Randomized Algorithms (RA-MIRI): Assignment #1

In this programming assignment you will have to write programs to simulate the outcomes of two random experiments and make empirical estimations of the constant π from the outcomes. It is fine to use C++ or Python; if you would like to use a different programming language, please check with the instructor. It is fine to embed the two simulations in one single program, or to write a different program for each simulation.

1 Throwing darts

The experiment that we want to simulate is that of throwing darts. The landing position of each dart is a random point in the unit square $[0, 1]^2$, each random point drawn uniformly and independently of the others.

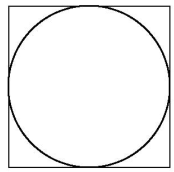
The area of the circle inscribed inside the unit square is $\pi/4 \approx 0.7854$, therefore the probability that a dart that is thrown at random in the unit square lands inside the circle is $\pi/4$.

Your program will be given a number N of darts and it will simulate the throwing of the N darts by generating their landing positions uniformly and independently—for each dart generate two random numbers x and y uniformly in $[0, 1]$ giving the coordinates of the position of the dart.

Count the number C of darts that fall inside the inscribed circle. Then C/N should be roughly $\pi/4$. In the document describing your work it will be very good that you graphically plot the evolution of the ratio

$$\frac{4 \times \text{darts-inside-circle}}{\text{darts-thrown}}$$

as we throw more and more darts (until darts-thrown reaches the value N). The ratio should approximate π, with increasingly better approximations as the number of throws increases.

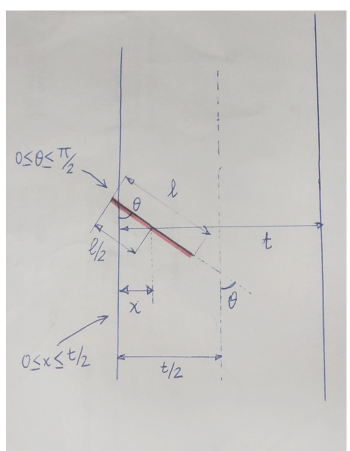


2 Buffon's needles

In Buffon's needles experiment there is a big surface divided into alternating black and white stripes of width t and a needle of length $\ell \leq t$ falls. Buffon wondered what was the probability that the needle landed across two contiguous stripes. The answer turns out to be quite simple:

$$\frac{2\ell}{t\pi}.$$

You should write a program that simulates the fall of N needles and counts how many of them would be crossing two stripes. Without loss of generality, for each needle we generate a random number x between 0 and $t/2$ that represents the distance between the center of the needle to the closest line that separates stripes, and another number θ, between 0 and $\pi/2$, that represents the angle (in radians) between the needle and the stripe line ($\theta = 0$ means the needle is parallel to the line, $\theta = \pi/2$ means it is perpendicular)—see the figure below.



In particular, the needle will cross the line if

$$x \leq \frac{\ell}{2}\sin\theta.$$

Let C the number of needles that cross a line; then

$$\frac{C}{N} \approx \frac{2\ell}{t\pi}$$

and

$$\frac{2\ell N}{tC} \approx \pi$$

Like in the previous section, make a graphical depiction of the evolution of the ratio $(2\ell N)/(tC)$. We can take $\ell = 1/2$ and $t = 1$, then $\ell = t/2 < t$, and the ratio N/C should approximate π as N grows.

3 Instructions to deliver your work

Submit your work using the FIB-Racó. The deadline for submission is October 2nd, 2022 at 23:59. It must consist of a zip or tar file containing all your source code, auxiliary files and your report in PDF format. Include a README file that briefly describes the contents of the zip/tar file and gives instructions on how to produce the executable program(s) used and how to reproduce the experiments. The PDF file with your report must be called `YourLastName_YourFirstName-1.pdf`,

N.B. I encourage you to use LaTeX to prepare your report. For the plots you can use any of the multiple packages that LaTeX has (in particular, the bundle TikZ+PGF) or use independent software such as matplotlib and then include the images/PDF plots thus generated into your document.