Present–day running of the cosmological constant and supernova data

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Abstract. A particle physics account of the cosmological constant is given through two different approaches. Both of them use the equations of quantum field theory and so the cosmological "constant" has its own renormalization group equation (RGE). The obtained running is then introduced into the theoretical expression for the magnitude–redshift relation, so that a minimization of the residuals with the observational data from supernovae (SN) allows us to fit some parameters. Among the latter are the lightest neutrino masses, for which the best value is $m_\nu = 0.004 - 0.005 \; eV$ (with the possible presence of a sterile light field). Future applications of the type of analysis presented here are finally pointed out.

1 Introduction

The cosmological constant, introduced 85 years ago by Einstein in his gravitational field equations in order to make a quasi–static global distribution of matter possible, also appears in the form of induced vacuum energy density in Quantum Field Theory. However, the values deduced from QFT within the framework of the Standard Model of the strong and electroweak interactions are approximately 120 of magnitude higher than the tiny values assumed by Einstein or now inferred from SN data on the accelerating expansion of the universe. A way to eliminate this discrepancy can be to treat the cosmological constant as a running parameter in a semiclassical formulation of the gravitational field equations. Quantum effects are computed via the Renormalization Group method. Then cancellation of terms can take place. This way, [1] combine the Spontaneous Symmetry Breaking term with the vacuum term, each of them following its own Renormalization Group Equation. They derive the equation for the low–energy regime using the decoupling theorem, which allows to take into account only the fields with particle masses below the relevant energy scale. In contrast, [2] arrive at the corresponding equation by assuming that the heaviest particles dominate the running of the cosmological constant at any energy scale. The implications of both approaches can be checked by comparison with current SN data at low redshifts ($z < 2$), which is the purpose of the present paper. The analysis presented here will be further applied to new, more consistent models within the framework of the Minimal Supersymmetric Standard Model.

2 Astroparticle background

The low–energy approximation for the running of the cosmological constant in the two cases above is now considered.

Case I. When only particles with a mass smaller than the present–day energy scale are relevant, only the lightest neutrinos have to be considered [1, 3]. Other approximations can be made: the one-loop RGE is used and since we are dealing with low energies the running of masses and couplings can be neglected. Putting all this information into the equations, a very useful cancellation between both contributions to the cosmological constant takes place and only the second–order term survives:

$$(4\pi)^2 \frac{d\Lambda_{ph}}{d\mu} = \beta_\Lambda = -4 \sum m_i^4 \tag{1}$$

Now the renormalization condition must be chosen. We adopt the value at the current energy so that it corresponds to what is suggested by the SN observations:

$$\Lambda_{ph}(\mu = \mu_0) = \Lambda_0 = \Omega_\Lambda \rho_c^0 \qquad \rho_c^0 = 34 \times 10^{-12}\ eV^4 \tag{2}$$

where μ is the renormalization scale (ρ_c^0 is the critical density at the present time). One may expect that only the lightest degrees of freedom of the SM (neutrinos) are involved to determine the running of Λ_{ph} at nearby points $\mu \gtrsim \mu_0$.

Case II. In this case one develops the RGE with the contribution of massive particles [2]. The other approximations and the renormalization condition are the same as in the previous case. This way the running of the cosmological constant becomes:

$$\begin{aligned} \Lambda_{ph}(\mu) = \Lambda_0 \ + \ & \frac{\mu^2}{(4\pi^2)} \frac{1}{4} \left[m_H^2 + 3m_Z^2 + 6m_W^2 - 4 \sum N_i m_i^2 \right] \\ & + \frac{\mu^4}{(4\pi^2)} \left[\frac{1}{2} \sum N_i - \frac{5}{4} \right] \end{aligned} \tag{3}$$

where the μ^2 term fixes m_H by imposing that the cosmological constant must be within the observed range. Besides, the value of $\sum N_i$ is fixed by the standard model, being N=3 for quarks and N=1 for leptons.

The evolution of both models depends on the beta function (β_Λ)in the RGE. In Case I, the beta function is negative when only neutrinos are considered and it can be positive when a sterile light field is also introduced as suggested in [1]. On the other hand, Case II has a fixed value of the beta function since it is imposed by the standard model. A supersymmetrical development of the equations could change this, β_Λ then becoming proportional to the number of fermions in the model.

