12. A voting system is said to be *partition consistent* if whenever some alternative wins in all the subelections that results in partitioning the voters into two disjoint groups, this alternative also wins the election as a whole. Here we assume that the voters keep their preferences in all the subelections.

Show that Plurality is partition consistent but that Borda and Copeland are not.

13. A *partial* election is an election in which the preferences of the voters over alternatives are partial orders. In this context we consider the following problem.

**Possible Condorcet Winner**: Given an election \((A, N, P)\) where \(P\) is a profile of partial orders over \(A\), and an alternative \(c\). Is it possible to extend every partial vote in \(P\) so that \(c\) is a Condorcet winner?

- For a profile \(T\) of linear orders over \(A\) and for any two alternatives \(x, y \in A\), let \(D_T(x, y)\) denote the number of voters that prefer \(x\) to \(y\) minus the number of voters that prefer \(y\) to \(x\).
- For a profile \(P\) of partial orders over \(A\) and for any two alternatives \(x, y \in A\), let \(D_R^{\max}(x, y)\) denote the maximum value of \(D_T(x, y)\), taken over all total extensions \(T\) of the partial order \(P\).

(a) Show that, for a profile \(P\) of partial orders over \(A\) and any two alternatives \(x, y \in A\),

\[
D_R^{\max}(x, y) = |\{i \mid \text{not}(y >_{P_i} x)\}| - |\{i \mid (y >_{P_i} x)\}|.
\]

(b) Show that \(x \in A\) is a possible Condorcet winner for \(P\) iff, for all \(y \neq x\), \(D_R^{\max}(x, y) > 0\).

(c) Show that **Possible Condorcet Winner** can be solved in polynomial time.

14. In a weighted election each player \(i\) has been assigned a weight \(w_i\). This weight is used as a multiplying factor to the number of points that player \(i\) assigns to an alternative. In this way any voting rule assigning points can be extended to the weighted case.

The **E-Coalitional-Weighted-Manipulation** problem \(E-CWM\) is defined as follows. The input is a weighted election \((A, N, w)\) where player \(i\) has weight \(w_i\), a set \(M \subset V\), a preference \(P_i\), for each player \(i \notin M\), and an alternative \(d \in A\). The question is whether it is possible to find preferences for the players in \(M\) so that in the joint preference profile \(c\) is a winner.

Given a **Partition** instance \((k_1, \ldots, k_n)\) with \(\sum_{i=1}^{n} k_i = 2K\), we construct the following instance of the Copeland-CWM problem.

- \(A = \{a, b, c, d\}\) and \(d\) is the distinguished alternative that we want to make a winner.
- The weights and preference of the 4 voters not in \(M\) are
<table>
<thead>
<tr>
<th>Weight</th>
<th>Preference</th>
</tr>
</thead>
<tbody>
<tr>
<td>2K+2</td>
<td>$d &gt; a &gt; b &gt; c$</td>
</tr>
<tr>
<td>2K+2</td>
<td>$c &gt; d &gt; b &gt; a$</td>
</tr>
<tr>
<td>K+1</td>
<td>$a &gt; b &gt; c &gt; d$</td>
</tr>
<tr>
<td>K+1</td>
<td>$b &gt; a &gt; c &gt; d$</td>
</tr>
</tbody>
</table>

- $M$ has $n$ players, player $m_i \in M$ has weight $k_i$.

Using this construction show that Copeland-CWM is NP-complete.

15. Consider the Constructive Coalitional Manipulation problem (CCM). We are given a set of alternatives $A$, the preferences on $A$ of a set of voters $S$ (the nonmanipulators’ votes), another set of voters $T$ whose preferences are still open (the manipulators’ votes), and a preferred candidate $p \in A$. We are asked whether there is a way to cast the votes in $T$ so that $p$ wins the election.

Let us consider another voting system: Cup (sequential binary comparisons). The cup is defined by a balanced binary tree $T$ with one leaf per candidate, and an assignment of candidates to leaves (each leaf gets one candidate). Each non-leaf node is assigned the winner of the pairwise election of the node’s children; the candidate assigned to the root wins.

The cup voting protocol assumes that $T$ and the assignment of candidates to leaves is known by the voters before they vote.

In the binary tree representing the cup, we can consider each node to be a subelection. When considering the CCM problem, we may say that the voters in $T$ only order the candidates in that subelection since the place of the other candidates in the order is irrelevant for the subelection.

We say that a candidate can obtain a particular result in the subelection if it does so for some coalitional vote on $T$. This defines the set of potential winners for each subelection.

(a) Show that a candidate can win a subelection at node $u$ in $T$ if and only if it can win in one of its children subelections, and it can defeat one of the potential winners of the sibling child in a pairwise election.

(b) Show that, for the cup voting protocol (given the tree and the assignment of candidates to leaves), CCM can be solved in polynomial time.