor-Critic learning: **Learn both Value Function and Policy**

Advantges of Policy over Value approach

- Advantages:
  - In some cases, computing Q-values is harder than picking optimal actions
  - **Better convergence properties**
  - **Effective in high dimensional or continuous action spaces**
  - Exploration can be directly controlled
  - **Can learn stochastic policies**
- Disadvantages:
  - Typically converge to a **local optimum** rather than a global optimum
  - Evaluating a policy is typically **data inefficient and high variance**

Stochastic Policies

- In general, two kinds of policies:
  - Deterministic policy
$$a = \pi_\theta(s)$$
  - Stochastic policy
$$P(a|s) = \pi_\theta(a|s)$$
- Nice thing is that they are **smoother** than greedy policies, and so, we can compute **gradients**!
- Not new: ϵ-greedy is stochastic...

Stochastic Policies

- In general, two kinds of policies:
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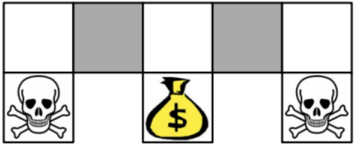
- Nice thing is that they are **smoother** than greedy policies, and so, we can compute **gradients**!
- Not new: ϵ-greedy is stochastic... but different idea. Stochastic policy is good on its own, not because it is an approx. of a greedy policy
- Any example where an stochastic policy could be better than a deterministic one?

Stochastic Policies: Rock-Paper-Scissors

- Two–player game of rock–paper–scissors:
  - Scissors beats paper
  - Rock beats scissors
  - Paper beats rock
- Consider policies for iterated rock–paper–scissors
  - A deterministic policy is easily exploited
  - A uniform random policy is optimal (i.e., Nash equilibrium)

Stochastic Policies when aliased states (POMDPs)



- The agent *cannot differentiate* the grey states
- Consider features of the following form:

$$\phi_d(s) = 1(\text{wall to } d) \quad \forall d \in \{N, E, S, W\}$$

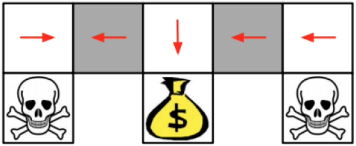
- Compare value-based RL, using an approximate value function

$$Q_\theta(s, a) = f_\theta(\phi(s, a))$$

- To policy-based RL, using a parametrized policy

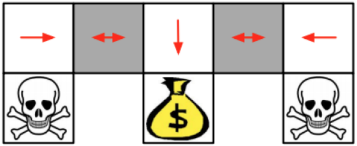
$$\pi_\theta(a|s) = g_\theta(\phi(s, a))$$

Stochastic Policies when aliased states (POMDPs)



- Under aliasing, an optimal deterministic policy will either
  - move W in both gray states
  - move E in both gray states
- Either way, it can get stuck and never reach the money
- So it will be stuck in the corridor for a long time
- Value–based RL learns a deterministic policy (or *near* deterministic when it explores)

Stochastic Policies when aliased states (POMDPs)



- An optimal stochastic policy will randomly move E or W in gray states
  - $\pi_\theta(\text{move E} \mid \text{wall to N and S}) = 0.5$
  - $\pi_\theta(\text{move W} \mid \text{wall to N and S}) = 0.5$
- It will reach the goal state in a few steps with high probability
- Policy–based RL can learn the optimal stochastic policy

Policy optimization

Policy Objective Functions

- Goal: given policy $\pi_\theta(a|s)$ with parameters θ, find best θ
- ... but how do we measure the quality of a policy π_θ?
- In episodic environments we can use the **start value**

$$J_1(\theta) = V^{\pi_\theta}(s_1)$$

Policy Objective Functions

- In continuing environments we can use the **average value**

$$J_{avV}(\theta) = \sum_s d^{\pi_\theta}(s) V^{\pi_\theta}(s)$$

  where $d^{\pi_\theta}(s)$ is stationary distribution of Markov chain for π_θ (can be estimated as the expected number of time steps on s in a randomly generated episode following π_θ divided by time steps of trial)
- Or the **average reward per time-step**

$$J_{avR}(\theta) = \sum_s d^{\pi_\theta}(s) \sum_a \pi_\theta(a|s) r(s,a)$$

- For simplicity, we will mostly discuss the episodic case, but can easily extend to the continuing / infinite horizon case

Policy optimization

- Goal: given policy $\pi_\theta(a|s)$ with parameters θ, find best θ
- Policy based reinforcement learning is an **optimization** problem
- Find policy parameters θ that maximize $J(\theta)$
- Two approaches for solving the optimization problem
  - Gradient-free
  - Policy-gradient

Subsection 1

Gradient Free Policy Optimization

Gradient Free Policy Optimization

- Goal: given parametrized method (with parameters θ) to approximate policy $\pi_\theta(a|s)$, find best values for θ
- Policy based reinforcement learning is an **optimization** problem
- Find policy parameters θ that maximize $J(\theta)$
- Some approaches do not use gradient
  - Hill climbing
  - Simplex / amoeba / Nelder Mead
  - Genetic algorithms
  - Cross-Entropy method (CEM)
  - Covariance Matrix Adaptation (CMA)

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Cross-Entropy Method (CEM)

- A simplified version of Evolutionary algorithm
- Works *embarrassingly* well in some problems, f.i.
  - Playing Tetris (Szita et al., 2006), (Gabillon et al., 2013)
  - A variant of CEM called Covariance Matrix Adaptation has become standard in graphics (Wampler et al., 2009)
- Very simple idea:
  1. From current policy, sample N trials (large)
  2. Take the M trials with larger *long-term return* (we call the **elite**)
  3. Fit new policy to behave as in M best sessions
  4. Repeat until satisfied
- Policy improves gradually

Tabular Cross-Entropy

Tabular Cross-Entropy Algorithm

Given M (f.i, 20), N (f.i. 200)
Initialize matrix policy $\pi(a|s) = A_{s,a}$ randomly
repeat
  Sample N roll-outs of the policy and collect for each R_t
  elite $= M$ best samples
  $$\pi(a|s) = \frac{[\text{times in M samples took a in s}] + \lambda}{[\text{times in M samples was at s}] + \lambda|A|}$$
until convergence
return π

Notice! No value functions!

