Syntactic Analysis (Parsing)

José Miguel Rivero

rivero@cs.upc.edu

Barcelona School of Informatics (FIB)

Universitat Politècnica de Catalunya BarcelonaTech (UPC)

Summary

- Objectives of Syntax Analysis
- Context Free Grammars. Applications
- Parsing in Compilers / Interpreters
- Syntax vs. Semantics
- Derivation. Parse Tree
- Cocke-Younger-Kasami Parsing Algorithm
- Parse Tree and Abstract Syntax Tree (AST)
- Ambiguous Grammars
- Linear Parsing Algorithms
  - Top-down $LL(1)$ parsers
  - Bottom-up $LR(1)$ parsers

Objectives of Syntax Analysis

- Given a CFG (Context Free Grammar) G, with start symbol S, and a word w (a sequence of tokens), can w be generated by G ?

$$w \in \mathcal{L}(G)\ ? \qquad S \Rightarrow^* w\ ?$$

- Analyze the sequence of tokens to determine their grammatical structure with respect to G.
  Compute the *parse tree* corresponding to the input
- Detect, diagnose, and recovery from *syntax errors*
- Accepts some invalid constructs, filtered out by the semantic analysis

Context Free Grammars

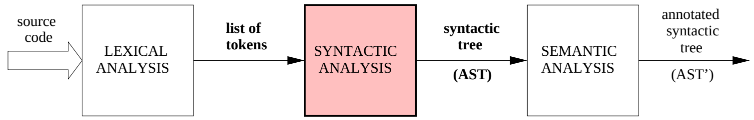
- Expressive power
  - $\mathcal{L}_1 = \{\ a^n\, b^n \mid n \geq 0\ \}$
  - $\mathcal{L}_2 = \{\ w\, c\, w^{-1} \mid w \in (a|b)^*\ \}$
  - $\mathcal{L}_3 = \{\ w\, c\, w \mid w \in (a|b)^*\ \}$
  - $\mathcal{L}_4 = \{\ a^n\, b^m\, c^n\, d^m \mid n, m \geq 0\ \}$
  - Algebraic expressions involving numbers, operations $+$ and $*$, and left and right parentheses
  - Constructions of programming languages such as declarations, statements (assignment, `if`, `while`, ...), expressions, etc.

Context Free Grammars

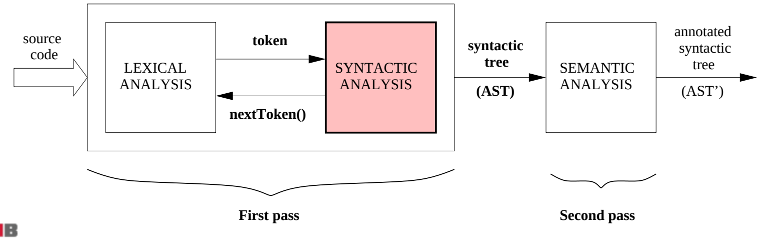
- Applications
  - Natural language processing (NLP)
  - Type-setting languages (nroff, postscript), document and extensible markup languages (LaTeX, SGML, XML, . . .)
  - Query for databases and information systems (SQL, DataLog, LDAP, . . .)
  - Logical synthesis and simulation of electronic circuits (VHDL)
  - Graphical or modelling languages (UML, Energy Systems Language)
  - Many applications come with their built-in language. For example, the scripting language for Adobe Flash (ActionScript)

Parsing in Compilers / Interpreters

Conceptual structure:

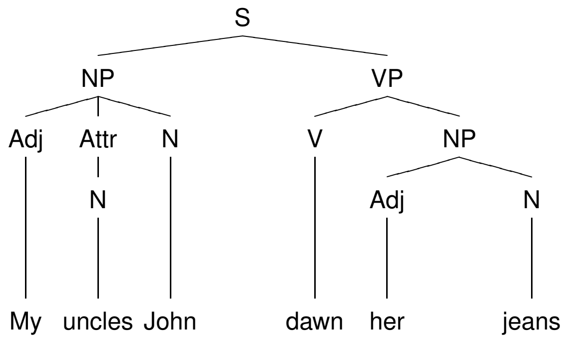


Usual structure:



