Lecture 6. Evolving data streams

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Contents

Time in data streams: Philosophical Intro

Time in data streams

Model up to now:

- All items seen since time 0 taken into account
- Order of the items in the stream is adversarial
- = Algorithms must good answers even for worst-case order

But:

- Most streams in real life change over time
- Most recent elements are more relevant

Time in data streams

“Most streams in real life change over time”

Q. What does this really mean?

Statistical answer. Occurrence of items now does not follow the same statistical laws as e.g. yesterday or last year.
In particular, there are statistical laws!

Metaphysical answer: Streams probabilistically reflects the current state of the world, which evolves over time

Time in data streams

"Most recent elements are more relevant"

Q. Really? Why?

My-boss-tells-me-so answer: I need to count packets sent during last month, for billing each customer

Machine learner answer: Because they help me better to predict the immediate future

This relies on the statistical interpretation!

Why should prediction be possible at all?

For discussion: *and on the metaphysical interpretation?*

A statistical (& metaphysical) setting

Rarely formalizd, but implicit in many works

- for each time t, distribution D_t on universe of items
- t-th stream item independently generated according to D_t
- each D_t possibly different from previous ones

- but either change occurs infrequently ("shift")
- or else they are very small ("drift")
- so that statistics has some time to converge
- and adversarial cannot be totally adversarial

A statistical (& metaphysical?) setting

Ignore at your own risk: Autocorrelation, bursts, varying rates of arrival…

Example:

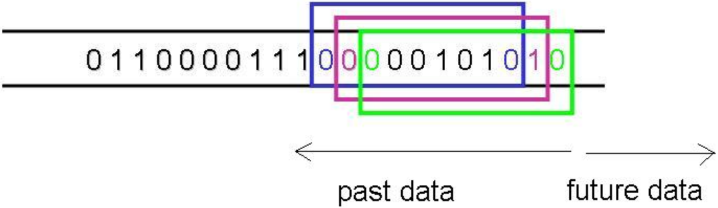
$$\Pr[\text{rain tomorrow} | \text{rain today}] > \Pr[\text{rain tomorrow}]$$

But even if $\Pr[\text{rain today}] = 1/5$,

$\Pr[\text{4 years with zero rain, followed by 1 year raining every day}] \simeq 0$

Sliding window model. Exponential histograms

The Sliding Window Model

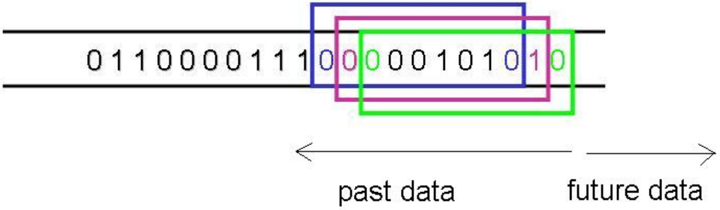


- Only last *n* items matter
- Clear way to bound memory
- Natural in applications: emphasizes most recent data
- Data that is too old does not affect our decisions

Examples:

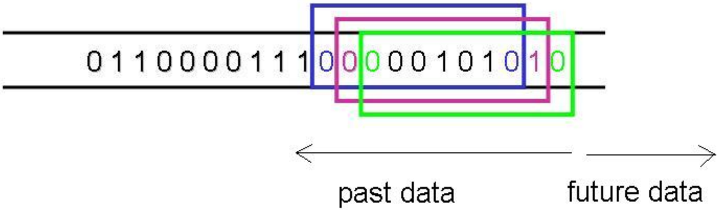
- Study network packets in the last day
- Detect top-10 queries in search engine in last month
- Analyze phone calls in last hours

Statistics on Sliding Windows



- Want to maintain mean, variance, histograms, frequency moments, hash tables, ...
- SQL on streams. Extension of relational algebra
- Want quick answers to queries at all times

Basic Problem: Counting 1's



Obvious algorithm, memory n:

- Keep window explicitly
- At each time t, add new bit b to head, remove oldest bit b' to tail,
- Add b and subtract b' from count

Fact:

$\Omega(n)$ memory bits are necessary to solve this problem exactly

Counting 1's

[Datar, Gionis, Indyk, Motwani, 2002]

Theorem:

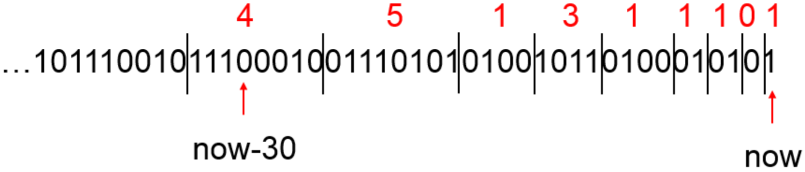
There is a *deterministic* algorithm that estimates the number of 1's in a window of length n with multiplicative error ε using $O(\frac{1}{\varepsilon}\log n)$ counters up to n

which means $O(\frac{1}{\varepsilon}(\log n)^2)$ bits of memory

Example:

