Syntactic Analysis (Parsing)

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Summary

- Linear Parsing Algorithms. (Counter)example
- Notations: BNF and Extended BNF
- Applying a Rule $A \rightarrow \alpha_i$
- Nullable, First and Follow
- Methods of Linear Parsing
  - Top-down Parsers
    - Grammar Restrictions in Top-down Parsing
      - Elimination of Left Recursion
      - Left Factoring
    - Types of Top-down Parsers
      - Table-driven Top-down Parser
      - Predictive Recursive Top-down Parser
  - Bottom-up Parsers

Linear Parsing Algorithms

- The list of tokens w will be visited only once, usually in a *left-to-right* traversal.
  The current token receives the name of *lookahead*
- Compute the derivation $S \Rightarrow^* w$ without *backtracking*.
  Sources of indeterminism at this point:

$$S \Rightarrow^* w_0 A_1 w_1 \dots A_n w_n \Rightarrow$$

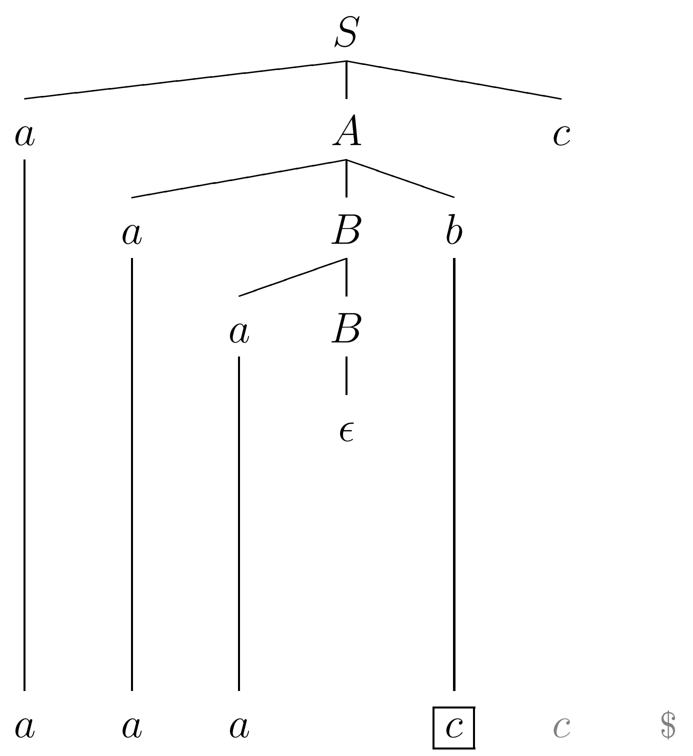
  - Which non-terminal A_i choose to expand?
  - If we have a rule of the form $A \rightarrow \gamma_1 | \dots | \gamma_k$,
    which γ_j —if any— will be used?
    $\Rightarrow$ bear in mind the *lookahead* token.

at most one γ_j may make sense

(Counter)example: backtracking

$$\begin{aligned} S &\rightarrow aAc \\ &\mid c \\ A &\rightarrow aBb \\ &\mid Bc \\ B &\rightarrow aB \\ &\mid \epsilon \end{aligned}$$

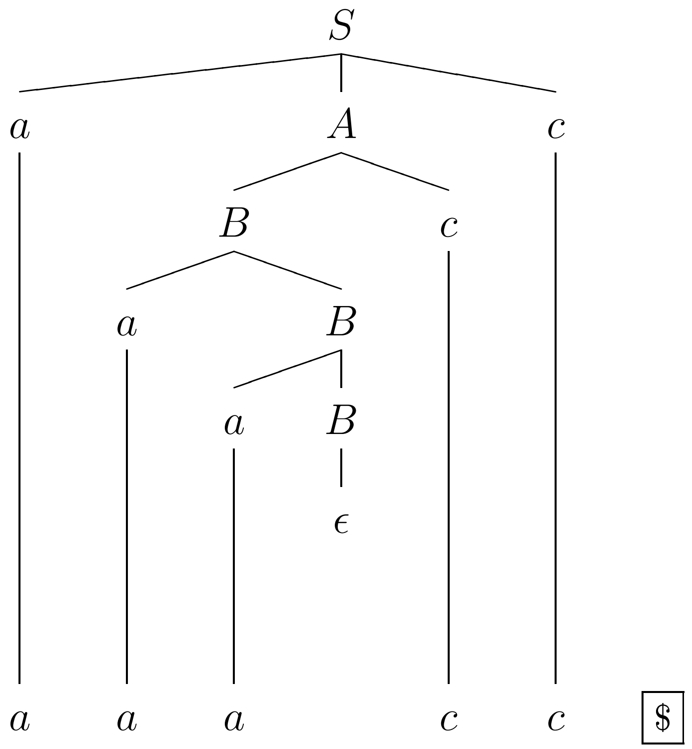
$w = a\,a\,a\,c\,c$



(Counter)example: backtracking

$$
\begin{aligned}
S &\rightarrow aAc \\
  &\mid\ c \\
A &\rightarrow aBb \\
  &\mid\ Bc \\
B &\rightarrow aB \\
  &\mid\ \epsilon
\end{aligned}
$$

$w = a\,a\,a\,c\,c$



Exercises

- Find another simpler grammar that cannot be parsed with a linear algorithm.
  Give an input that demonstrate it.
- Characterize some conflicting grammars.