Figure 1 represents those different evolutions. Case I with a neutrino mass lower than 0.005 eV is close to a nonevolutionary cosmological constant. Higher masses (farther away from the expected ones) cause a more pronounced evolution to take place. Although the largest running occurs in Case II, it is only important from z = 2 on, which makes difficult an observational discrimination.

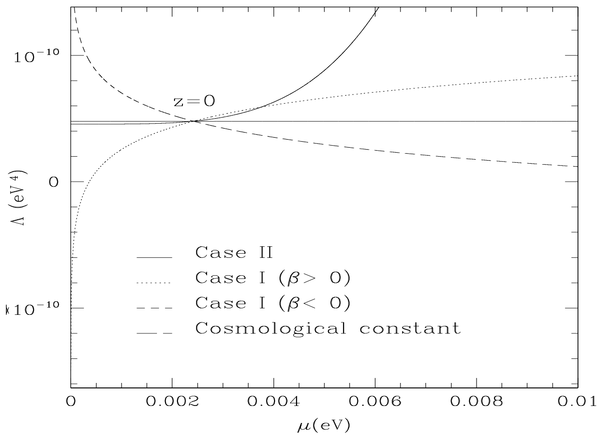

Figure 1: Λ–evolution for different modes of running of the cosmological constant.

3 Magnitude-redshift relation

The analysis of SN magnitudes allows to check different cosmological models, since magnitude depends on the density evolution in the universe [4].

$$m(d); \qquad d(z, \Omega_M, \Omega_\Lambda) = c(1+z) \int_0^z \frac{dz'}{\sqrt{\rho_M(z') + \Lambda_{ph}(z')}} \tag{4}$$

where Λ_{ph} evolves following the equations given in the previous section and ρ_M is that obtained from $\dot{\rho}_m + 3\rho_M H + \dot{\Lambda}_{ph} = 0$.

Using the observational data now available [6], a χ^2 test can be made using the analytical expressions for the running of Λ for the different possibilities considered. At present there is a relatively small number of data, and different models yield undistinguishable predictions (see Figure 2). With improved data, such as those that the SNAP mission will provide, more constraining tests will be possible [5].

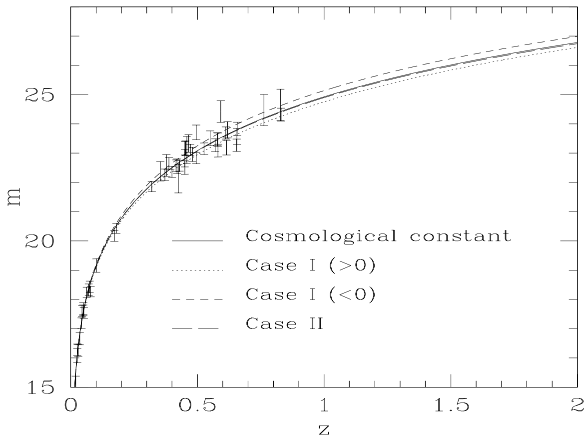

Figure 2: Magnitude-redshift relation for de two cases discussed here. Points represent the 54 high–z and low–z SN used by the Supernova Cosmology Project [6].

4 Testing with supernova data

We want to check both cases with the current SN data. To do this we fit the magnitude–distance relation to the observed magnitudes, where a choice of the energy scale is needed to determine the running of the cosmological constant. We have adopted $\mu = \rho^{\frac{1}{4}}$ as the corresponding parameter (renormalization scale set by the energy density), although the Hubble parameter can also be used to set the scale (then $\mu \sim H(t)$: see the discussion on this point in [3]). With this change of variable the expressions for the running become

$$\Lambda^{I}_{ph}(z) \; = \; \Lambda_0 \; - \; \frac{4\sum m_i^4}{2(4\pi)^2} \; ln(1+z) = \; \Lambda_0 \; - \; \frac{\sum m_i^4}{(4\pi)^2} \; ln\left(\frac{\rho(z)}{\rho_0}\right) \tag{5}$$

$$\Lambda^{II}_{ph}(z) \; = \; \Lambda_0 \; + \; \frac{\frac{1}{2}\sum N_i - \frac{5}{4}}{(4\pi)^2} \; (\; \rho(z) \; - \; \rho_0 \;) \tag{6}$$

where $\rho(z)$ is the sum of the matter plus cosmological "constant" density terms. As it has been seen in the previous section, the theoretical SN magnitudes can be written as function of the evolution of Λ_{ph} and ρ_M.

