1. Compute (if any) the dominant strategies and the pure Nash equilibria of the following game:

		Player 2		
		0	1	2
Player 1	0	2,3	0,4	2,3
	1	1,-1	2,4	1,2
	2	2,3	1,2	-2,1

2. Compute the Nash equilibria (pure and mixed) of the following game:

		Player 2	
		A	B
Player 1	A	6,6	2,7
	B	7,2	0,0

3. Consider the following strategic game:

		Player 2		
		R	S	T
Player 1	A	6,6	2,7	2,6
	B	7,2	3,4	0,0

Determine whether the strategy profile

$$(0.6, 0.4), (0.2, 0.4, 0.4)$$

is a Nash equilibrium.

4. Study the pure Nash equilibria in the Koutsoupias Papadimitriou model which has,

- two links ℓ_1, ℓ_2 with identical capacities and initial loads L_1 and L_2 and,
- two agents $1, 2$ having amounts of traffic w_1 and w_2.

We assume that $w_1 > w_2 > 0$ and $L_1 > L_2 > 0$. The players are the agents and every agent has two strategies ℓ_1, ℓ_2. When agent i chosses ℓ_j means that i sends w_i traffic through link ℓ_j. The cost of a strategy profile is the total load of the selected link. For instance, when both agents send to ℓ_1 the the total load on the selected link is $w_1 + w_2 + L_1$ and this will be the cost for both players.

- Provide a bimatrix representation of the game as function of the parameters $(\ell_1, \ell_2, L_1, L_2)$.
- Prove that when $w_2 > L_1 - L_2$ this game has two pure Nash equilibria.
- What happens in relation with pure Nash equilibria when $w_2 < L_1 - L_2 < w_1$?

5. The *cooperation* game is defined as follows. There is a group of n people and a task to be performed. To perform correctly the task requires that exactly k persons cooperate. Each player can decide whether to cooperate (1) or not (0). The utility of a strategy profile $x \in \{1,0\}^i$ for player i is defined as

$$u_i(x) = \begin{cases} 1 & \text{the task is performed and } x_i = 1. \\ 0 & \text{otherwise} \end{cases}$$

Characterize the pure Nash equilibrium of the cooperation game and analyze the complexity of the problems related to pure Nash equilibria for this family of games.

6. The *matching* game is played in a bipartite graph $G = (V_1, V_2, E)$ in which edges are connect only vertices V_1 to vertices in V_2. The players are the vertices in the graph that is $V_1 \cup V_2$. Each player has to select one of its neighbors. Player i gets utility 1 when the selection is mutual (player i selects j and player j selects i) otherwise he gets 0. Characterize the pure Nash equilibrium of the matching game and analyze the complexity of the problems related to pure Nash equilibria for this family of games.

7. We have two printers and three users. Both printers are equal and work under the same conditions. A printer is able to print a job in time proportional to the size of the job without unnecessary delays between jobs. Each user has a job to print and has to select a printer machine. The users have jobs of sizes 4, 6 and 10 respectively. Once the selection of printers is done both printers start at the same time processing their corresponding assignment and the printing is made available to the user just when the printer finishes the allocated work.

   - Assuming that the speed factor of the printers is one, provide a table with the utility for each player and pure strategy profile.
   - Find all the Nash equilibria of the game (pure and mixed).

8. A *fully mixed strategy* is a mixed strategy that assigns positive probability to all the possible actions. Consider the two player's game described by the following bi-matrix.

		Player 2 A	Player 2 B
Player 1	C	1,1	4,2
	D	3,3	1,1
	D	2,2	2,3

Find all the NE for the game having a fully mixed strategy for each player or show that such Nash equilibria do not exists.

9. Consider a set of n players that must be partitioned into two groups. However, there is a set of bad pairings and the two players in such a pair do not want to be in the same group. Moreover, each player is free to choose which of the two groups to be in. We can model this by a graph $G = (V, E)$ where each player i is a vertex. There is an edge (i, j) if i and j form a bad pair. The private objective of player i is to maximize the number of its neighbors that are in the other group.

   Characterize the PNE and the complexity of the associated problems.

10. The Max 2SAT game is defined by a weighted 2-CNF formula on n variables. In a weighted formula each clause has a weight. The game has n players. Player i controls the i-th variable and can decide the value assigned to this variable. A strategy profile is a truth assignment $x \in \{0,1\}^n$. Player i gets 1/3 of the weight of the clauses that are satisfied due to its bit selection.

    Characterize the PNE and the complexity of the associated problems.

11. Assume that we have fixed a finite set K of k colors. Consider a graph $G = (V, E)$ with a labeling function $\ell : V \to 2^K$ and define an associated *coloring game* $\Gamma(G, \ell)$ as follows

    - the players are $V(G)$,
    - the set of strategies for player v is $\ell(v)$,
    - the payoff function of player v is $u_v(s) = |\{u \in N(v) \mid s_u = s_v\}|$.

    Characterize the PNE and the complexity of the associated problems.