Tabular Cross-Entropy

Some possible problems and solutions:

- If you were in an state only *once*, you only took *one* action and probabilities become $0/1$
- Solution: Introduction of λ, a parameter to smooth probabilities

Tabular Cross-Entropy

Some possible problems and solutions:

- If you were in an state only *once*, you only took *one* action and probabilities become $0/1$
- Solution: Introduction of λ, a parameter to smooth probabilities
- Due to randomness, algorithm will prefer "lucky" sessions (training on lucky sessions is no good)
- Solution: run several simulations with these state-action pairs and average the results.

Approximated Cross-Entropy Method (CEM)

Approximated Cross-Entropy Method

Given M (f.i, 20), N (f.i. 200) and function approximation (f.i. NN) depending on θ
Initialize θ randomly
repeat
  Sample N roll-outs of the policy and collect for each R_t
  elite = M best samples
  $\theta = \theta + \alpha \nabla \left[\sum_{s,a \in elite} \log \pi_\theta(a|s) \right]$
until convergence
return π_θ

Approximated Cross-Entropy Method (CEM)

- No Value function involved
- Notice that best policy is:

$$\arg\max_{\pi_\theta} \sum_{s,a \in elite} \log \pi_\theta(a|s) = \arg\max_{\pi_\theta} \prod_{s,a \in elite} \pi_\theta(a|s)$$

  so gradient goes in that direction (some theory about Entropy behind)
- Intuitively, is the policy that maximizes similarity with behavior of successful samples
- Tabular case is a particular case of this algorithm
- I promised no gradient, but notice that gradient is for the approximation, not for the rewards of the policy
- Can easily be extended to continuous action spaces (f.i. robotics)

Gradient-Free methods

- Often a great simple baseline to try
- Benefits
  - Can work with any policy parameterizations, including non-differentiable
  - Frequently very easy to parallelize (faster wall-clock *training* time)
- Limitations
  - Typically not very *sample* efficient because it ignores temporal structure

Subsection 2

Policy gradient

Policy gradient methods

- Policy based reinforcement learning is an **optimization** problem
- Find policy parameters θ that maximize V^{π_θ}
- We have seen gradient-free methods, but greater efficiency often possible using gradient in the optimization
- Pletora of methods:
  - Gradient descent
  - Conjugate gradient
  - Quasi-newton
- We focus on *gradient ascent*, many extensions possible
- And on methods that exploit sequential structure

Policy gradient differences wrt Value methods

- With Value functions we use Greedy updates:

$$\theta_{\pi'} = \arg\max_{\theta} \mathbb{E}_{\pi_\theta} \left[Q^\pi(s, a) \right]$$

- $V^{\pi_0} \xrightarrow{\text{small change}} \pi_1 \xrightarrow{\text{large change}} V^{\pi_1} \xrightarrow{\text{small change}} \pi_2 \xrightarrow{\text{large change}} V^{\pi_2}$
- Potentially unstable learning process with large policy jumps because $\arg\max$ is not differentiable
- On the other hand, Policy Gradient updates are:

$$\theta_{\pi'} = \theta_{\pi'} + \alpha \frac{\partial J(\theta)}{\partial \theta}$$

- Stable learning process with smooth policy improvement

Policy gradient method

- Define $J(\theta) = J^{\pi_\theta}$ to make explicit the dependence of the evaluation policy on the policy parameters
- Assume episodic MDPs
- Policy gradient algorithms search for a local **maximum** in $J(\theta)$ by **ascending** the gradient of the policy, w.r.t parameters θ

$$\nabla\theta = \alpha\nabla_\theta J(\theta)$$

- Where $\nabla_\theta J(\theta)$ is the *policy gradient* and α is a step-size parameter

Computing the gradient analytically

- We now compute the policy gradient analytically
- **Assume policy is differentiable whenever it is non-zero**

Computing the gradient analytically

- We now compute the policy gradient analytically
- **Assume policy is differentiable whenever it is non-zero**
- and that we know the gradient $\nabla_\theta \pi_\theta(a|s)$
- Denote a state-action **trajectory** (or trial) τ as

$$\tau = (s_0, a_0, r_1, s_1, a_1, r_2, \ldots s_{T-1}, a_{T-1}, r_T, s_T)$$

- Define long-term-reward to be the sum of rewards for the trajectory $(R(\tau))$

$$R(\tau) = \sum_{t=1}^{T} r(s_t)$$

- It can be discounted or not. Now not important because we will not use Bellman equations.

Computing the gradient analytically

- The value of the policy $J(\theta)$ is:

$$J(\theta) = \mathbb{E}_{\pi_\theta}\left[R(\tau)\right] = \sum_{\tau} P(\tau|\theta)R(\tau)$$

  where $P(\tau|\theta)$ denotes the probability of trajectory τ when following policy π_θ
- Notice that sum is for all possible trajectories
- In this new notation, our goal is to find the policy parameters *theta*) that:

$$\arg\max_{\theta} J(\theta) = \arg\max_{\theta} \sum_{\tau} P(\tau|\theta)R(\tau)$$

[Log-trick: a convenient equality]

- In general, assume we want to compute $\nabla \log f(x)$:

$$\begin{aligned}\nabla \log f(x) &= \frac{1}{f(x)} \nabla f(x) \\ f(x) \nabla \log f(x) &= \nabla f(x)\end{aligned}$$

