Lógica en la Informática / Logic in Computer Science

Thursday May 10th, 2018

Time: 1h30min. No books, lecture notes or formula sheets allowed.

1) (3 points)
1a) Let F, G, H be propositional formulas. Is it true that always $(F \wedge G) \wedge H \equiv F \wedge (G \wedge H)$? Prove it using only the definition of propositional logic.

1b) Let F, G, H be propositional formulas. Is it true that always $F \wedge (G \vee H) \equiv F \vee (G \wedge H)$? Prove it using only the definition of propositional logic.

2) (2 points) Write all clauses obtained by applying Tseitin's transformation to the formula $(p \wedge (q \vee \neg r)) \vee q$. Use auxiliary variables named $a_0, a_1, a_2, \ldots$ (where a_0 is for the root).

3) (4 points) John wants to buy a subset of Amazon's n products (and, as you know, with a very large n). But he has the following $1 + p + q$ constraints, where all M, I_i, L_j, R_j denote subsets of $\{1 \ldots n\}$:

- he *must* buy all products of M
- $I_1 \ldots I_p$ are *incompatibility sets*: for each I_i, John cannot buy *all* products in I_i
- constraints $L_1 \to R_1 \quad \ldots \quad L_q \to R_q$, where $L_i \to R_i$ means that if John buys *all* products of L_i, then he must also buy *all* products of R_i.

3a) Answer all three questions **very briefly**. What would you recommend John to do for *efficiently* finding a set S of products that he can buy without violating any of the constraints?

3b) Same question for finding a set S with minimal $|S|$.

3c) Is the minimal set S of 3b) unique or can there be several distinct minimal sets?

4) Consider the following problem, called *model counting*:
Input: a natural number k and a set of propositional clauses S over symbols $\mathcal{P}$.
Question: does S have at least k different models $I : \mathcal{P} \to \{0, 1\}$?
We want to analyze the computational complexity of model counting, that is, determine if it is polynomial, NP-complete, or perhaps even harder, etc. Answer all four questions **very briefly** (max. 10 words per question).

4a) (1 point) Is model counting at least as hard as SAT? (that is, can we express SAT as a model counting problem?) Why?

4b) (4b,c,d: 1 bonus point, if short and correct) What do you think, is SAT at least as hard as model counting? Why?

4c,4d) Same questions if S is a set of Horn clauses.