Case I. Using a χ^2 test, the best value for the neutrino mass can be obtained. Here we have considered two possibilities: a universe which is just flat and a universe with a constraint on the mass density: $\Omega_M = 0.3$. Although the confidence regions are quite wide with the present–day number of SN, an estimate of the neutrino mass has been made. To this effect we have assumed the two lightest neutrinos to have the same mass and the heaviest neutrinos to be decoupled in the range of energy corresponding to the SN data. The results are $m_\nu = 0.004\ eV$ for the flat case and $m_\nu = 0.005\ eV$ for the $\Omega_M = 0.3$ case. Although we have obtained this value from the minimum χ^2, the results do not discard the presence of the sterile light field mentioned above.

Case II. The standard model fixes the number of families to 3, so if N=3 for quarks, N=1 for leptons and the sum is extended over fermions, $\sum N_i$ takes the value of 24. We can then verify with these values the goodness of the fit to the observational data. In Figure 3 (Right) we see that the confidence regions are very similar to those found for a pure cosmological constant, and thus when we look for the best values of the cosmological parametres for a flat universe very similar results are also obtained:

$$\Omega_M = 0.32 \qquad \Omega_\Lambda = 0.68 \qquad (\chi^2/\mathrm{dof} = 1.13)$$

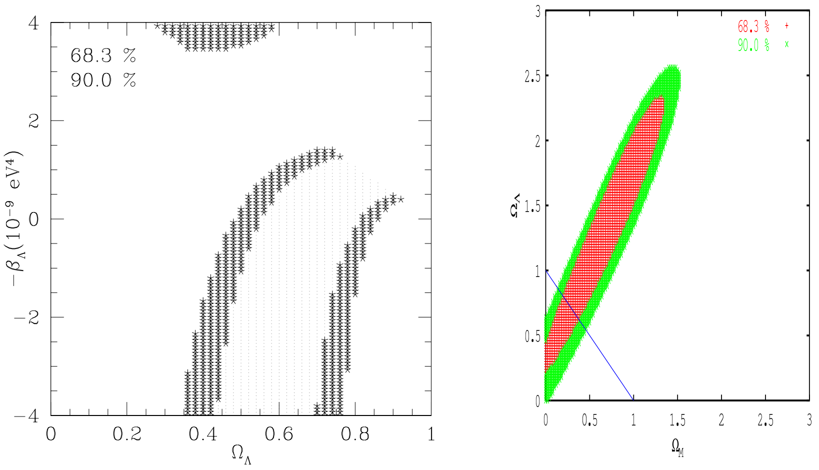

Figure 3: (Left) Confidence regions for the beta function in a flat universe. The beta function is defined in this case by $\beta_\Lambda = -\sum m_j^4$. It must be noted that SN data a low z alone would not allow to determine any regions in this diagram but that the results mostly depend on the high–z SN data. (Right) Confidence regions for a universe with $\sum N_i$ determined by the standard model

5 Conclusions

The preceding, still preliminary, analysis shows that the approach to solve the problems of the low value of the cosmological constant and of its approximate coincidence with that of the current matter density, based on the running of the cosmological constant, is compatible with the SN results, although the present data do not yet allow to discriminate between this approach and a pure, phenomenological cosmological constant.

Adopting the running of Λ determined by the lightest degrees of freedom, the neutrino masses giving the best fit to the SN data can be determined. Although the value obtained is higher than the expected one (0.004–0.005 eV *vs* 0.001–0.002 eV), the uncertainties of the data are large enough that the order of magnitude agreement is acceptable.

The running of Λ with contribution from massive particles is also compatible with the observations but, again, from current data it cannot be preferred to the presence of a pure cosmological constant alone.

It is possible to explore the running of the cosmological constant, not only within the framework of the SM but also beyond it, with a large enough sample of high–redshift SN. That emphasizes the interest of new and more precise data.

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