- It can be applied to any function and we can use the equality in any direction
- The term $\frac{\nabla f(x)}{f(x)}$ is called *likelihood ratio* and is used to analytically compute the gradients
- Btw. Notice the caveat... *Assume policy is differentiable whenever it is non-zero.*

Computing the gradient analytically

- In this new notation, our goal is to find the policy parameters θ that:

$$\arg\max_{\theta} J(\theta) = \arg\max_{\theta} \sum_{\tau} P(\tau|\theta) R(\tau)$$

- So, taken the gradient wrt θ

$$\begin{aligned}
\nabla_\theta J(\theta) &= \nabla_\theta \sum_{\tau} P(\tau|\theta) R(\tau) \\
&= \sum_{\tau} \nabla_\theta P(\tau|\theta) R(\tau) \\
&= \sum_{\tau} \frac{P(\tau|\theta)}{P(\tau|\theta)} \nabla_\theta P(\tau|\theta) R(\tau) \\
&= \sum_{\tau} P(\tau|\theta) R(\tau) \frac{\nabla_\theta P(\tau|\theta)}{P(\tau|\theta)} \\
&= \sum_{\tau} P(\tau|\theta) R(\tau) \nabla_\theta \log P(\tau|\theta)
\end{aligned}$$

Computing the gradient analytically

- Goal is to find the policy parameters θ that:

$$\arg\max_\theta J(\theta) = \arg\max_\theta \sum_\tau P(\tau|\theta)R(\tau)$$

- So, taken the gradient wrt θ

$$\nabla_\theta J(\theta) = \sum_\tau {\color{red}P(\tau|\theta)} R(\tau) \nabla_\theta \log P(\tau|\theta)$$

- Of course we cannot compute all trajectories...

Computing the gradient analytically

- Goal is to find the policy parameters θ that:

$$\arg\max_{\theta} J(\theta) = \arg\max_{\theta} \sum_{\tau} P(\tau|\theta) R(\tau)$$

- So, taken the gradient wrt θ

$$\nabla_{\theta} J(\theta) = \sum_{\tau} {\color{red}P(\tau|\theta)} R(\tau) \nabla_{\theta} \log P(\tau|\theta)$$

- Of course we cannot compute all trajectories...but we can sample m trajectories because of the form of the equation

$$\nabla_{\theta} J(\theta) \approx (1/m) \sum_{i=1}^{m} R(\tau_i) \nabla_{\theta} \log P(\tau_i|\theta)$$

Computing the gradient analytically: at last!

- Sample m trajectories:

$$\nabla_\theta J(\theta) \approx (1/m) \sum_{i=1}^{m} R(\tau_i) \nabla_\theta \log P(\tau_i|\theta)$$

- However, we still have a problem, we don't know the how to compute $\nabla_\theta \log P(\tau|\theta)$
- Fortunately, we can derive it from the stochastic policy

$$\begin{aligned}
\nabla_\theta \log P(\tau|\theta) &= \nabla_\theta \log \left[\mu(s_0) \prod_{i=0}^{T-1} \pi_\theta(a_i|s_i) P(s_{i+1}|s_i, a_i) \right] \\
&= \nabla_\theta \left[\log \mu(s_0) + \sum_{i=0}^{T-1} \log \pi_\theta(a_i|s_i) + \log P(s_{i+1}|s_i, a_i) \right] \\
&= \sum_{i=0}^{T-1} \underbrace{\nabla_\theta \log \pi_\theta(a_i|s_i)}_{\text{No dynamics model required!}}
\end{aligned}$$

Computing the gradient analytically

- We assumed at the beginning that policy is differentiable and that we now the derivative wrt parameters θ
- So, we have the desired solution:

$$\nabla_\theta J(\theta) \approx (1/m) \sum_{i=1}^{m} R(\tau_i) \sum_{i=0}^{T-1} \nabla_\theta \log \pi_\theta(a_i|s_i)$$

Differentiable policies? Soft-max

- One popular way to do action selection instead of using ϵ-greedy is to assign probabilities to actions according to values:

$$\pi(a|s) = \frac{e^{Q(s,a)/\tau}}{\sum_{a'} e^{Q(s,a')/\tau}} \propto e^{Q(s,a)/\tau}$$

where τ is parameter that controls exploration. Let's assume $\tau = 1$

- Let's consider the case where $Q(s,a) = \phi^T(s,a)\theta$ is approximated by a linear function

$$\begin{aligned}
\nabla_\theta \log \pi_\theta(a_i|s_i) &= \nabla_\theta \log \frac{e^{\phi^T(s,a)\theta}}{\sum_{a'} e^{\phi^T(s,a)\theta}} \\
&= \nabla_\theta \;\phi^T(s,a)\theta - \nabla_\theta \sum_{a'} \phi^T(s,a)\theta \\
&= \phi(s,a) - \mathbb{E}_{\pi_\theta}\left[\phi(s,\cdot)\right]
\end{aligned}$$

Differentiable policies? Gaussian Policy

- **In continuous spaces of actions**, action is generated by a random distribution with parameters (f.i. Gaussian distribution)
- Parameter of the Gaussian is a linear combination of feature vector ($\mu = \phi^T(s)\,\theta$). Variance σ can be fixed or approximated.
- This approach allows to consider actions vectors of continuous values (two actions same time!).
- Policy select actions following Gaussian distribution:

$$a \sim \mathcal{N}(\mu_\theta(s), \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(a-\phi^T(s)\theta)^2}{2\sigma^2}}$$

- In this case,

$$\nabla_\theta \log \pi_\theta(a|s) = \nabla_\theta \left[-\frac{(a-\phi^T(s)\,\theta)^2}{2\sigma^2} \right] = \frac{(a-\phi^T(s)\,\theta)\,\phi(s)}{\sigma^2}$$