Notation. Backus-Naur Form (BNF)

- S, S' are the initial symbol of the grammar
- A, B, C, ..., are non-terminal symbols
- a, b, c, ..., and $\$$ are terminal symbols (tokens)
- X, Y, Z are terminal or non-terminal symbols
- u, v, w are words formed by terminal symbols, possibly empty (ϵ)
- α, β, γ are words formed by terminal and non-terminal symbols, possibly empty (ϵ)
- $A \rightarrow \alpha$ is a production rule.
- $A \rightarrow \alpha_1 \mid \ldots \mid \alpha_n$ is the group of rules for the non-terminal symbol A

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Notation. Backus-Naur Form (BNF)

- $\gamma_1 A \gamma_2 \Rightarrow \gamma_1 \alpha \gamma_2$ is a derivation step with the rule $A \rightarrow \alpha$
- $\Rightarrow^*$ is the reflexive transitive closure of $\Rightarrow$
- Leftmost-derivation ($\Rightarrow^*_{ld}$) if every step:

$$wA\beta \Rightarrow_{A\rightarrow\alpha} w\alpha\beta$$

- Rightmost-derivation ($\Rightarrow^*_{rd}$) if every step:

$$\beta A w \Rightarrow_{A\rightarrow\alpha} \beta\alpha w$$

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Extended Backus-Naur Form (EBNF)

Rules $A \to \alpha$ where α can take the form of a regular expression:

- $\alpha_1 \cdots \alpha_n$: concatenation, or ϵ
- $\alpha_1 | \ldots | \alpha_n$: alternatives
- α_1^* : 0 or more times α_1
- α_1^+ : 1 or more times α_1 ($\alpha_1 \alpha_1^*$)
- $\alpha_1?$: 0 or 1 times α_1 ($\alpha_1 | \epsilon$)
- (α_1) : parenthesis for breaking the standard precedence between operators
  $$\{\,|\,\} \prec_p \{\,\cdot\,\} \prec_p \{\,^*, \,^+, \,?\,\}$$
- a non-terminal symbol A , or a terminal symbol a

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Extended Backus-Naur Form (EBNF)

In *ANTLR*, identifiers in capital letters denote terminals symbols, and identifiers beginning in lower case denote non-terminals.

Rules take the form id : reg_exp ;

Example:

```
expr : term ( (PLUS | MINUS) term)* ;

term : factor ( (TIMES | QUOTIENT) factor)* ;

factor : IDENT ( LEFT_BRK expr RIGHT_BRK )?
       | NUM
       | LEFT_PAR expr RIGHT_PAR
       ;
```

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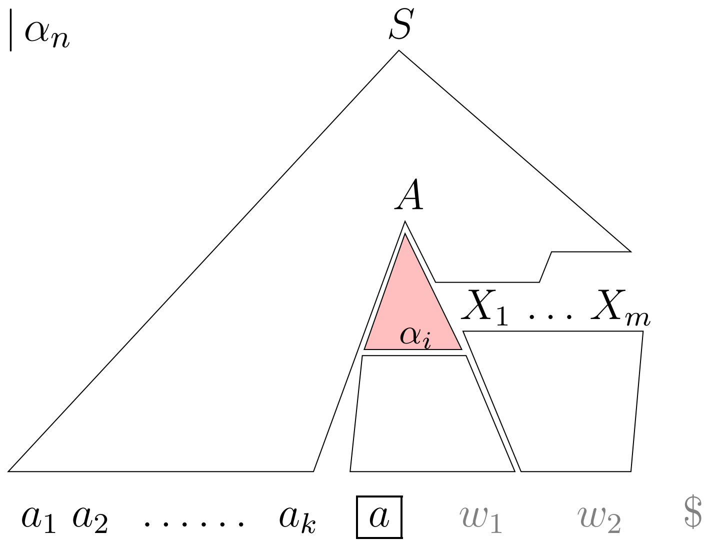
Applying a Rule $A \to \alpha_i$

