Performance of SAT Solvers

- The most important tasks that a SAT solver performs are:
  - Choose which variable to Decide on
  - Apply unit propagation exhaustively
  - Analyze conflicts

- When profiling a state-of-the-art SAT solver we get:
  - Variable selection (≈ 10%)
  - Unit propagation application (≈ 80%)
  - Conflict analysis (≈ 10%)

- Hence, the most important thing to optimize is unit propagation, aka BCP (Boolean Constraint Propagation)
BCP with Occur Lists

- BCP only has to detect propagating or conflicting clauses
- There is no need to detect that all clauses are true
- Instead of traversing the whole clause set again and again:
  - For each literal, store the clauses where it appears in occur lists
  - Every time a new lit $l$ is added to the assignment, only clauses containing $\overline{l}$ need to be visited
- Let’s see how it would work with an example
BCP with Occur Lists

1. \( p_1 \, \overline{p}_2 \, \overline{p}_6 \)

2. \( p_2 \, \overline{p}_3 \, p_5 \, \overline{p}_1 \, \overline{p}_6 \)

3. \( p_6 \, p_2 \, p_4 \)

4. \( p_1 \, p_2 \)

5. \( \overline{p}_6 \, \overline{p}_1 \, p_3 \)

6. \( \overline{p}_5 \, p_4 \, p_2 \)

Model

\[
\begin{array}{ccccccc}
& p_1 & p_2 & p_3 & p_4 & p_5 & p_6 \\
U & U & U & U & U & U & U \\
\end{array}
\]

To Propagate

Clauses With

Current assignment: \( \emptyset \)
Current assignment: $\overline{p}_2^d$

Now, we propagate $\overline{p}_2$ visiting $\text{ClausesWith}[p_2]$
BCP with Occur Lists

Model

\begin{array}{ccccccc}
& p_1 & p_2 & p_3 & p_4 & p_5 & p_6 \\
\hline
U & F & U & U & U & U & U \\
\end{array}

ToPropagate

\begin{array}{|cccc|}
\hline
p_1 & 1 & 4 & \\
p_2 & 2 & 3 & 4 & 6 \\
p_3 & 5 & \\
p_4 & 3 & 6 & \\
p_5 & 2 & \\
p_6 & 3 & \\
\hline
\end{array}

ClausesWith

\begin{array}{|cccc|}
\hline
p_1 & 2 & 5 & \\
p_2 & 1 & \\
p_3 & 2 & \\
p_4 & 6 & \\
p_5 & 1 & 2 & 5 & \\
p_6 & 1 & 2 & 5 & \\
\hline
\end{array}

Current assignment: $\overline{p_2^d}$

Literal $p_1$ has to be added to the assignment
BCP with Occur Lists

1. $p_1 \; \overline{p}_2 \; \overline{p}_6$
2. $\overline{p}_2 \; p_3 \; p_5 \; p_1 \; \overline{p}_6$
3. $p_6 \; p_2 \; p_4$
4. $p_1 \; p_2$
5. $\overline{p}_6 \; p_1 \; p_3$
6. $\overline{p}_5 \; p_4 \; p_2$

Current assignment: $\overline{p}_2 \; p_1$
Now, we propagate $p_1$ visiting $\text{ClausesWith}[^{\overline{p}_1}]$
BCP with Occur Lists

1. \( p_1 \quad \overline{p}_2 \quad \overline{p}_6 \)
2. \( p_2 \quad \overline{p}_3 \quad p_5 \quad \overline{p}_1 \quad \overline{p}_6 \)
3. \( p_6 \quad p_2 \quad p_4 \)
4. \( p_1 \quad p_2 \)
5. \( \overline{p}_6 \quad \overline{p}_1 \quad p_3 \)
6. \( \overline{p}_5 \quad p_4 \quad p_2 \)

Model:
\[
\begin{array}{cccccc}
p_1 & p_2 & p_3 & p_4 & p_5 & p_6 \\
T & F & U & U & U & U \\
\end{array}
\]

To Propagate:

Clauses With:
\[
\begin{array}{c|c|c|c|c|c|c}
p_1 & 1 & 4 & \multicolumn{4}{c}{\overline{p}_1} \\
\hline
p_2 & 2 & 3 & 4 & 6 & \multicolumn{2}{c}{\overline{p}_2} \\
\hline
p_3 & 5 & \multicolumn{4}{c}{\overline{p}_3} & \overline{p}_3 \\
\hline
p_4 & 3 & 6 & \multicolumn{4}{c}{\overline{p}_4} \\
\hline
p_5 & 2 & \multicolumn{4}{c}{\overline{p}_5} & \overline{p}_5 \\
\hline
p_6 & 3 & \multicolumn{4}{c}{\overline{p}_6} & \overline{p}_6 \\
\end{array}
\]

Current assignment: \( \overline{p}_2 d p_1 \)
No lit is propagated, we have to decide
Current assignment: $\overline{p}_2 \overline{d} p_1 \overline{p}_4 d$

Now, we propagate $\overline{p}_4$ visiting ClausesWith[$p_4$]
BCP with Occur Lists

1. \( p_1 \quad \overline{p_2} \quad \overline{p_6} \)
2. \( p_2 \quad \overline{p_3} \quad p_5 \quad \overline{p_1} \quad \overline{p_6} \)
3. \( p_6 \quad p_2 \quad p_4 \)
4. \( p_1 \quad p_2 \)
5. \( \overline{p_6} \quad \overline{p_1} \quad p_3 \)
6. \( \overline{p_5} \quad p_4 \quad p_2 \)

Model

\[
\begin{array}{cccccc}
  & p_1 & p_2 & p_3 & p_4 & p_5 & p_6 \\
T & F & U & F & F & U & U
\end{array}
\]

To Propagate

Clauses With

Current assignment: \( \overline{p_2}^d \ p_1 \overline{p_4}^d \)

Literals \( p_6, \overline{p_5} \) have to be added to the assignment
BCP with Occur Lists

Model:

\[
\begin{array}{cccccc}
\ p_1 \ & \ p_2 \ & \ p_3 \ & \ p_4 \ & \ p_5 \ & \ p_6 \\
T \ & \ F \ & \ U \ & \ F \ & \ F \ & \ T \\
\end{array}
\]

ToPropagate:

\[
\begin{array}{c}
1 \\
2 \\
3 \\
4 \\
5 \\
6 \\
\end{array}
\]

ClausesWith:

\[
\begin{array}{c}
1 \ 4 \\
2 \ 3 \ 4 \ 6 \\
5 \\
3 \ 6 \\
2 \\
3 \ 6 \\
1 \ 2 \ 5 \\
\end{array}
\]

Current assignment: \( p_2^d \ p_1 \ p_4^d \ p_6 \ p_5 \)

Now, we propagate \( p_5 \) visiting ClausesWith\([p_5]\)
BCP with Occur Lists

Current assignment: $\overline{p}_2 \overline{p}_1 p_4 \overline{p}_6 \overline{p}_5$

Literal $\overline{p}_3$ has to be added to the assignment
BCP with Occur Lists

Current assignment: $\overline{p_2}\overline{d} p_1\overline{p_4}\overline{d} p_6\overline{p_5}\overline{p_3}$

Now, we propagate $\overline{p_3}$ visiting ClausesWith[$p_3$]
BCP with Occur Lists

1. \( p_1 \quad \bar{p}_2 \quad \bar{p}_6 \)

2. \( p_2 \quad \bar{p}_3 \quad p_5 \quad \bar{p}_1 \quad \bar{p}_6 \)

3. \( p_6 \quad p_2 \quad p_4 \)

4. \( p_1 \quad p_2 \)

5. \( \bar{p}_6 \quad p_1 \quad p_3 \)

6. \( \bar{p}_5 \quad p_4 \quad p_2 \)

Current assignment: \( \bar{p}_2 \quad p_1 \quad \bar{p}_4 \quad p_6 \quad \bar{p}_5 \quad \bar{p}_3 \)

Clause 5 indicates a conflict. Backtrack/backjump is called.