Differentiable policies? Deep Neural Network

- A very popular way to approximate the policy is to use a Deep NN with soft-max last layer with so many neurons as actions.
- In this case, use *autodiff* of the neural network package you use! In tensorflow:

  ```
  loss = - tf.reduce_mean(tf.log(prob_outputs) * reward)
  ```

  where `prob_outputs` is the output layer of the DNN
- Backpropagation implemented will do the work for you.
- Common approaches:
  - Last *softmax* layer in discrete case
  - Last layer with μ and $\log \sigma$ in continuous case

Vanilla Policy Gradient

Vanilla Policy Gradient

Given architecture with parameters θ to implement π_θ
Initialize θ randomly
repeat
  Generate episode $\{s_1, a_1, r_2, \dots s_{T-1}, a_{T-1}, r_T, s_T\} \sim \pi_\theta$
  Get $R \leftarrow$ long-term return for episode
  for all time steps $t = 1$ to $T - 1$ **do**
    $\theta \leftarrow \theta + \alpha \nabla_\theta \log \pi_\theta(a_t|s_t) R$
  end for
until convergence

Substitute $\nabla_\theta \log \pi_\theta(a_t|s_t)$ with appropriate equation.

Btw, notice no explicit exploration mechanism needed when policies are stochastic (all on policy)!

Vanilla Policy Gradient

- Remember:

$$\nabla_\theta J(\theta) \approx (1/m) \sum_{i=1}^{m} R(\tau_i) \sum_{i=0}^{T-1} \nabla_\theta \log \pi_\theta(a_i|s_i)$$

- Unbiased but very noisy
- Fixes that can make it practical
  - Temporal structure
  - Baseline

Subsection 3

Reduce variance using temporal structure: Reinforce and Actor-Critic architectures

Policy Gradient using Temporal structure

- Instead on focusing on reward of trajectories,

$$J(\theta) = \mathbb{E}_{\pi_\theta}\left[R(\tau)\right] = \sum_{\tau} P(\tau|\theta)R(\tau)$$

- We want to optimize the expected return

$$J_{avV}(\theta) = \sum_{s} d^{\pi_\theta}(s)V(s) = \sum_{s} d^{\pi_\theta} \sum_{a} \pi_\theta(a|s)Q(s,a)$$

  where $d^{\pi_\theta}(s)$ is the expected number of time steps on s in a randomly generated episode following π_θ divided by time steps of trial
- Let's start with and MDP with one single step.

$$J_{avR}(\theta) = \sum_{s} d^{\pi_\theta}(s)V(s) = \sum_{s} d^{\pi_\theta} \sum_{a} \pi_\theta(a|s)r(s,a)$$

Policy Gradient using Temporal structure

$$
\begin{aligned}
J_{avR}(\theta) &= \sum_s d^{\pi_\theta} \sum_a \pi_\theta(a|s) r(s,a) \\
\nabla_\theta J_{avR}(\theta) &= \nabla_\theta \sum_s d^{\pi_\theta} \sum_a \pi_\theta(a|s) r(s,a) \\
&= \sum_s d^{\pi_\theta} \sum_a \nabla_\theta (\pi_\theta(a|s) r(s,a)) \\
&= \sum_s d^{\pi_\theta} \sum_a \nabla_\theta \pi_\theta(a|s) \log \pi_\theta(a|s) r(s,a) \\
&= \mathbb{E}_{\pi_\theta} \left[\nabla_\theta \log \pi_\theta(a|s) r(s,a) \right]
\end{aligned}
$$

- And this expectation can be sampled

Policy Gradient theorem!

- The policy gradient theorem generalize the likelihood ratio approach to multi–step MDPs
- Replaces instantaneous reward r with long–term value $Q(s, a)$
- Policy gradient theorem applies to all objective functions we have seen

Policy gradient theorem

For any differentiable policy $\pi_\theta(s, a)$, for any of the policy objective functions $J = J_1, J_{avR}$ or J_{avV}, the policy gradient is:

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(a|s) Q^{\pi_\theta}(s, a)\right]$$

Simple proof in pag. 325 of (Sutton 2018)

REINFORCE algorithm

- REINFORCE algorithm (also called Monte–Carlo Policy Gradient) use reward R as unbiased sample of $Q^{\pi_\theta}(s, a)$.

REINFORCE algorithm

Given architecture with parameters θ to implement π_θ
Initialize θ randomly
repeat
  Generate episode $\{s_1, a_1, r_2, \ldots s_{T-1}, a_{T-1}, r_T, s_T\} \sim \pi_\theta$
  for all time steps $t = 1$ to $T - 1$ **do**
    Get $R_t \leftarrow$ long-term return from step t to T[a]
    $\theta \leftarrow \theta + \alpha \nabla_\theta \log \pi_\theta(a_t|s_t) R_t$
  end for
until convergence

[a] See proof from Don't Let the Past Distract You if you are not convinced.

REINFORCE algorithm

- Let's analyze the update:

$$\theta \leftarrow \theta + \alpha \nabla_\theta \log \pi_\theta(a_t|s_t) R_t$$

- Let's us rewrite is as follows

$$\theta \leftarrow \theta + \alpha \frac{\nabla_\theta \pi_\theta(a_t|s_t)}{\pi_\theta(a_t|s_t)} R_t$$

- Update is proportional to:
  - the product of a return R_t and
  - the gradient of the probability of taking the action actually taken,
  - divided by the probability of taking that action.

REINFORCE algorithm

- Update:

$$\theta \leftarrow \theta + \alpha \frac{\nabla_\theta \pi_\theta(a_t|s_t)}{\pi_\theta(a_t|s_t)} R_t$$

  - ...move most in the directions that favor actions that yield the highest return
  - ...is inversely proportional to the action probability (actions that are selected frequently are at an advantage (the updates will be more often in their direction))
- Is it necessary to change something in the algorithm for continuous actions?