$A \;\to\; \alpha_1 \,|\, \ldots \,|\, \alpha_n$

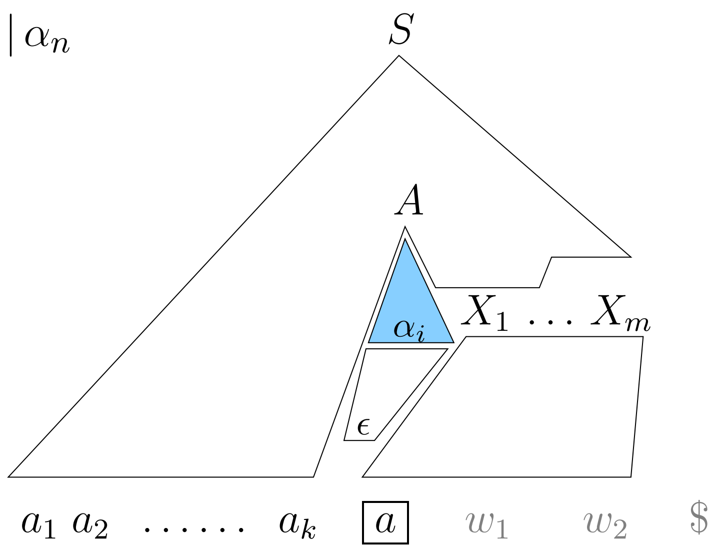


$A \Rightarrow \alpha_i \Rightarrow^* a\,w_1$ $\qquad X_1 \ldots X_m \Rightarrow^* w_2$

$a \in first(\alpha_i)$

Applying a Rule $A \to \alpha_i$

$A \;\to\; \alpha_1 \,|\, \ldots \,|\, \alpha_n$

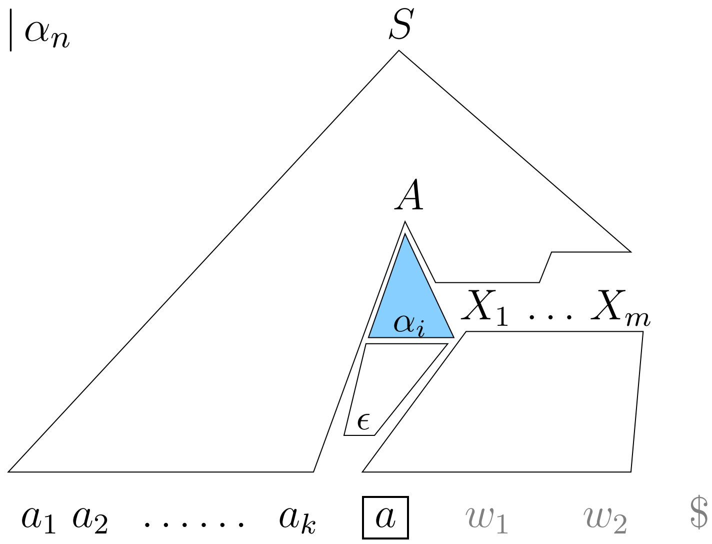


$A \Rightarrow \alpha_i \Rightarrow^* \epsilon$ $\qquad X_1 \ldots X_m \Rightarrow^* a\,w_1\,w_2$

$nullable?(\alpha_i)$ $\qquad S\,\$ \;\Rightarrow^*\; a_1 \ldots a_k\,A\,a\,w_1\,w_2\,\$$

Applying a Rule $A \to \alpha_i$

$A \;\to\; \alpha_1 \,|\, \ldots \,|\, \alpha_n$



$A \Rightarrow \alpha_i \Rightarrow^* \epsilon$

$nullable?(\alpha_i)$

$S\,\$ \Rightarrow^* \gamma_1\, A\, a\, \gamma_2$

$a \in follow(A)$

Nullable, First, Follow

- Can α derive ϵ?

$$nullable?(\alpha) \text{ iff } \alpha \Rightarrow^* \epsilon$$

- Set of terminals a that can be the first symbol of words derived from α

$$first(\alpha) \;=\; \{\, a \mid \alpha \Rightarrow^* a\, w \,\}$$

- Set of terminals a that can follow A in a derivation from $S\,\$$

$$follow(A) \;=\; \{\, a \mid S\,\$ \Rightarrow^* \gamma_1 A\, a\, \gamma_2 \,\}$$

To Sum Up . . .

$$
\begin{aligned}
S &\rightarrow aAc \\
&\mid c \\
A &\rightarrow aBb \\
&\mid Bc \\
B &\rightarrow aB \\
&\mid \epsilon
\end{aligned}
$$

$$
\begin{aligned}
first(aB) &= \{a\} \\
first(\epsilon) &= \emptyset \\
first(B) &= first(aB) \cup first(\epsilon) \\
&= \{a\} \\
first(aBb) &= \{a\} \\
first(Bc) &= \{a, c\} \\
first(A) &= first(aBb) \cup first(Bc) \\
&= \{a, c\} \\
first(aAc) &= \{a\} \\
first(c) &= \{c\} \\
first(S) &= first(aAc) \cup first(c) \\
&= \{a, c\}
\end{aligned}
$$

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To Sum Up . . .