Syntax vs. Semantics

- "My uncles John dawn her jeans"



- "Marketplace for : input the a ."
- "They are hunting dogs"

Syntax vs. Semantics

- Programming languages.
  Syntax errors:

```
public class {
    int static i;
    boolean j;
    public double i(UnknownClass k {
        i..x = "hello";
        if (i / 2)
            i + 1 = j;
        return i > ;
    }
    }
}
```

Syntax vs. Semantics

- Programming languages.
  Semantic errors:

```
public class MyFirstClass {
    static int i;
    boolean j;
    public double i(UnknownClass k) {
        i.x[10] = "hello";
        if (i / 2)
            i + 1 = j;
        return i > 3.14;
    }
}
```


Syntax vs. Semantics

- Programming languages.
  Ambiguity:

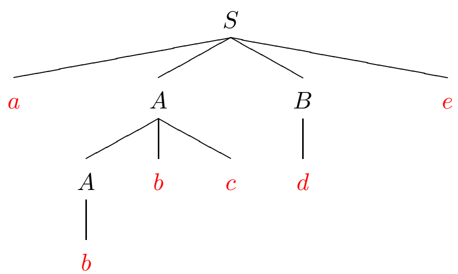
```
class Bar {
    public float foo( ) {
        return 1;
    }

    public int foo( ) {
        return 2;
    }
};

float x = Bar::foo( );
```


Derivation. Parse Tree

Grammar G: $\begin{cases} S \to aABe \\ A \to Abc \mid b \\ B \to d \end{cases}$ $\qquad w = abbcde$

$$S \Rightarrow aABe \Rightarrow aAbcBe \Rightarrow abbcBe \Rightarrow abbcde = w$$

Parse tree:



Cocke-Younger-Kasami Algorithm

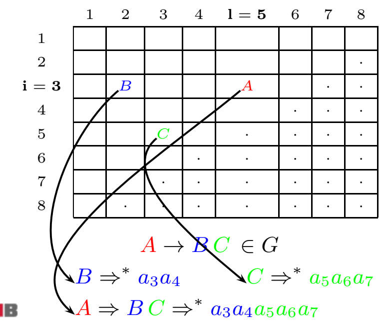
Input : A CFG in *Chomsky Normal Form* G, and a word $w = a_1 \ldots a_n$

Only rules of the form $A \to a$ or $A \to B\,C$

Output : An $n \times n$ table T, where $T[i, l]$ is the set of nonterminals A

that generate the substring $a_i \ldots a_{i+l-1}$

Example : $w = a_1 a_2 \ldots a_8$



Cocke-Younger-Kasami Algorithm

Input : A CFG in *Chomsky Normal Form* G, and a word $w = a_1 \ldots a_n$

Only rules of the form $A \to a$ or $A \to B\,C$

Output : An $n \times n$ table T, where $T[i,\ l]$ is the set of nonterminals A that generate the substring $a_i \ldots a_{i+l-1}$

Algorithm :

```
for all i ∈ [1..n] : T[i, 1] := {A | A → a_i ∈ G}
for all l ∈ [2..n] :
  for all i ∈ [1..n − l] :
    for every A → B C ∈ G :
      if there is a l_1 ∈ [1..l − 1] such that
        B ∈ T[i, l_1] and C ∈ T[i + l_1, l − l_1]
      then
        T[i, l] := T[i, l] ∪ {A}
```

Using dynamic programming, each table entry can be filled in $O(n)$ time.

The algorithm runs in $O(n^3)$ time.