REINFORCE algorithm

- Update:

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  - ...move most in the directions that favor actions that yield the highest return
  - ...is inversely proportional to the action probability (actions that are selected frequently are at an advantage (the updates will be more often in their direction))
- Is it necessary to change something in the algorithm for continuous actions?
- No! Just uses a continuous action policy mechanism and everything is the same!

REINFORCE algorithm with baseline

- Monte-Carlo policy gradient still has high variance because R_t has a lot of variance
- We can **reduce variance subtracting** a **baseline** to the estimator

$$\theta \leftarrow \theta + \alpha \nabla_\theta \log \pi_\theta(a_t|s_t)(R_t - b(s_t))$$

- without introducing any bias *when baseline does not depend on actions* taken
- A good baseline is $b(s_t) = V^{\pi_\theta}(s_t)$ so we will use that

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- without introducing any bias *when baseline does not depend on actions* taken
- A good baseline is $b(s_t) = V^{\pi_\theta}(s_t)$ so we will use that
- **How to estimate** V^{π_θ}?
- **We'll use another set of parameters** w to approximate

REINFORCE algorithm with baseline

REINFORCE algorithm with baseline (aka MC Actor Critic)

Given architecture with parameters θ to implement π_θ and parameters w to approximate V

Initialize θ randomly

repeat

  Generate episode $\{s_1, a_1, r_2, \dots s_{T-1}, a_{T-1}, r_T, s_T\} \sim \pi_\theta$

  for all time steps $t = 1$ to $T-1$ **do**

    Get $R_t \leftarrow$ long-term return from step t to T

    $\delta \leftarrow R_t - V_w(s_t)$

    $w \leftarrow w + \beta\delta\nabla_w V_w(s_t)$

    $\theta \leftarrow \theta + \alpha\delta\nabla_\theta \log \pi_\theta(a_t|s_t)$

  end for

until convergence

Actor-Critic Architectures

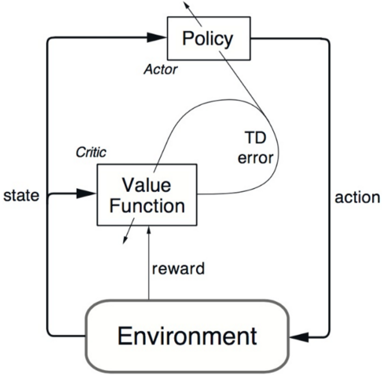
- Monte-Carlo policy gradient has high variance
- So we used a baseline to reduce the variance $R_t - V(s_t)$
- Can we do something to speed up learning like we did with MC using TD?

Actor-Critic Architectures

- Monte-Carlo policy gradient has high variance
- So we used a baseline to reduce the variance $R_t - V(s_t)$
- Can we do something to speed up learning like we did with MC using TD?
- Yes, **use different estimators of R_t that do bootstrapping** f.i. TD(0), n-steps, etc.
- These algorithms are called **Actor Critic**

Actor-Critic Architectures

- The **Critic**, *evaluates the current policy* and the result is used in the policy training
- The **Actor** *implements the policy* and is trained using Policy Gradient with estimations from the critic



Actor-Critic Architectures

- Actor-critic algorithms maintain two sets of parameters (like in REINFORCE with baseline):

  Critic parameters: approximation parameters w for action-value function under current policy

  Actor parameters: policy parameters θ

- Actor-critic algorithms follow an approximate policy gradient:

  Critic: Updates action-value function parameters w like in *policy evaluation* updates (you can apply everything we saw in FA for prediction)

  Actor: Updates policy gradient θ, in direction suggested by critic

Actor-Critic Architectures

- Actor updates are always in the same way:

$$\theta \leftarrow \theta + \alpha \nabla_\theta \log \pi_\theta(a_t|s_t) G_t$$

  where G_t is the evaluation of long-term returned by the critic for s_t

- Critic updates are done to evaluate the current policy

$$w \leftarrow w + \alpha \delta \nabla_\theta V_w(a_t|s_t)$$

  where δ is the estimated error in evaluating the s state and that implements the kind of bootstrapping done.

One step Actor Critic (QAC)

One step actor-critic: $\delta \leftarrow r + Q_w(s', a') - Q_w(s, a)$

One step Actor Critic

Given architecture with parameters θ to implement π_θ and parameters w to approximate Q
Initialize θ randomly
repeat
  Set s to initial state
  Get a from π_θ
  repeat
    Take action a and observe reward r and new state s'
    Get a' from π_θ
    $\delta \leftarrow r + Q_w(s', a') - Q_w(s, a)$ // TD-error (Bellman equation)
    $w \leftarrow w + \beta \delta \nabla_w Q_w(s, a)$ // critic update
    $\theta \leftarrow \theta + \alpha \nabla_\theta \log \pi_\theta(a|s) Q_w(s, a)$ // Actor update
    $s \leftarrow s'$
  until s is terminal
until convergence

Advantage Actor Critic (AAC or A2C)

- In this critic Advantage value function is used:

$$A^{\pi_\theta}(s,a) = Q^{\pi_\theta}(s,a) - V^{\pi_\theta}(s)$$

- The advantage function can significantly reduce variance of policy gradient
- So the critic should really estimate the advantage function, for instance, estimating **both** V(s) and Q using two function approximators and two parameter vectors:

$$V^{\pi_\theta}(s) \approx V_v(s) \quad (1)$$
$$Q^{\pi_\theta}(s,a) \approx Q_w(s,a) \quad (2)$$
$$A(s,a) = Q_w(s,a) - V_v(s) \quad (3)$$

- And updating both value functions by e.g. TD learning
- Nice thing, you only punish policy when not optimal (why?)