$$
\begin{aligned}
S &\rightarrow aAc \\
&\mid c \\
A &\rightarrow aBb \\
&\mid Bc \\
B &\rightarrow aB \\
&\mid \epsilon
\end{aligned}
$$

$$
\begin{aligned}
first(aB) &= \{a\} \\
first(\epsilon) &= \emptyset \\
nullable?(\epsilon) &= true \\
follow(B) &= \{b, c\}
\end{aligned}
$$

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$nullable?(\alpha)$ (BNF)

$$nullable?(\alpha) \text{ iff } \alpha \Rightarrow^* \epsilon$$

- $nullable?(a) = false$
- $nullable?(A) = true$ if $\exists\, A \rightarrow \alpha \in G \wedge nullable(\alpha)$
  $= false$ otherwise
- $nullable?(X_1 \ldots X_n) = true$ if $\forall i : 1 \leq i \leq n : nullable(X_i)$
  $= false$ otherwise

$nullable?(\alpha)$ (Extended BNF)

$$nullable?(\alpha) \text{ iff } \alpha \Rightarrow^* \epsilon$$

- $nullable?(\alpha_1\, \alpha_2) = true$ if $nullable?(\alpha_1) \wedge nullable?(\alpha_2)$
  $= false$ otherwise
- $nullable?(\alpha_1 \mid \alpha_2) = true$ if $nullable?(\alpha_1) \vee nullable?(\alpha_2)$
  $= false$ otherwise
- $nullable?(\alpha_1^*) = true$
- $nullable?(\alpha_1^+) = true$ if $nullable?(\alpha_1)$
  $= false$ otherwise
- $nullable?(\alpha_1?) = true$

$first(\alpha)$ (BNF)

$$first(\alpha) \;=\; \{\, a \mid \alpha \Rightarrow^{*} a\, w \,\}$$

- $first(a) \;=\; \{a\}$
- $first(A) \;\supseteq\; first(\alpha_j)$ if $\exists\, A \rightarrow \alpha_j \in G$
- $first(X_1 \ldots X_n) \;\supseteq\; first(X_i)$ if $nullable?(X_1 \ldots X_{i-1})$

$first(\alpha)$ (Extended BNF)

$$first(\alpha) \;=\; \{\, a \mid \alpha \Rightarrow^{*} a\, w \,\}$$

- $first(\alpha_1\, \alpha_2) \;=\; first(\alpha_1)$ if $\neg\, nullable?(\alpha_1)$
  $first(\alpha_1\, \alpha_2) \;=\; first(\alpha_1) \cup first(\alpha_2)$ if $nullable?(\alpha_1)$
- $first(\alpha_1 \mid \alpha_2) \;=\; first(\alpha_1) \cup first(\alpha_2)$
- $first(\alpha_1^{*}) \;=\; first(\alpha_1)$
- $first(\alpha_1^{+}) \;=\; first(\alpha_1)$
- $first(\alpha_1?) \;=\; first(\alpha_1)$

$follow(A)$ (BNF)

$$follow(A) \;=\; \{\, a \mid S\,\$ \Rightarrow^* \gamma_1 A\, a\, \gamma_2 \,\}$$

- $follow(S) \supseteq \{\,\$\,\}$
- $follow(B) \supseteq first(\beta)$ if $\exists\, A \rightarrow \alpha B \beta \in G$
- $follow(B) \supseteq follow(A)$ if $\exists\, A \rightarrow \alpha B \in G$
  $follow(B) \supseteq follow(A)$ if $\exists\, A \rightarrow \alpha B \beta \in G \,\wedge$ $nullable?(\beta)$

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$follow(\alpha)$ (Extended BNF)

If α occurs in G: $follow(\alpha) \;=\; \{\, a \mid S\,\$ \Rightarrow^* \gamma_1 \alpha\, a\, \gamma_2 \,\}$

- $follow(\alpha) \supseteq follow(A)$ if $\exists\, A \rightarrow \alpha \in G$
- $follow(\alpha_1) \supseteq first(\alpha_2)$ if $\alpha_1\, \alpha_2$ occurs in G
- $follow(\alpha_2) \supseteq follow(\alpha_1\, \alpha_2)$ if $\alpha_1\, \alpha_2$ occurs in G
- $follow(\alpha_1) \supseteq follow(\alpha_1\, \alpha_2)$ if $\alpha_1\, \alpha_2$ occurs in G $\wedge\, nullable?(\alpha_2)$
- $follow(\alpha_1) \supseteq follow(\alpha_1 \mid \alpha_2)$ if $\alpha_1 \mid \alpha_2$ occurs in G
- $follow(\alpha_2) \supseteq follow(\alpha_1 \mid \alpha_2)$ if $\alpha_1 \mid \alpha_2$ occurs in G
- $follow(\alpha_1) \supseteq first(\alpha_1) \cup follow(\alpha_1^*)$ if α_1^* occurs in G
- $follow(\alpha_1) \supseteq first(\alpha_1) \cup follow(\alpha_1^+)$ if α_1^+ occurs in G
- $follow(\alpha_1) \supseteq follow(\alpha_1?)$ if $\alpha_1?$ occurs in G

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Methods of Linear Parsing

The list of tokens will be traversed *left-to-right*. Decisions to proceed take into account one token of lookahead.