FIB

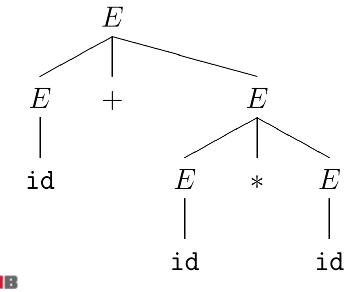
Parsing expressions

$G: \quad E \to E + E \mid E * E \mid (E) \mid \mathtt{id}$ $\qquad w = id_1 + id_2 * id_3$

(Leftmost) Derivation #1 :

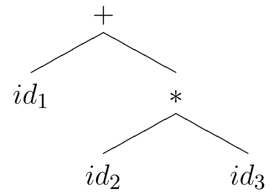
$E \Rightarrow_{lm} E + E \Rightarrow_{lm} \mathtt{id} + E \Rightarrow_{lm} \mathtt{id} + E * E \Rightarrow_{lm} \mathtt{id} + \mathtt{id} * E \Rightarrow_{lm} \mathtt{id} + \mathtt{id} * \mathtt{id}$

Parse Tree:



$id_1 + (id_2 * id_3)$

Abstract Syntax Tree (AST):



FIB

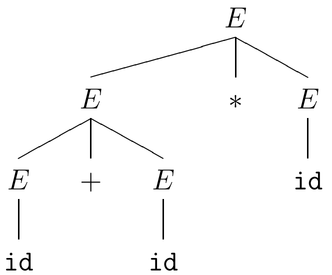
Parsing expressions

$G: \quad E \to E + E \mid E * E \mid (E) \mid \mathtt{id}$ $\qquad w = id_1 + id_2 * id_3$

(Leftmost) Derivation #2 :

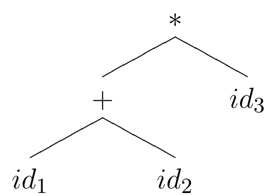
$E \Rightarrow_{lm} E * E \Rightarrow_{lm} E + E * E \Rightarrow_{lm} \mathtt{id} + E * E \Rightarrow_{lm} \mathtt{id} + \mathtt{id} * E \Rightarrow_{lm} \mathtt{id} + \mathtt{id} * \mathtt{id}$

Parse Tree:



$(id_1 + id_2) * id_3$

Abstract Syntax Tree (AST):



FIB

Ambiguous grammar

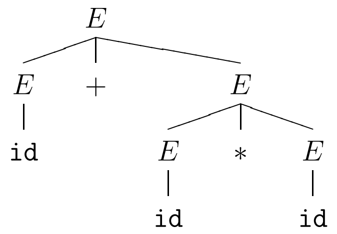
A grammar G is said to be *ambiguous* if there is some string w that has more than one parse tree or more than one leftmost derivation.

FIB

Ambiguous grammar

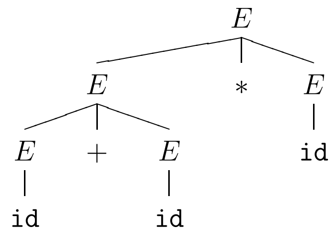
$G: \;\; E \rightarrow E + E \mid E * E \mid (\,E\,) \mid \texttt{id}$
$w = id_1 + id_2 * id_3$

Parse Tree A:



$id_1 + (id_2 * id_3)$

Parse Tree B:



$(id_1 + id_2) * id_3$

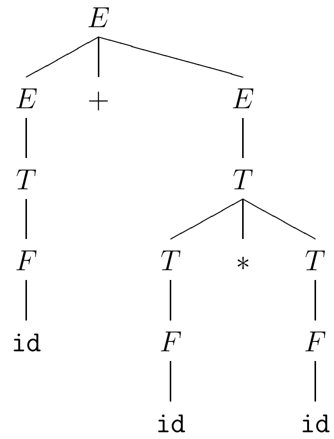
FIB

Operator's precedence

- $id_1 + (id_2 * id_3 * id_4) + id_5$
- Precedence: $+ \prec_p *$
- This other grammar G' reflects the operator's precedence:

  $E \rightarrow E + E \mid T$
  $T \rightarrow T * T \mid F$
  $F \rightarrow (\,E\,) \mid \texttt{id}$
- Now $w = id_1 + id_2 * id_3$ only has this parse tree:

  $id_1 + (id_2 * id_3)$



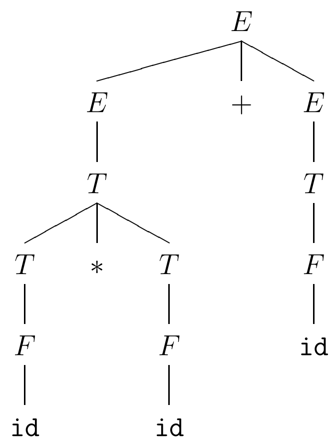
FIB

Operator's precedence

- $id_1 + (id_2 * id_3 * id_4) + id_5$
- Precedence: $+ \prec_p *$
- This other grammar G' reflects the operator's precedence:

  $E \rightarrow E + E \mid T$
  $T \rightarrow T * T \mid F$
  $F \rightarrow (\,E\,) \mid \texttt{id}$
- Now $w = id_1 * id_2 + id_3$ only has this parse tree:

  $(id_1 * id_2) + id_3$



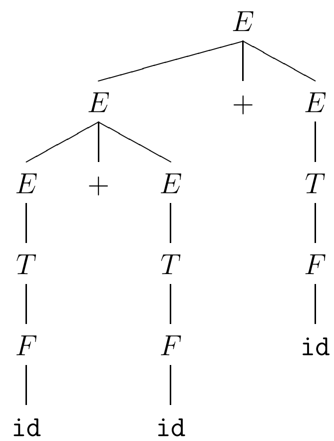
FIB

Operator's precedence

- $id_1 + (id_2 * id_3 * id_4) + id_5$
- Precedence: $+ \prec_p *$
- This other grammar G' reflects the operator's precedence:

  $E \rightarrow E + E \mid T$
  $T \rightarrow T * T \mid F$
  $F \rightarrow (\,E\,) \mid \texttt{id}$
- But $w = id_1 + id_2 + id_3$ still has two parse trees:

  $(id_1 + id_2) + id_3$

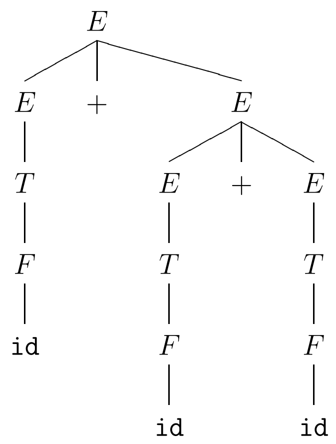


FIB

Operator's precedence

- $id_1 + (id_2 * id_3 * id_4) + id_5$
- Precedence: $+ \prec_p *$
- This other grammar G' reflects the operator's precedence:

  $E \rightarrow E + E \mid T$
  $T \rightarrow T * T \mid F$
  $F \rightarrow (\,E\,) \mid \texttt{id}$
- But $w = id_1 + id_2 + id_3$ still has two parse trees:



$id_1 + (id_2 + id_3)$

FIB

Operator's associativity

Left-associative:

$e_1 \otimes e_2 \otimes e_3$ means $(e_1 \otimes e_2) \otimes e_3$

Right-associative:

$e_1 \otimes e_2 \otimes e_3$ means $e_1 \otimes (e_2 \otimes e_3)$

Non-associative:

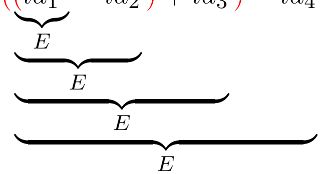
$e_1 \otimes e_2 \otimes e_3$ is not correct

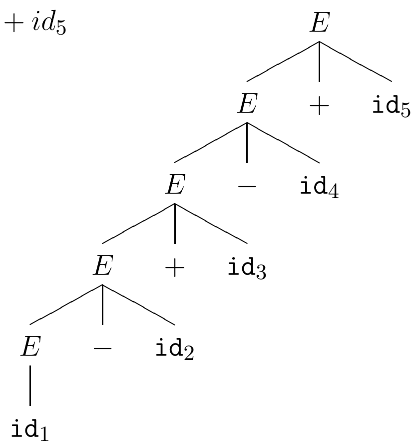
Usually arithmetic ($+$, $-$, $*$, $/$) and logical operators (`&&`, `||`) are left-associative, comparison operators ($>$, $=$, $<$, $\ldots$) are non-associative, and exponentiation (ˆ) is right-associative.