From A2C to REINFORCE with baseline

- One way to implement A2C method without two different networks to estimate $Q_w(s, a)$ and $V_v(s)$ is the following.
- For the true value function $V^{\pi_\theta(s)}$, the TD error $\delta^{\pi_\theta(s)}$

$$\delta^{\pi_\theta(s)} = r + \gamma V^{\pi_\theta}(s') - V^{\pi_\theta}(s)$$

- ...that it is an unbiased estimate of the advantage function:

$$\begin{aligned} \mathbb{E}_{\pi_\theta}\left[\delta_\theta^\pi | s, a\right] &= \mathbb{E}_{\pi_\theta}\left[r + \gamma V^{\pi_\theta}(s') | s, a\right] - V^{\pi_\theta}(s) \\ &= Q^{\pi_\theta}(s, a) - V^{\pi_\theta}(s) = A^{\pi_\theta}(s, a) \end{aligned}$$

- So we can use the TD error to compute the policy gradient

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(a|s) \delta_\theta^\pi\right]$$

- In practice this approach only requires one set of critic parameters v to approximate TD error

$$\delta_v = r + \gamma V_v(s') - V_v(s)$$

- Notice this algorithm *resemblance* with REINFORCE with baseline

Asyncrhonous Advantage Actor Critic (A3C)

- A3C (Mnih et al. 2016) idea: Sample for data can be parallelized using several copies of the same agent
  - use N copies of the agents (workers) working in parallel collecting samples and computing gradients for policy and value function
  - After some time, pass gradients to a main network that updates actor and critic using the gradients of all
  - After some time the worker copy the weights of the global network
- This parallelism decorrelates the agents' data, so **no Experience Replay Buffer needed**
- Even one can explicitly use different exploration policies in each actor-learner to maximize diversity
- Asynchronism can be extended to other update mechanisms (Sarsa, Q-learning...) but it works better in Advantage Actor critic setting

Generalized Advantage Estimator (GAE)

- Generalized Advantage Estimator (Schulman et al. 2016). [nice review]
- Use a version of Advantage that consider weighted average of n-steps estimators of advantage like in TD(λ):

$$A^{\pi}_{GAE} = \sum_{t'=t}^{\infty} (\lambda\gamma)^{t'-t} \underbrace{[r_{t'+1} + \gamma V^{\pi}_{\theta}(s_{t'+1}) - V^{\pi}_{\theta}(s_{t'})]}_{\text{t'-step advantage}}$$

- Used in continuous setting for locomotion tasks

Subsection 4

Conclusions and other approaches

Summary

- The policy gradient has many equivalent forms

$$
\begin{aligned}
\nabla_\theta J(\theta) &= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(a|s) R_t\right] && \text{REINFORCE} \\
&= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(a|s) Q_w(s,a)\right] && \text{Actor-Critic} \\
&= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(a|s) A_w(s,a)]\right] && \text{Advantage Actor-Critic} \\
&= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(a|s) \delta\right] && \text{TD Actor-Critic} \\
&= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(a|s) \delta e\right] && \text{TD}(\lambda)\text{ Actor-Critic}
\end{aligned}
$$

- Each leads a stochastic gradient ascent algorithm
- Critic uses **policy evaluation** (e.g. MC or TD learning) to estimate $Q^\pi(s,a)$, $A^\pi(s,a)$ or $V^\pi(s)$

Compatible Function Approximation: Bias in AC

- Approximating the policy gradient with critic can introduce bias
- A biased policy gradient may not find the right solution
- Luckily, if we choose value function approximation carefully, then we can avoid bias
- If the following two conditions are satisfied:
  1. Value function approximator is compatible to the policy

$$\nabla_w Q_w(s,a) = \nabla_\theta log \pi_\theta(a|s)$$

  2. Value function parameters w minimize the mean-squared error

$$\nabla_w \mathbb{E}_{\pi_\theta}\left[(Q^{\pi_\theta}(s,a) - Q_w(s,a))^2\right] = 0$$

- Then the policy gradient is **without bias**

Problems with Policy Gradient Directions

- Goal: Each step of policy gradient yields an updated policy π' whose value is greater than or equal to the prior policy π: $V^{\pi'} \geq V^{\pi}$
- Several inefficiencies:
  - Gradient ascent approaches update the weights a **small step** in direction of gradient
  - Gradient ascent algorithms can follow *any* ascent direction (a good ascent direction can significantly **speed** convergence)
  - Gradient is First order / linear approximation of the value function's dependence on the **policy parameterization** instead of **actual policy**[1]

[1] A policy can often be re–parameterized without changing action probabilities (f.i., increasing score of all actions in a softmax policy). Vanilla gradient is sensitive to these re–parameterizations.

About step size

- Step size is important in any problem involving finding the optima of a function
- Supervised learning: Step too far $\rightarrow$ next updates will fix it
- But in Reinforcement learning
  - Step too far $\rightarrow$ bad policy
  - Next batch: collected under bad policy
  - **Policy is determining data collect!** Essentially controlling exploration and exploitation trade o due to particular policy parameters and the stochasticity of the policy
  - May not be able to recover from a bad choice, collapse in performance!

Better Policy Gradient Directions: Natural Gradient

- A more efficient gradient in learning problems is the **natural gradient**
- It corresponds to **steepest ascent in policy space and not in the parameter space** with **right step size**
- Also, the natural policy gradient is *parametrization independent*
- Convergence to a local minimum is guaranteed
- It finds ascent direction that is closest to vanilla gradient, when changing policy by a small, fixed amount

$$\nabla_\theta^{nat} \pi_\theta(a|s) = G_\theta^{-1} \nabla_\theta \pi_\theta(a|s)$$

- Where G_θ is the Fisher information matrix

$$G_\theta = \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(a|s) \nabla_\theta \log \pi_\theta(a|s)^T\right]$$

Natural Actor Critic (Peters et al 2005)