- Top-down parsers (LL(1))
  - Build the AST from the root to the leaves (top-down)
  - Follow a left-most derivation in forward direction
  - More intuitive: can be *manually* written
  - **Grammars may need preprocessing**
- Bottom-up parsers (LR(1))
  - Build the AST from the leaves to the root (bottom-up)
  - Follow a right-most derivation in *backward* direction
  - Less intuitive than top-down parsers
  - Slightly more powerful

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Elimination of Left Recursion

Grammar G:

$$E \rightarrow E + \texttt{id} \mid \texttt{id}$$

$$first(E + \texttt{id}) = \{\texttt{id}\}$$
$$first(\texttt{id}) = \{\texttt{id}\}$$
$$first(E + \texttt{id}) \cap first(\texttt{id}) \neq \emptyset$$

$w = \texttt{id} + \texttt{id} + \texttt{id}$

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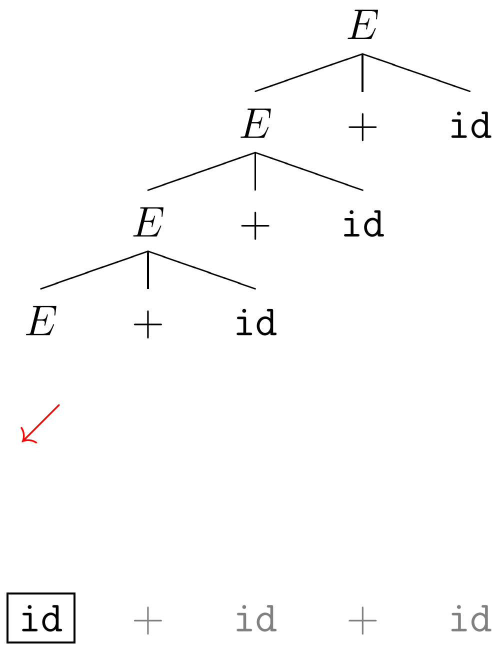
Elimination of Left Recursion

Grammar G:

$$\begin{aligned} E &\rightarrow E + \texttt{id} \\ &\mid \texttt{id} \end{aligned}$$

$w = \texttt{id} + \texttt{id} + \texttt{id}$



Elimination of Left Recursion (BNF)

$$\begin{aligned} A &\rightarrow A\,\alpha_1 \\ &\mid \ldots \\ &\mid A\,\alpha_n \\ &\mid \beta_1 \\ &\mid \ldots \\ &\mid \beta_m \end{aligned}$$

Transform into right recursion:

$$\begin{aligned} A &\rightarrow \beta_1\,A' \\ &\mid \ldots \\ &\mid \beta_m\,A' \end{aligned}$$

$$\begin{aligned} A' &\rightarrow \alpha_1\,A' \\ &\mid \ldots \\ &\mid \alpha_n\,A' \\ &\mid \epsilon \end{aligned}$$

Elimination of Left Recursion (EBNF)

$$
\begin{aligned}
A \;\rightarrow\;& A\,\alpha_1 \\
|\;& \ldots \\
|\;& A\,\alpha_n \\
|\;& \beta_1 \\
|\;& \ldots \\
|\;& \beta_m
\end{aligned}
$$

Extended BNF: use regular expressions

$$A \rightarrow (\,\beta_1 \,|\, \ldots \,|\, \beta_m\,)\;(\,\alpha_1 \,|\, \ldots \,|\, \alpha_n\,)^*$$

$$
\begin{aligned}
A &\rightarrow B\,(A')^* \\
B &\rightarrow \beta_1 \,|\, \ldots \,|\, \beta_m \\
A' &\rightarrow \alpha_1 \,|\, \ldots \,|\, \alpha_n
\end{aligned}
$$

Exercises

$$
\begin{aligned}
LI \;\rightarrow\;& LI\;\; I \\
|\;& I
\end{aligned}
$$

$$
\begin{aligned}
E \;\rightarrow\;& E + T \\
|\;& T \\
T \;\rightarrow\;& T * F \\
|\;& F \\
F \;\rightarrow\;& (\,E\,) \\
|\;& \texttt{id}
\end{aligned}
$$

Indirect left recursion:

$$
\begin{aligned}
A \;\rightarrow\;& B\,d \\
B \;\rightarrow\;& C\,e \\
C \;\rightarrow\;& A\,f \\
|\;& g
\end{aligned}
$$