FIB

Operator's associativity

$(((id_1 - id_2) + id_3) - id_4) + id_5$





FIB

Operator's associativity

Left-associativity ($+$, $-$):

$$E \rightarrow E + \texttt{id} \mid E - \texttt{id} \mid \texttt{id}$$

Right-associativity ($+$, $-$):

$$E \rightarrow \texttt{id} + E \mid \texttt{id} - E \mid \texttt{id}$$

Left-associativity ($+$, $*$):

$$E \rightarrow E + T \mid T$$
$$T \rightarrow T * F \mid F$$
$$F \rightarrow (\,E\,) \mid \texttt{id}$$

Non-associativity ($>$):

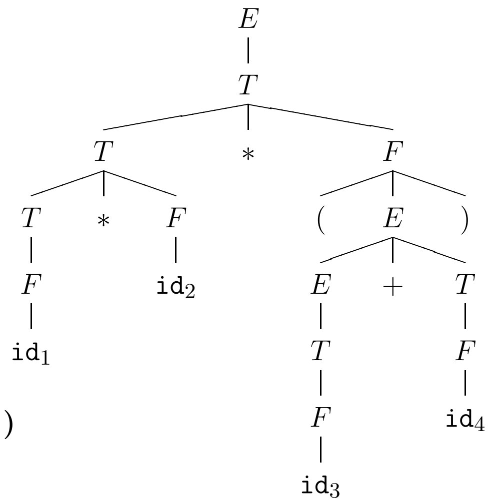
$$E \rightarrow T > T \mid T$$
$$T \rightarrow \texttt{id} \mid \texttt{num}$$

FIB

Grammar for expressions

Grammar G:

$$
\begin{aligned}
E &\rightarrow E + T \\
&\mid T \\
T &\rightarrow T * F \\
&\mid F \\
F &\rightarrow (E) \\
&\mid \texttt{id}
\end{aligned}
$$



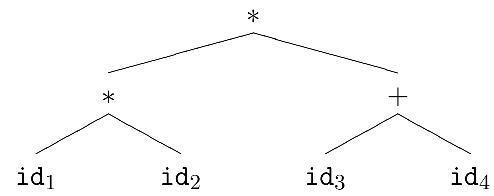
$$w = \underbrace{id_1 * id_2} * (\underbrace{id_3 + id_4})$$

Grammar for expressions

Grammar G:

$$
\begin{aligned}
E &\rightarrow E + T \\
&\mid T \\
T &\rightarrow T * F \\
&\mid F \\
F &\rightarrow (E) \\
&\mid \texttt{id}
\end{aligned}
$$

Abstract Syntax Tree (AST) :



$$w = \underbrace{id_1 * id_2} * (\underbrace{id_3 + id_4})$$

Exercises

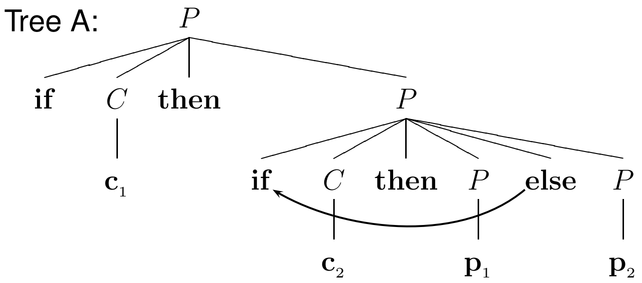
- Extend the grammar for expressions with the following operators and precedence:
  $\{>, <, =\} \prec_p \{+, -\} \prec_p \{*, /\} \prec_p \{-_u\}$
  where $-_u$ denotes unary minus ($-e_1$).
  All the binary operators are left-associative.
- Extend the previous grammar incorporating the access-to-array operator [ ] (with the usual syntax ($e_1[e_2]$), the access-to-struct operator ., and the postfix access-to-pointed operator ˆ (e_1ˆ).
  All of them have maximal precedence.