- Under linear model modelization of critic:

$$A^{\pi_\theta}(s,a) = \phi(s,a)^T w$$

- Using compatible function approximation,

$$\nabla_w A_w(s,a) = \nabla_\theta \log \pi_\theta(a|s)$$

- The natural policy gradient nicely simplifies,

$$\begin{aligned}
\nabla_\theta J(\theta) &= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(a|s) A^{\pi_\theta}(s,a)\right] \\
&= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(a|s) \nabla_\theta \log \pi_\theta(a|s)^T w\right] \\
&= G_\theta w \\
\nabla_\theta^{nat} J(\theta) &= w
\end{aligned}$$

- i.e. update actor parameters in direction of critic parameters

TRPO (Schulman et al 2017)

- *Trust Region Policy Optimization* (TRPO) maximize parameters that change the policy increasing advantage in action over wrt. old policy in proximal spaces to avoid too large step size.

$$\arg\max_{\theta} L_{\theta_{old}}(\theta) = \arg\max_{\theta} \mathbb{E}_{s_{0:\infty}}\left[\sum_{t=0}^{T-1} \mathbb{E}_{a\sim\theta}\left[\frac{\pi_\theta(a_t|s_t)}{\pi_{\theta_{old}}(a_t|s_t)} A_\theta(s_t, a_t)\right]\right]$$

- Under penalizing constraint (using KL divergence of θ and θ_{old}) that ensures improvement of the policy in the proximity (small step size)
- Solves using Natural Gradient

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- Under penalizing constraint (using KL divergence of θ and θ_{old}) that ensures improvement of the policy in the proximity (small step size)
- Solves using Natural Gradient
- Some TRPO videos here.
- Proximal Policy Optimization PPO inspired in TRPO simplifies computation

Subsection 5

New off-policy AC methods

DDPG: Deep Determ. PG (Lillicrap et al. 2016)

- DDPG is an extension of **Q-learning** for **continuous action spaces**.
  - Therefore, it is an **off-policy algorithm** (we can use ER!)
- It is also an **actor-critic** algorithm (has networks Q_ϕ and π_θ.)
- Uses Q and π **target** networks for stability.
- Differently from other critic algorithms, **policy is deterministic**,
- noise added for exploration: $a_t = \pi_\theta(s_t) + \epsilon$ (where $\epsilon \sim \mathcal{N}$)

DDPG: Deep Determ. PG (Lillicrap et al. 2016)

- Q_ϕ network is trained using standard loss function:

$$L(\phi, \mathcal{D}) = \underset{(s,a,r,s')\sim\mathcal{D}}{\mathbb{E}}\left[\left(Q_\phi(s,a) - \left(r + \gamma Q_{\phi_{\text{targ}}}(s', \pi_{\theta_{\text{targ}}}(s'))\right)\right)^2\right]$$

- As action is *deterministic* and *continuous* (NN), we can easily follow the gradient in policy network to increase future reward:

$$\max_\theta \underset{s\sim\mathcal{D}}{\mathbb{E}}\left[Q_\phi(s, \pi_\theta(s))\right] \rightarrow \nabla_\theta \underset{s\sim\mathcal{D}}{\mathbb{E}}\left[Q_\phi(s, \pi_\theta(s))\right] \approx \frac{1}{N}\sum_{i=1}^{N} \nabla_a Q_\phi(s,a) \nabla_\theta \pi_\theta(s)$$

DDPG: Deep Determ. PG (Lillicrap et al. 2016)

Algorithm 1 DDPG algorithm

Randomly initialize critic network $Q(s,a|\theta^Q)$ and actor $\mu(s|\theta^\mu)$ with weights θ^Q and θ^μ.
Initialize target network Q' and μ' with weights $\theta^{Q'} \leftarrow \theta^Q$, $\theta^{\mu'} \leftarrow \theta^\mu$
Initialize replay buffer R
for episode = 1, M **do**
  Initialize a random process $\mathcal{N}$ for action exploration
  Receive initial observation state s_1
  for t = 1, T **do**
    Select action $a_t = \mu(s_t|\theta^\mu) + \mathcal{N}_t$ according to the current policy and exploration noise
    Execute action a_t and observe reward r_t and observe new state s_{t+1}
    Store transition (s_t, a_t, r_t, s_{t+1}) in R
    Sample a random minibatch of N transitions (s_i, a_i, r_i, s_{i+1}) from R
    Set $y_i = r_i + \gamma Q'(s_{i+1}, \mu'(s_{i+1}|\theta^{\mu'})|\theta^{Q'})$
    Update critic by minimizing the loss: $L = \frac{1}{N}\sum_i (y_i - Q(s_i, a_i|\theta^Q))^2$
    Update the actor policy using the sampled policy gradient:

$$\nabla_{\theta^\mu} J \approx \frac{1}{N} \sum_i \nabla_a Q(s,a|\theta^Q)|_{s=s_i, a=\mu(s_i)} \nabla_{\theta^\mu} \mu(s|\theta^\mu)|_{s_i}$$

    Update the target networks:

$$\theta^{Q'} \leftarrow \tau\theta^Q + (1-\tau)\theta^{Q'}$$

$$\theta^{\mu'} \leftarrow \tau\theta^\mu + (1-\tau)\theta^{\mu'}$$

  end for
end for

TD3: Twin Delayed DDPG (Fujimoto et al, 2018)

- Similar to DDPG but with the following changes:
  1. *Clipped action exploration*: noise added like DDPG but noise bounded to fixed range.

$$a'(s') = \text{clip}\left(\pi_{\theta_{\text{targ}}}(s') + \text{clip}(\epsilon, -c, c), a_{Low}, a_{High}\right), \qquad \epsilon \sim \mathcal{N}(0, \sigma)$$

