

Lógica en la Informática / Logic in Computer Science

Tuesday April 30th, 2019

Time: 1h30min. No books, lecture notes or formula sheets allowed.

1) (4 points)

1a) Let F, G, H be formulas. Is it true that if $F \vee G \models H$ then $F \wedge \neg H$ is unsatisfiable? Prove it using only the definition of propositional logic.

1B) Let F and G be propositional formulas. Is it true that if $F \rightarrow G$ is satisfiable and F is satisfiable, then G is satisfiable? Prove it using only the definition of propositional logic.

2) (2 points) Let \mathcal{P} be the set of four predicate symbols $\{p, q, r, s\}$.

2a) How many propositional formulas F built over \mathcal{P} exist?

2b) My friend John has a list $L = \{F_1, F_2, \dots, F_{100000}\}$ of one hundred thousand formulas over $\{p, q, r, s\}$. He says that they are all logically non-equivalent, that is, $F_i \not\equiv F_j$ for all i, j with $1 \leq i < j \leq 100000$. What is the most efficient way to check whether John is right for a given L ? Why? Your answer cannot be longer than 20 words.

3) (4 points) Let C be the atleast-1 constraint $l_1 + l_2 + l_3 \geq 1$, where l_1, l_2, l_3 are literals, and let S be the set of five exactly-1 constraints

$\{ l_1 + a_1 + a_4 = 1, \quad l_2 + a_2 + a_4 = 1, \quad l_3 + a_3 = 1, \quad a_1 + a_2 + a_5 = 1, \quad a_3 + a_4 + a_6 = 1 \}$
where $a_1 \dots a_6$ are distinct propositional symbols not occurring in C .

3A) Is it true that $S \models C$? Why? (answer in at most two lines).

3B) Is it true that any model I of C can be extended to a model I' of S ?

Here, by "extending" I to I' we mean that $eval_I(l_i) = eval_{I'}(l_i)$ and adequately defining the $I'(a_j)$.

Answer by just listing I' for the 7 cases of I , completing the table:

l_1	l_2	l_3	a_1	a_2	a_3	a_4	a_5	a_6
0	0	1	.	.	0	1	.	.
0	1	0	.	.				
.	.	.						

3C) *Exactly-1-SAT* is the problem of deciding the satisfiability of a given set S of exactly-1 constraints. What do you think is the computational complexity of exactly-1-SAT? (polynomial?, NP-complete?, harder?). Why?

3D) Same question if all exactly-1 constraints in S have the form $l + l' = 1$ for literals l and l' .