Ambiguous If-then-else (dangling else)

Grammar: $w =$ **if c₁ then if c₂ then p₁ else p₂**

$$
\begin{aligned}
P &\rightarrow \textbf{if}\ C\ \textbf{then}\ P \\
&\mid \textbf{if}\ C\ \textbf{then}\ P\ \textbf{else}\ P \\
&\mid \textbf{p} \\
C &\rightarrow \textbf{c}
\end{aligned}
$$

Parse Tree A:

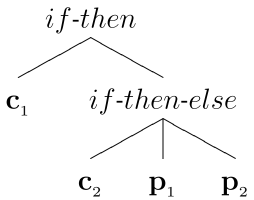


Ambiguous If-then-else (dangling else)

Grammar: $w = \textbf{if}\ \textbf{c}_1\ \textbf{then}\ \textbf{if}\ \textbf{c}_2\ \textbf{then}\ \textbf{p}_1\ \textbf{else}\ \textbf{p}_2$

$$
\begin{aligned}
P &\rightarrow \textbf{if}\ C\ \textbf{then}\ P \\
&\mid\ \textbf{if}\ C\ \textbf{then}\ P\ \textbf{else}\ P \\
&\mid\ \textbf{p} \\
C &\rightarrow \textbf{c}
\end{aligned}
$$

Abstract Syntax Tree (AST) A:



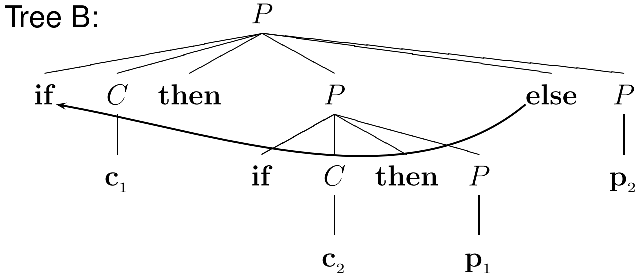
FIB

Ambiguous If-then-else (dangling else)

Grammar: $w = \textbf{if}\ \textbf{c}_1\ \textbf{then}\ \textbf{if}\ \textbf{c}_2\ \textbf{then}\ \textbf{p}_1\ \textbf{else}\ \textbf{p}_2$

$$
\begin{aligned}
P &\rightarrow \textbf{if}\ C\ \textbf{then}\ P \\
&\mid\ \textbf{if}\ C\ \textbf{then}\ P\ \textbf{else}\ P \\
&\mid\ \textbf{p} \\
C &\rightarrow \textbf{c}
\end{aligned}
$$

Parse Tree B:



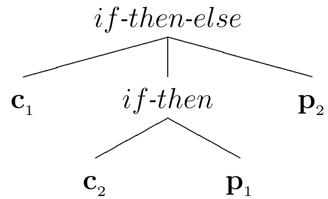
FIB

Ambiguous If-then-else (dangling else)

Grammar: $w = \textbf{if}\ \textbf{c}_1\ \textbf{then}\ \textbf{if}\ \textbf{c}_2\ \textbf{then}\ \textbf{p}_1\ \textbf{else}\ \textbf{p}_2$

$$
\begin{aligned}
P &\rightarrow \textbf{if}\ C\ \textbf{then}\ P \\
&\mid\ \textbf{if}\ C\ \textbf{then}\ P\ \textbf{else}\ P \\
&\mid\ \textbf{p} \\
C &\rightarrow \textbf{c}
\end{aligned}
$$

Abstract Syntax Tree (AST) B:



FIB

Exercise

- Find an alternative non-ambiguous grammar for *this* language.
- Give a clue?
  There are two kind of propositions:
  - *closed*, where a following `else` cannot correspond to them.
  - *open*, where a following `else` will correspond to them (to their `if`).

FIB