

Manipulation

AGT-MIRI

Fall 2020

- 1 Strategy-proofness
- 2 Manipulation
- 3 Some manipulable rules
- 4 A hard to manipulate voting rule

Strategy-proofness

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- Assume \succ is a preference profile so that \succ_i is the true preferences of voter i .
- A voting rule F is **strategy-proof** if for every preference profile $\succ' = (\succ_{-i}, \succ'_i)$, it is not the case that $F(\succ') \succ_i F(\succ)$

Strategy-proofness: Borda count

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- Yes, but not very satisfactory!

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- **Onto:** Every alternative can win under some preference profile.
- **Non-dictatorial:** There is no voter i such that $F(\succ)$ is always the top alternative for voter i .

Gibbard-Satterthwaite

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For $m \geq 3$, no deterministic social choice function can be strategyproof, onto, and nondictatorial simultaneously.

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In ☺ words, for $m \geq 3$, any deterministic social choice function must be at least one of the following:

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In 😊 words, for $m \geq 3$, any deterministic social choice function must be at least one of the following:

- ***dictatorial**: there exists a single fixed voter whose most-preferred alternative is chosen for every profile;*
- ***imposing**: there is at least one alternative that does not win under any profile;*
- ***manipulable** (i.e., not strategyproof).*

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- *manipulable* (i.e., not strategyproof).

The first two properties are not acceptable in most voting settings.

So, we need to assume that the voters have an incentive to misreport true preferences.

Strategy-proofness

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Some manipulable rules

A hard to manipulate voting rule

Impossibility result

Circumventing G-S: Complexity approach

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- Maybe the rule is manipulable, but it is NP-hard to find a successful manipulation.

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- Maybe the rule is manipulable, but it is NP-hard to find a successful manipulation.
For once NP-hardness can be good!!

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- The problem belongs to NP provided F is computable in polynomial time.
- For **plurality**, this problem is computationally trivial:
- The only sensible manipulation is to put a as your most preferred candidate!

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 - **Responsive**: the candidate with the largest score wins (in the voting under the joint profile)
 - **Monotone**: for any two preference orders P and P' and for any candidate a , if for each voter i , $\{b \mid a P b\} \subseteq \{b \mid a P' b\}$, then $S(P, a) \leq S(P', a)$.

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The problem

Greedy-Manipulation

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Determine whether a candidate b can be placed in the next lower position (independent of remaining choices) without preventing c from winning.

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Determine whether a candidate b can be placed in the next lower position (independent of remaining choices) without preventing c from winning.
If so, place b in the next position, otherwise terminate claiming that order does not exist.

Manipulable rules by Greedy

Theorem

For any voting rule F satisfying the BTT conditions, G-Man solves the F -MANIPULATION problem.

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- If Greedy-Manipulation succeeds, it constructs a preference order that guarantees that under the joint profile c wins.

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- If Greedy-Manipulation succeeds, it constructs a preference order that guarantees that under the joint profile c wins.
- Assume that such an order exists and that Greedy-Manipulation terminates without providing an ordering. Let us reach a contradiction.

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- Consider any completion P of the preference order started by G-Man that places u in the first unassigned place.

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- By initialization $S(P, c) \geq S(P', c)$.
- So, $S(P, c) \geq S(P, u)$.
- But G-Man did not assign u , so $S(P, c) < S(P, u)$ and we get the contradiction.

Manipulable rules by Greedy

Corollary

For any voting rule F satisfying the BTT conditions, and for which the scoring rule can be computed in polynomial time G -Man solves the F -MANIPULATION problem in polynomial time.

By monotonicity, it should be possible to compute the score to of the alternative ranked "first" among a set of unranked alternatives

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Plurality

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- **Plurality is polynomial time manipulable.**

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- If $\{b \mid a P b\} \subseteq \{b \mid a P' b\}$, the score cannot decrease. So we get monotonicity.
- Both, Copeland vote and the score can be computed in polynomial time.
- Copeland is polynomial time manipulable.

Maximin

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- Maximin
 - $Score(x) = \min_y n_{x \succ y}$
 - elect x^* with the maximum score
- Working in a similar way, Maximin is polynomial time manipulable.

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STV

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- The NP-hardness follows by a reduction from the 3-cover problems which is NP-complete problem (3-Cover).
- The basic idea is to build a large election instance introducing all sorts of constraints on the ballot of the manipulator, such that finding a ballot meeting those constraints solves a given instance of 3-Cover.