Lógica en la Informática / Logic in Computer Science

Monday Nov 5th, 2018

Time: 1h30min. No books, lecture notes or formula sheets allowed.

1) (2 points)

1a) Is it true that for any two propositional formulas F and G, we have that $\neg F \vee G$ is a tautology if and only if $F \models G$? Prove it using only the definition of propositional logic.

1b) Is it true that for any three propositional formulas F, G, H, we have that $F \wedge (G \vee H)$ is a tautology iff $(\neg G \wedge H) \vee \neg F$ is unsatisfiable? Prove it using only the definition of propositional logic.

2) (2 points)
Our friend John has invented something he calls a "propositional database", which consists of a set of positive unit clauses (propositional symbols) and of a set of "rules" of the form:
$Condition \rightarrow Consequence$, where $Condition$ and $Consequence$ are sets of propositional symbols.

I asked John what he considers to be true facts, or simply "facts", in his database, and he said: "the minimal set of facts such that all positive unit clauses are facts, and, for every rule $Condition \rightarrow Consequence$, if all elements of $Condition$ are facts, then also all elements of $Consequence$ are facts".

2a) Given such a database D, we want to know if a certain symbol p is a fact in D. Explain very briefly. What is the cost of deciding this? How?

2b) I want to know how many facts are true in D. Explain very briefly. What is the cost of counting this? How?

3) (4 points) MaxSAT is a problem related to SAT. It takes as input a natural number k and a set S of n propositional clauses over propositional symbols $\mathcal{P}$, and it asks whether there is any interpretation $I : \mathcal{P} \rightarrow \{0, 1\}$ that satisfies at least k clauses of S.

3a) Do you think that MaxSAT is polynomial? NP-complete? Exponential? Why?

3b) Is it true that, using auxiliary variables, one can decide MaxSAT in a single call to a SAT solver? Explain why.

3c) How would you use a SAT solver to solve the optimization version of MaxSAT, that is, how to find the I that satisfies as many of the clauses of S as possible? Give one single (and short) explanation.

3d) We want to 3-color a given graph with n nodes and m edges: assign one of the 3 colors to each node such that for no edge (u, v) nodes u and v get the same color. Of course this may be impossible, so we will allow some nodes to remain uncolored: they get *no* color. How can we use the ideas of 3b,c) to compute the 3-coloring with the *minimal* number of such uncolored nodes?

4) (2 points) Is 3-SAT NP-complete? Explain your answer very briefly, using the fact that SAT (deciding the satisfiability of an arbitrary propositional formula F) is NP-complete.