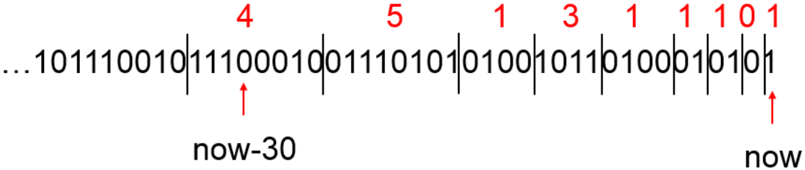
- $n = 10^6$; $\varepsilon = 0.1 \to 200$ counters, 4000 bits

Idea: Exponential Histograms



- Each bit has a timestamp - time at which it arrived
- At time t, bits with timestamp $\leq t - n$ are *expired*
- We have up to k buckets of size 1, 2, 4, 8 …
- We keep the counts of 1s in each bucket
- Errors: expired bits in the last bucket
- Last bucket $\leq$ all buckets / $k \simeq n/k$

Exponential Histograms

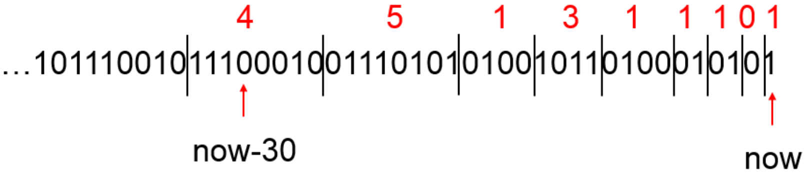


Questions:

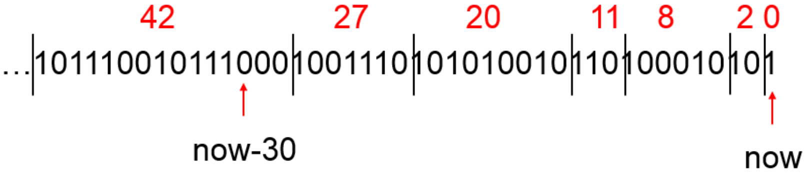
- How do we maintain the buckets when new items arrive
- How do we obtain error (# 1's)/k, rather than n/k

Exponential Histograms

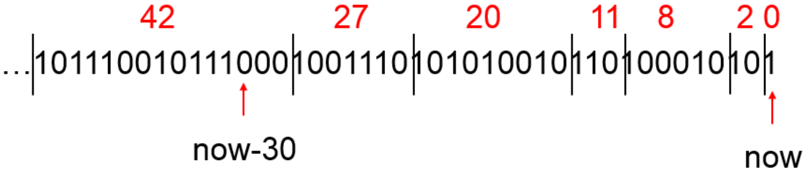
Instead of keeping buckets with 2^i bits, with number of ones,



Store buckets with 2^i *ones*, with timestamps,



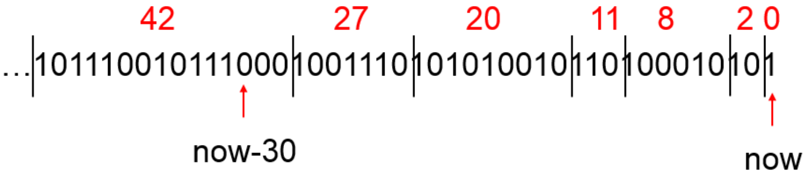
Exponential Histograms



Init: Create empty set of buckets

Query: Return total number of bits in buckets - last bucket / 2

Exponential Histograms



Insert rule(bit):

- If bit is a 0, ignore it. Otherwise, if it's a 1:
- Add a bucket with 1 bit and current timestamp t to the front
- for $i = 0, 1, \ldots$

  If more than k buckets of capacity 2^i,

  merge two oldest as newest bucket of capacity 2^{i+1},

  with timestamp of the older one
- if oldest bucket timestamp $< t - n$, drop it (all expired)

Why Does This Work?