Left Factoring

$$
\begin{aligned}
E &\rightarrow T + E \\
&\mid T \\
T &\rightarrow \texttt{id} \\
&\mid (E)
\end{aligned}
$$

$$first(T + E) = first(T) = \{ \texttt{id}, (\}$$

$$first(T) = \{ \texttt{id}, (\}$$

$$first(T + E) \cap first(T) \neq \emptyset$$

$$first(\texttt{id}) = \{ \texttt{id} \}$$

$$first((E)) = \{ (\}$$

$$first(\texttt{id}) \cap first((E)) = \emptyset$$

Left Factoring (BNF)

$$
\begin{aligned}
A &\rightarrow \beta\, \alpha_1 \\
&\mid \ldots \\
&\mid \beta\, \alpha_n \\
&\mid \gamma_1 \\
&\mid \ldots \\
&\mid \gamma_m
\end{aligned}
\qquad
\begin{aligned}
A &\rightarrow \beta\, A' \\
&\mid \gamma_1 \\
&\mid \ldots \\
&\mid \gamma_m \\
A' &\rightarrow \alpha_1 \\
&\mid \ldots \\
&\mid \alpha_n
\end{aligned}
$$

Left Factoring (EBNF)

$$
\begin{aligned}
A &\rightarrow \beta\,\alpha_1 \\
&\mid \ \dots \\
&\mid \ \beta\,\alpha_n \\
&\mid \ \gamma_1 \\
&\mid \ \dots \\
&\mid \ \gamma_m
\end{aligned}
$$

Extended BNF:

$$A \rightarrow \beta\,(\,\alpha_1 \mid \dots \mid \alpha_n\,) \mid \gamma_1 \mid \dots \mid \gamma_m$$

$$
\begin{aligned}
A &\rightarrow \beta\,A' \mid \gamma_1 \mid \dots \mid \gamma_m \\
A' &\rightarrow \alpha_1 \mid \dots \mid \alpha_n
\end{aligned}
$$

Exercises

$$
\begin{aligned}
P &\rightarrow \texttt{if}\ C\ \texttt{then}\ P\ \texttt{endif} \\
&\mid \ \texttt{if}\ C\ \texttt{then}\ P\ \texttt{else}\ P\ \texttt{endif} \\
&\mid \ \texttt{p} \\
C &\rightarrow \texttt{c}
\end{aligned}
$$

$$
\begin{aligned}
I &\rightarrow LE\ \texttt{':='}\ E \\
&\mid \ \texttt{write}\ \texttt{'('}\ E\ \texttt{')'} \\
&\mid \ \texttt{id}\ \texttt{'('}\ E\ \texttt{')'} \\
E &\rightarrow \texttt{id} \\
&\mid \ \texttt{num} \\
LE &\rightarrow \texttt{id}
\end{aligned}
$$

Types of Top-down Parsers

- Table Driven parsers (iterative)
  - Parsing algorithm is fixed, driven by a decision table
  - Table M is built from the grammar G.
    Empty boxes correspond to syntax errors

M	a_1	$\dots$	a	$\dots$	a_n	$\$$
A_1						
$\vdots$						
A			$A \to \alpha_k$			
$\vdots$						
A_m						

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Types of Top-down Parsers

- Table Driven parsers (iterative)
  - Parsing algorithm is fixed, driven by a decision table
  - Table M is built from the grammar G.
    Empty boxes correspond to syntax errors
- Recursive predictive parsers
  - Parsing algorithm is formed by a set of mutually recursive functions
  - Each rule $A \to \alpha$ generates the code of its function

```
void A(void) {
    // Code generated from α
}
```

  - `Gencode` describes how to translate a rule to the associated function

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Table-driven Top-down Parser

M	a_1	$\dots$	a	$\dots$	a_n	$\$$
A_1						
$\vdots$						
A			$A \rightarrow \alpha_k$			
$\vdots$						
A_m						

$$A \rightarrow \alpha_1 \mid \dots \mid \alpha_k \mid \dots \mid \alpha_o$$

$A \rightarrow \alpha_k \in M[A, a]$ if :

- $a \in first(\alpha_k)$, or
- $nullable?(\alpha_k)$ and $a \in follow(A)$

Table-driven Top-down Parser

Algorithm to build the parser table $M[A, a]$

for all rule $A \rightarrow \alpha \in G$ do
  add $A \rightarrow \alpha$ to $M[A, a]$ if :
  - $a \in first(\alpha)$ or,
  - $nullable?(\alpha)$ and $a \in follow(A)$

Exercises

Complete the table M for the following grammars, indicating the possible problems.

Grammar G_1 :

$$\begin{aligned} E &\rightarrow E + T \\ &\mid T \\ T &\rightarrow T * F \\ &\mid F \\ F &\rightarrow \texttt{id} \\ &\mid (E) \end{aligned}$$

Grammar G_2 :

$$\begin{aligned} E &\rightarrow T\, E' \\ E' &\rightarrow +\, T\, E' \\ &\mid \epsilon \\ T &\rightarrow F\, T' \\ T' &\rightarrow *\, F\, T' \\ &\mid \epsilon \\ F &\rightarrow \texttt{id} \\ &\mid (E) \end{aligned}$$

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Exercises

Complete the table M for the following grammars, indicating the possible problems.

