9. Assume that a WVG is described by $\Gamma = (q; w_1, \dots, w_n)$. Analyze the computational complexity of the problemS

   - Compute the smallest number of players that can form a wining coalition in Γ.
   - Compute the biggest number of players that can form a losing coalition in Γ.

10. **The diameter game.** Consider a cooperative game which is defined on an undirected connected graph $G = (V, E)$. The players are the edges in the graph. For $X \subseteq E$, let $G_X = (V, X)$ be the graph formed by V and the edges in X. The valuation function is the following

$$v(X) = \begin{cases} 2|X| - diam(G_X) & \text{if } G_X \text{ is connected} \\ \frac{|X|}{2} & \text{otherwise,} \end{cases}$$

   where $diam(H)$ is the diameter of the graph H.

   (a) Is the valuation function monotone? supperadditive? supermodular?

   (b) Are there connected graphs such that the core of the associated diameter game is non-empty?

11. **Games on social networks** One of the criticism to simple games is the fact of assuming that any coalition can be formed. In the context in which the players participate in a social networks, a natural restriction on a coalition to take effect is that the members of a should at least be able to establish some level of communication among themselves.

   For simplicity you can assume that a simple game $\Gamma = (N, \mathcal{W})$ is defined and that the social network is an undirected graph $H = (N, E)$.

   On top of that we can came out with different combinations for defining winning coalitions in an associated *social game*, Γ_s on N. Consider the following options:

   (a) A coalition X is winning in Γ_s iff X wins in Γ and $H[X]$ has no isolated vertices.

   (b) A coalition X is winning in Γ_s iff X wins in Γ and $H[X]$ is connected.

   (c) A coalition X is winning in Γ_s iff there is $Y \subseteq X$, so that Y wins in Γ and $H[Y]$ is connected.

   Under which of the options (a), (b) or (c) is Γ_s a simple game?

   For those cases in which a simple game is defined, assuming that you have access to a polynomial time algorithm that given X decides whether $X \in \mathcal{W}$, analyze the computational complexity of the problem of deciding whether Γ_s has an empty core.

12. **Vertex cover games** For a given undirected graph $G = (V, E)$, the associated *vertex cover game* has $N = V$ and in it a coalition wins iff and only if X is a vertex cover in G.

   (a) Show that vertex cover games are simple games.

   (b) Are there games in which the core is non-empty?

   (c) Analyze the computational complexity of the IsProper and IsStrong problem on vertex cover games

13. Show that Kemeney, Copeland and Maximin are Condorcet consistent and that Borda is not.

14. A voting system is said to be *partition consistent* if whenever some alternative wins in all the subelections that results in partitioning the voters into two disjoints groups, this alternative also wins the election as a whole. Here we assume that the voters keep their preferences in all the subelections.

   Show that Plurality is partition consistent but that Borda and Copeland are not.

15. A *partial* election is an election in which the preferences of the voters over alternatives are partial orders. In this context we consider the folowing proble.

   Possible Condorcer Winner: Given an election (A, N, P) where P is a profile of partial orders over A, and an alternative c. Is it possible to extend every partial vote in P so that c is a condorcet winner?

   - For a profile T of linear orders over A and for any two alternatives $x, y \in A$, let $D_T(x, y)$ denote the number of voters that prefer x to y minus the number of voters that prefer y to x.
   - For a profile P of partial orders over A and for any two alternatives $x, y \in A$, let $D_R^{\max}(x, y)$ denote the maximum value of $D_T(x, y)$, taken over all total extensions T of the partial order P.

   (a) Show that, for a profile P of partial orders over A and any two alternatives $x, y \in A$,

   $$D_R^{\max}(x, y) = |\{i \mid \text{ not}(y >_{P_i} x\}| - |\{\{i \mid (y >_{P_i} x\}\}|.$$

   (b) Show that $x \in A$ is a possible Condorcet winner for P iff, for all $y \neq x$, $D_R^{\max}(x, y) > 0$.

   (c) Show that Possible Condorcet Winner can be solved in polynomial time.

16. In a weighthed election each layer i has been assigned a weight w_i. This weight is used as a multiplying factor to the number of points that player i assigns to an alternative. In this way any voting rule assigning points can be extended to the weighted case.

   The E-Coalitional-Weighted-Manipulation problem E-CWMis defined as follows. The input is a weighthed election (A, N, w) where player i has weight w_i, a set $M \subset V$,a preference P_i, for each player $i \not{i}nM$, and an alternative $d \in A$. The question is whether it is possible to find preferences for the palyers in M so that in the joint preference profile c is a winner.

   Given a Partition instance $(k_1, \ldots, k_n)$ with $\sum_{i=1}^{n} = 2K$, we construct the following instance of the Copeland-CWM problem.

- $A = \{a, b, c, d\}$ and d is the distinguished alternative that we want to make a winner.
- The weights and preference of the 4 voters not in M are

Weight	Preference
2K+2	$d > a > b > c$
2K+2	$c > d > b > a$
K+1	$a > b > c > d$
K+1	$b > a > c > d$

- M has n palyers, player $m_i \in M$ has weight k_i.

Using this construction show that Copeland-CWM is NP-complete.