Lógica en la Informática / Logic in Computer Science

Friday November 10th, 2019

Time: 1h30min. No books, lecture notes or formula sheets allowed.

1) (4 points)
Consider the following statement. For all propositional formulas F, G, H,

$(F \to G) \land (H \to G)$ is satisfiable iff $\neg G \models \neg F \land \neg H$.

Prove the following using only the definitions of propositional logic.
1a) Is the $\Longrightarrow$ implication of this iff statement true?
1b) Is the $\Longleftarrow$ implication of this iff statement true?
1c) Is it true that if $\neg G \models \neg F \land \neg H$, then $(F \to G) \land (H \to G)$ is a tautology?
(hint for 1c: use what you did in 1b).

2) (4 points) Let S_1, S_2 be the two sets of clauses given below. How many models does each one of them have? Give a very short and simple answer, based on what these sets encode.

$$\begin{array}{llllllll} S_1 = \{ & \neg x_0 \lor \neg x_1, & \neg x_0 \lor \neg x_2, & \neg x_0 \lor \neg a_1, & \neg x_1 \lor \neg x_2, & \neg x_1 \lor \neg a_1, & \neg x_2 \lor \neg a_1, \\ & a_1 \lor \neg x_3, & a_1 \lor \neg x_4, & \neg x_3 \lor \neg x_4 & \} \end{array}$$

$$\begin{array}{lllll} S_2 = \{ & \neg x_0 \lor \neg a_2, & \neg x_0 \lor \neg a_1, & \neg x_0 \lor \neg a_0 & \\ & \neg x_1 \lor \neg a_2, & \neg x_1 \lor \neg a_1, & \neg x_1 \lor a_0 & \\ & \neg x_2 \lor \neg a_2, & \neg x_2 \lor a_1, & \neg x_2 \lor \neg a_0 & \\ & \neg x_3 \lor \neg a_2, & \neg x_3 \lor a_1, & \neg x_3 \lor a_0 & \\ & \neg x_4 \lor a_2, & \neg x_4 \lor \neg a_1, & \neg x_4 \lor \neg a_0 & \} \end{array}$$

3) (2 points) Given a graph, we want to decide whether it is 2-colorable, that is, if we can assign one of 2 colors to each node such that, for every edge (u, v), nodes u and v get different colors. Give a short and simple answer *based on propositional logic* of the following: what is the computational complexity of this problem? Is it polynomial, or NP-complete, or ... ?