Grammar G_3 :

$$\begin{aligned} P &\rightarrow \texttt{if}\ C\ \texttt{then}\ P \\ &\mid \texttt{if}\ C\ \texttt{then}\ P\ \texttt{else}\ P \\ &\mid \texttt{p} \\ C &\rightarrow \texttt{c} \end{aligned}$$

Grammar G_4 :

$$\begin{aligned} P &\rightarrow \texttt{if}\ C\ \texttt{then}\ P\ P' \\ &\mid \texttt{p} \\ P' &\rightarrow \epsilon \\ &\mid \texttt{else}\ P \\ C &\rightarrow \texttt{c} \end{aligned}$$

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Exercises

Complete the table M for the following grammars, indicating the possible problems.

Grammar G_5 :

$P \to$ `if` C `then` P P'
$\quad |$ `p`
$P' \to$ `endif`
$\quad |$ `else` P `endif`
$C \to$ `c`

Table-driven Top-down Parser Algorithm

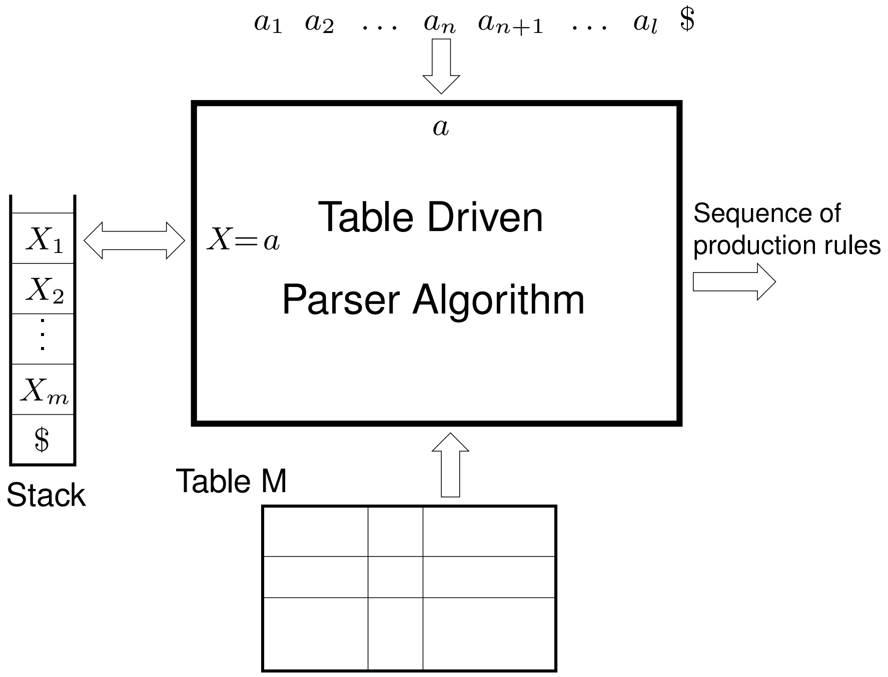

Table-driven Top-down Parser Algorithm