  2. *Pessimistic Double-Q Learning*: It uses two (twin) Q networks and uses the "pessimistic" one for current state for updating the networks

$$L(\phi_i, \mathcal{D}) = \underset{(s,a,r,s')\sim\mathcal{D}}{\mathbb{E}}\left(Q_{\phi_i}(s,a) - \min_{i=1,2} Q_{\phi_{i,\text{targ}}}(s', a'(s'))\right)^2$$

  3. *Delayed Policy Updates*: Updates of Critic are more frequent than of policy (fi. 2 or 3 times)

SAC: Soft Actor Critic (Haarnoja et al, 2018)

- Policy Entropy-regularized: we will look for *maximum entropy* policies with given data (in SAC we go back to stochastic π).

$$\mathcal{H}(\pi(\cdot|s)) = \mathop{\mathbb{E}}_{a\sim\pi(s)}[-\log \pi(a|s)]$$

- So we search for policy:

$$\pi^* = \arg\max_{\pi} \mathop{\mathbb{E}}_{\tau\sim\pi}\left[\sum_{t=0}^{\infty} \gamma^t \left(R(s_{t+1}) + \alpha \mathcal{H}\left(\pi(\cdot|s_t)\right)\right)\right]$$

  where α is the trade-off between reward and entropy.
- Entropy enforces exploration, so no need to add noise to actions.
- Usually α decreases during learning and is disabled to test performance.

SAC: Soft Actor Critic (Haarnoja et al, 2018)

- Let's define value functions in this case:

$$
\begin{aligned}
V^{\pi}(s) &= \mathop{\mathbb{E}}_{\tau\sim\pi}\left[\sum_{t=0}^{\infty}\gamma^{t}\left(R(s_{t+1})+\alpha\mathcal{H}\left(\pi(\cdot|s_t)\right)\right)\middle|s_0=s\right]\\
Q^{\pi}(s,a) &= \mathop{\mathbb{E}}_{\tau\sim\pi}\left[\sum_{t=0}^{\infty}\gamma^{t}R(s_{t+1})+\alpha\sum_{t=1}^{\infty}\gamma^{t}\mathcal{H}\left(\pi(\cdot|s_t)\right)\middle|s_0=s,a_0=a\right]
\end{aligned}
$$

- So Bellman equations can be written as:

$$
\begin{aligned}
V^{\pi}(s) &= \mathop{\mathbb{E}}_{\tau\sim\pi}\left[Q^{\pi}(s,a)+\alpha\mathcal{H}\left(\pi(\cdot|s)\right)\right]\\
Q^{\pi}(s,a) &= \mathop{\mathbb{E}}_{s'\sim P,a'\sim\pi}\left[R(s')+\gamma\left(Q^{\pi}(s',a')+\alpha\mathcal{H}\left(\pi(\cdot|s')\right)\right)\right]\\
&= \mathop{\mathbb{E}}_{s'\sim P}\left[R(s,a,s')+\gamma V^{\pi}(s')\right]
\end{aligned}
$$

SAC: Soft Actor Critic (Haarnoja et al, 2018)

- Architecture: Networks and loss functions for each one:
  - Q-value functions: $Q_{\theta_1}(s,a), Q_{\theta_2}(s,a)$ (*twin* like TD3)

$$L(\theta_i, \mathcal{D}) = \mathop{\mathbb{E}}_{(s,a,r,s',d)\sim\mathcal{D}} \left[\left(Q_{\theta_i}(s,a) - \left(r + \gamma V_{\psi_{\text{targ}}}(s')\right) \right)^2 \right]$$

  - Value functions $V_\psi(s)$, $V_{\psi_{targ}}(s)$:

$$L(\psi, \mathcal{D}) = \mathop{\mathbb{E}}_{s\sim\mathcal{D}, a\sim\pi_\phi} \left(V_\psi(s) - \left(\min_{i=1,2} Q_{\theta_i}(s,a) - \alpha \log \pi_\phi(a|s) \right) \right)^2$$

  - Policy $\pi_\phi(a|s)$. Maximize:

$$\mathop{\mathbb{E}}_{a\sim\pi} \left(Q^\pi(s,a) - \alpha \log \pi(a|s) \right)$$

which maximize V value function... but how to compute gradients?

Reparametrization trick see here or here

- Problematic because in ∇_ϕ, expectation follow **stochastic** π_ϕ.

$$\mathop{\mathbb{E}}_{a\sim\pi_\phi}\left[Q^{\pi_\phi}(s,a) - \alpha \log \pi_\phi(a|s)\right]$$

- It can be done using the *log-trick* like REINFORCE... but high variance.
- Authors use a reparametrizarion trick. It can be done **when we define the stochastic π_ϕ as Gaussian** by adding noise to the action:

$$\tilde{a}_\phi(s,\xi) = \tanh\left(\mu_\phi(s) + \sigma_\phi(s) \odot \xi\right), \qquad \xi \sim \mathcal{N}(0,I)$$

- Now we can rewrite the term as:

$$\mathop{\mathbb{E}}_{a\sim\pi_\phi}\left[Q^{\pi_\phi}(s,a) - \alpha \log \pi_\phi(a|s)\right] =$$
$$\mathop{\mathbb{E}}_{\xi\sim\mathcal{N}}\left[Q^{\pi_\phi}(s,\tilde{a}_\phi(s,\xi)) - \alpha \log \pi_\phi(\tilde{a}_\phi(s,\xi)|s)\right]$$

- Now we can optimize the policy according to

$$\max_\phi \mathop{\mathbb{E}}_{s\sim\mathcal{D},\xi\sim\mathcal{N}}\left[Q_{\theta_1}(s,\tilde{a}_\phi(s,\xi)) - \alpha \log \pi_\phi(\tilde{a}_\phi(s,\xi)|s)\right]$$

Recommended resources

- Nice review of Policy Gradient Algorithms in Lil'Log blog
- Good description of algorithms in Spinning Up with implementation in Pytorch and Tensorflow
- Understable implementations of Actor Critic methods in RL-Adventure-2