1. Consider the CSP with variables \( x \in \{1, \ldots, 4\} \), \( y \in \{3, \ldots, 4\} \), \( z \in \{3, \ldots, 4\} \) and constraints \( c_1 \equiv y < z \), \( c_2 \equiv x = z \). Write the solution space when using Forward Checking with the static ordering of variables \( x, y, z \), and value ordering from least value to largest value. Explain which propagations happen in each node.

2. Repeat the same exercise but using Maintaining Arc Consistency instead of Forward Checking.

3. In the naive backtracking algorithm, a node in the search tree corresponds to a partial assignment \( x_1 = a_1, \ldots, x_j = a_j \). A more general approach, which we will refer to as generalized backtracking, consists in associating to each node of the search tree a set of branching constraints \( b_1, \ldots, b_j \): a node \( p \) is extended by adding \( k \) children \( p \cup \{b_{j+1}^1, \ldots, b_{j+1}^k\} \), where the \( b_{j+1}^1, \ldots, b_{j+1}^k \) are constraints that are mutually exclusive (i.e., \( b_{j+1}^r \land b_{j+1}^s \) has no solution for \( 1 \leq r < s \leq k \)) and exhaustive (i.e., \( \bigwedge_{s=1}^k b_{j+1}^s \) is always true). Show that, by adding auxiliary variables, one can mimic generalized backtracking by means of naive backtracking.

4. Given a set \( x_1, \ldots, x_n \) of Boolean variables (i.e., with domain \( \{0, 1\} \)), a literal is a variable (positive literal) or the negation of a variable (negative literal), and a clause is a disjunction of literals (i.e., of the form \( \neg x_i \lor \cdots \lor \neg x_r \lor x_j \lor \cdots \lor x_s \), for \( r, s \geq 0 \)). Let us consider a CSP consisting of a conjunction of clauses. The unit propagation rule states that, if \( A \) is a partial assignment and \( C \lor l \) is a clause such that for all literal \( l' \in C \) we have \( A(l') = 0 \), then \( l \) can be added to the partial assignment. Show that the unit propagation rule corresponds to Forward Checking.

5. Consider the 2-Coloring problem (i.e., is it possible to color a given graph with 2 colors?). Prove that an implementation of Forward Checking that always selects first the variables with the smallest domain solves the problem only visiting a polynomial number of nodes of the search tree. Assume that the graph is connected.

6. Given a set \( x_1, \ldots, x_n \) of Boolean variables, a Horn clause is a clause with at most one positive literal. Let us consider a CSP \( S \) consisting of a conjunction of Horn clauses. Show that Forward Checking decides if \( S \) has a solution without backtracking.