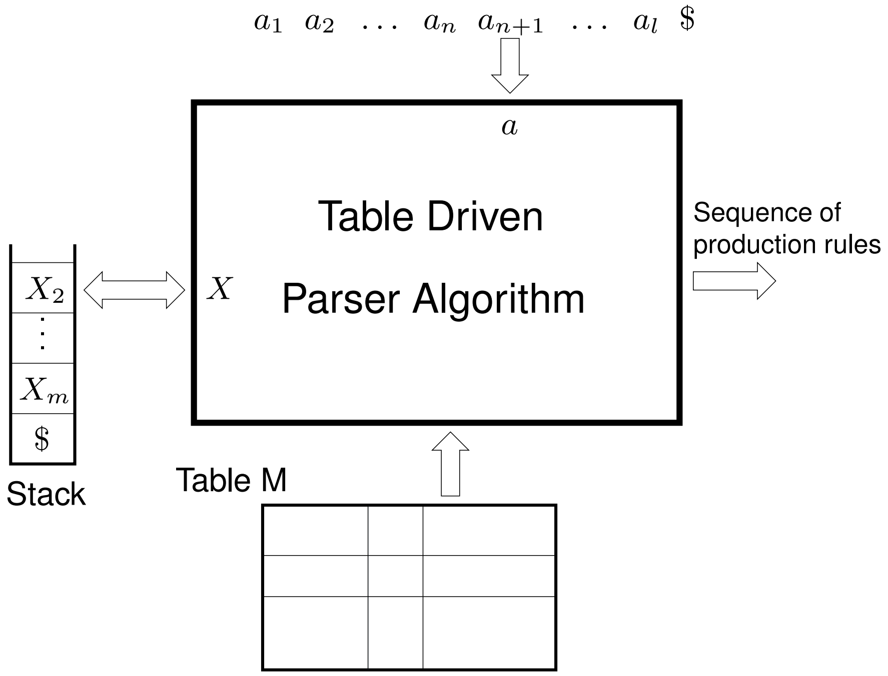

Table-driven Top-down Parser Algorithm

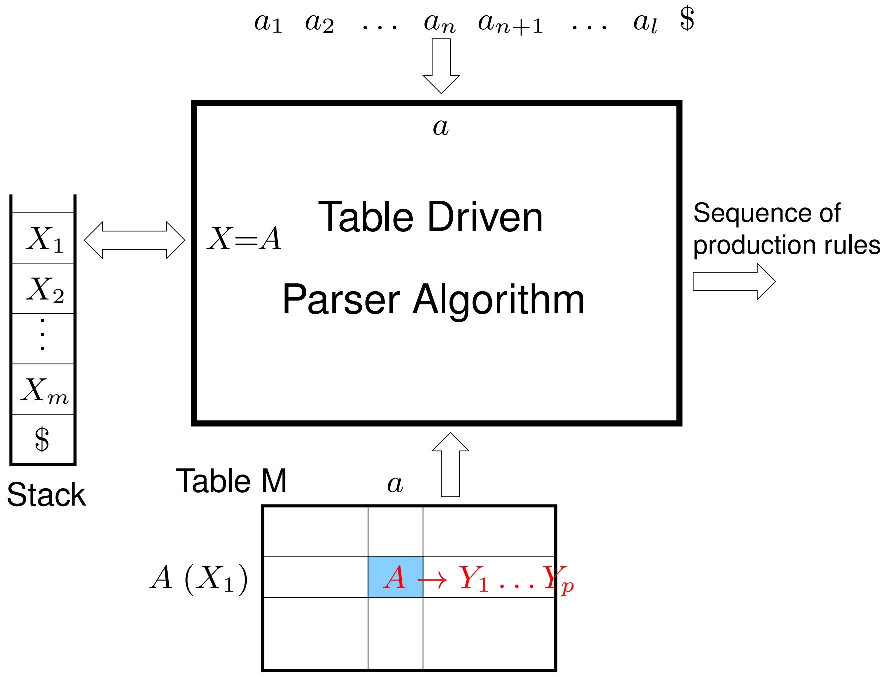

Table-driven Top-down Parser Algorithm

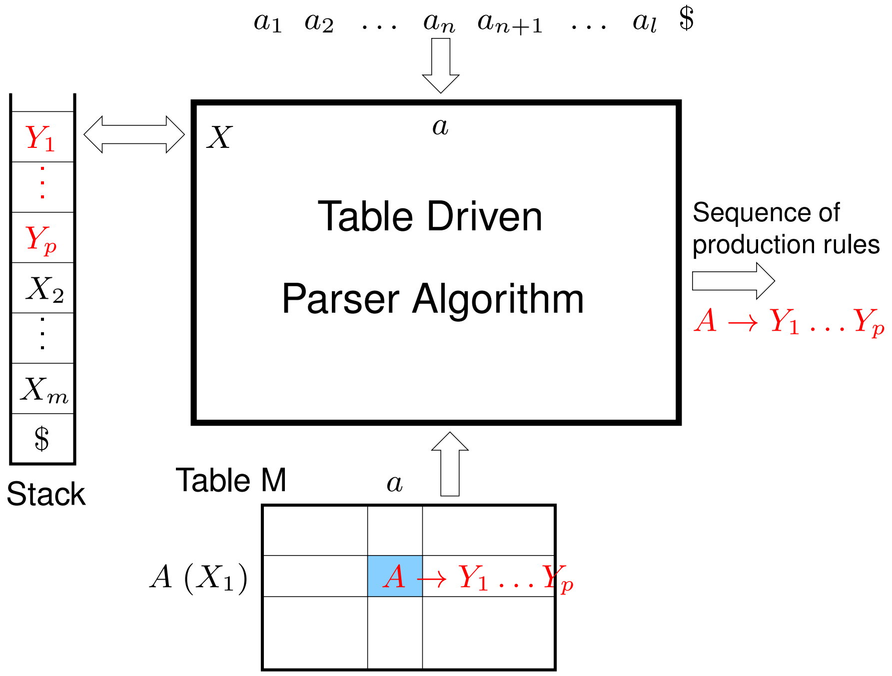

Table-driven Top-down Parser Algorithm

```
Stk := EmptyStack(); PushStack(Stk, $); PushStack(Stk, S);
X := TopStack(Stk); a := FirstToken();
while X ≠ $ do
    if X is terminal then
        if X = a then
            PopStack(Stk); a := NextToken();
        else
            throw syntax error
    else // X is non−terminal
        if M[X, a] is empty (is error) then
            throw syntax error
        else // M[X, a] = X → Y1 ... Yp
            emit production rule X → Y1 ... Yp
            PopStack(Stk); for i := p downto 1 do PushStack(Stk, Yi);
    X := TopStack(Stk);
endwhile
```