Let 2^C be capacity of largest (oldest) bucket:

- Claim 1: All buckets except oldest one are totally full with non-expired bits
- Claim 2: Oldest bucket contains between ≥ 1 and $\leq 2^C$ non-expired bits
- Claim 3: For each capacity except that of the largest bucket there are k or $k-1$ buckets
- Claim 4: Sum of all buckets except those of largest capacity is in $[(k-1)(2^C-1), k(2^C-1)]$
- Claim 5: Estimate is within $[1-\frac{1}{2k}, 1+\frac{1}{2k}]$ of correct number of 1s

So take $k = \frac{1}{2\varepsilon}$, we're done

Memory Estimate

- Largest bucket needed: $k\sum_{i=0}^{C} 2^i \simeq n \to C \simeq \log(n/k)$
- Total number of buckets: $k \cdot (C+1) \simeq k \log(n/k)$
- Each bucket contains a timestamp only (perhaps its capacity, dep. on implementation)
- timestamps are in $t-n \dots t$: recycle timestamps mod n
- Memory is $O(k \log(n/k) \log n)$ bits, $k = 1/\varepsilon$

Generalizations

Technique can be applied to maintain many natural aggregates, those satisfying well-defined conditions:

- Distinct elements
- Max, min
- Histograms
- Hash tables
- Frequency moments

Simpler version: “Linear” histograms

A simpler version:

- Queue of n/k buckets
- each of equal capacity k
- Memory $O(n/k \cdot \log k)$ bits

Exercise 1

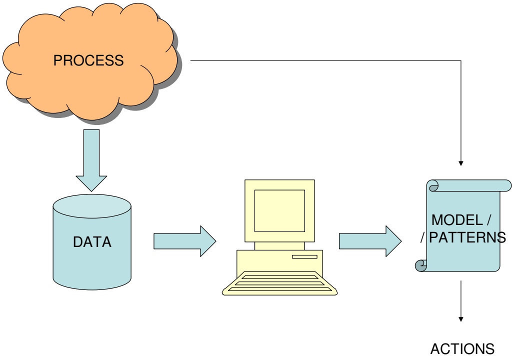
Analyze this simpler version:

- k to obtain error ε?
- Memory to obtain error ε? (relative to n? relative to number of 1’s?)
- When is it worth using the exponential histograms?

Tracking statistics

Tracking Statistics

Many mining / learning algorithms compute statistics on data and combine them



Tracking Statistics

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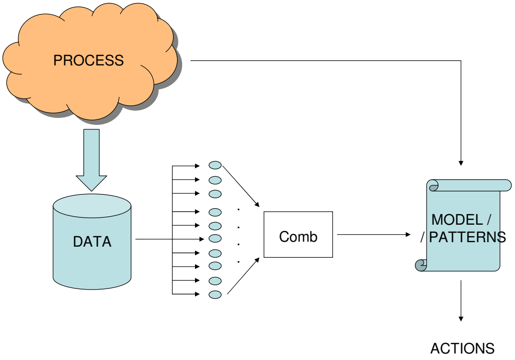


Tracking Statistics

- Fix a statistic ϕ on distributions
- Want to compute ϕ on a stream $S = \{x_t\}_t$, making recent samples more important
- General method:
  - Choose a decay function $f : \mathbb{N} \to [0,1]$, of sum 1
  - At time t, assign importance $f(i)$ to x_{t-i}
  - Compute $y_t = \phi(\{f(i) \cdot x_{t-i}\}_i)$

Tracking Statistics

Special cases, for $x_t \in \mathbb{R}$, $\phi = \text{avg}$:

- Memory-based: Sliding window of size W
  - $y_t = \text{avg}(x_t, \ldots, x_{t-W+1})$
  - $f(i) = \begin{cases} 1/W & \text{for } i < W \\ 0 & \text{for } i \geq W \end{cases}$
- Memoryless: EWMA: Exponentially Weighted Moving Average
  - $y_1 = x_1$,
  - $y_t = \lambda \cdot x_t + (1-\lambda) \cdot y_{t-1}$, for fixed $\lambda < 1$
  - inductively, $y_t = \sum_i \lambda (1-\lambda)^i x_{t-i}$
  - $f(i) = \lambda (1-\lambda)^i$

Change detection

Tracking Statistics, reviewed

Stream of numbers, want to keep mean of most recent ones
My-boss-told-me-so setting:

- Sliding window: $y_t = \text{avg}(x_t, \ldots, x_{t-W+1})$
- EWMA: $y_t = \sum_i \lambda(1-\lambda)^i x_{t-i}$

Statistical setting:

- Each x_t is the realization of a random variable $X_t \simeq D_t$
- We want to have an estimation of $E[X_t]$
- We can average x_t, x_{t-1}, x_{t-2} ... if $D_t \simeq D_{t-1}, D_{t-2}, \ldots$

The variance / reaction time tradeoff

Average $(x_t + x_{t-1} + \cdots + x_{t-W+1})/W$

Two sources of inaccuracy:

- Variance of each x_{t-i}
- Decreases as we average more values

- Drift as we consider older values
- Increases as we average over more values

- Where is the sweet spot?
- Depends on variance and change rate, unknown

Change detectors

A simple, memoryless change detector is CUSUM, CUmulative SUM test:

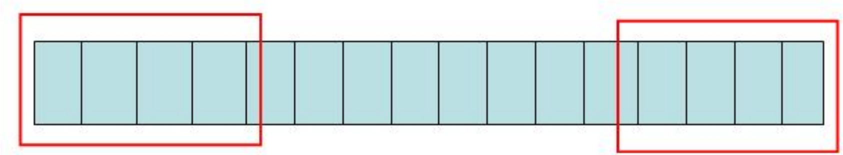
- $g_0 = 0$
- $g_t = \max(0, g_{t-1} - v + \varepsilon_t)$
- if $g_t > h$ declare change, set $g_t = 0$, reset ε_t

Idea: v, h, ε control sensitivity

- $v \simeq \sigma$, “normal” deviation
- $h = k\sigma$, “alarming” deviation
- $\varepsilon_t \to 0$

Similar to: Page-Hinkley test

Window-based change detection



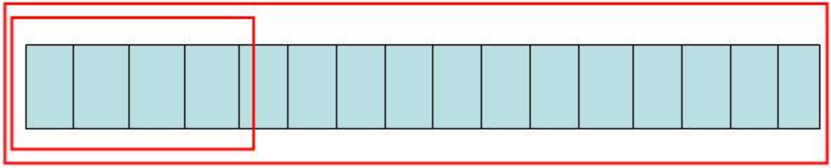
"If $d(W_1, W_2) > \varepsilon$ then declare change"

[Kifer - Ben-David - Gehrke 04]:

W_1 reference, W_2 sliding, fixed size

Window size choice is an issue

Window-based change detection



"If $d(W_1, W_2) > \varepsilon$ then declare change"
[Gama et al, 04]: Specific for classifier error monitoring
W_1 achieves min error, W_2 all data, varying size
Slow to react after long stretches with no change

Window-based change detection: ADWIN [Bifet-G 07]

ADaptive Windowing: Look at the data to find the optimal tradeoff variance - current rate of change

No need to guess parameters a priori, or assume that "optimal parameters" are optimal forever

Idea: Keep W as long as possible while consistent with

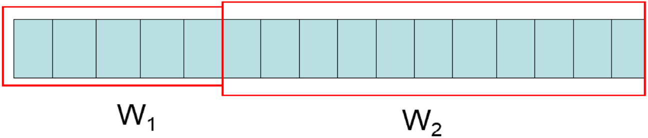
"there is no clear evidence of change within W"

ADWIN [Bifet-G 07]

Fix a distance among probability distributions d and a statistical test *test* such that for two samples S_1, S_2 from distributions D_1, D_2,

$$\Pr[test(S_1, S_2, \delta) = \text{true}] = \begin{cases} \leq \delta & \text{if } D_1 = D_2 \\ > 1 - \delta & \text{if } d(D_1, D_2) \geq \varepsilon \\ ? & \text{otherwise} \end{cases}$$

(Actually, $\varepsilon = \varepsilon(|S_1|, |S_2|, \delta)$)



Try $test(W_1, W_2, \delta)$ for every partition of $W = W_1 + W_2$

If the test for W_1, W_2 returns true, shrink W to W_2

ADWIN: The Guarantees

Theorem

At every time t,

- (No false positives) If there is no change in average within W, ADWIN does not cut W, with prob $\geq 1 - \delta$
- (No false negatives) If there is a split $W = W_1 + W_2$ such that $d(D_1, D_2) \geq \varepsilon(\dots)$ then ADWIN shrinks W with probability $\geq 1 - \delta$

For general sets of items, Kolmogorov-Smirnov test
For windows of bounded real numbers, a simple test using $d(D_1, D_2) = |\text{avg}(D_1) - \text{avg}(D_2)|$ can be based on Hoeffding

In practice, use heuristics variants

ADWIN2: Going for efficiency

Built on top of exponential histograms:

Initialization:

Empty all buckets (=empty window)

At each time t,

for all partitions of W into $W_1 + W_2$,

where W_2 comprises exactly a number of buckets **do**

if test$(W_1, W_2, \delta) \geq \varepsilon(|W_1|, |W_2|, \delta)$ **then**

declare change; drop oldest bucket

end if

end for

ADWIN: The Guarantees

- ADWIN uses memory $O(\log W)$ for a window of length W
- Time per item is $O(\log W)$; can be made amortized $O(1)$

- Can be used both as estimator and as change detector
- It cuts W if and only if change has occurred
- Keeps longest window statistically consistent with the hypothesis “no change within window”
- Autonomously solves the variance - reaction time tradeoff
- Parameter-free (only δ), scale-free (no guessing)

ADWIN: The Use

Pluses:

- Black box, guesswork-free module to deal with change
- Change detector + “intelligent counter”
- $O(\log W)$ memory & time
- Parameter-free, scale-free algorithm

Cons:

- Memory is not $O(1)$ like EWMA+CUSUM
- Much slower than EWMA+CUSUM (100-500 times) in direct implementation
- Optimizations should be possible
- Still, OK when in the middle of other computation or disk access