Clustering of exchange rates and their dynamics under different dependence measures

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Argimiro Arratia and Martí Renedo

Universitat Politècnica de Catalunya, SPAIN

Outline

Introduction

- We analyze the clustering method of currency exchange networks developed by Fenn et al., *Dynamical clustering of exchange rates*[1]
- Our aims are:
  - Implement the clustering method described by Fenn et al.
  - Propose potential improvements to the method and study how they compare.

[1] *Quantitative Finance*, 12 (10) 2012, pp.1493-1520

Choice of Currencies

The currencies included in the study are 13 of the 15 most traded as of April 2013:

- US dollar (USD)
- euro (EUR)
- yen (JPY)
- pound sterling (GBP)
- Australian dollar (AUD)
- Swiss franc (CHF)
- Canadian dollar (CAD)
- Mexican peso (MXN)
- Chinese yuan (CNY)
- New Zealand dollar (NZD)
- Swedish krona (SEK)
- Hong Kong dollar (HKD)
- Singapore dollar (SGD)

The Forex Network

- nodes will consist of every pair of currencies exchange rate return series (resulting in $n = 78$ nodes);
- edges between exchange-nodes weighted accordingly to their similarity.
- The data was sourced from the Federal Reserve Economic Data and downloaded into R using the Quandl package.
- Using the USD exchanges data, the exchange rate of every other pair of currencies is defined: $XXX/YYY = \frac{XXX/USD}{YYY/USD}$.

Pearson Correlation

Fenn et al. use Pearson correlation between two returns r_i, r_j:

$$\rho(r^i, r^j) = \frac{\mathrm{Cov}(r^i, r^j)}{\sqrt{\mathrm{Var}(r^i)\mathrm{Var}(r^j)}} \tag{1}$$

the weighted similarity matrix A^ρ is given by:

$$A^\rho_{ij} = \frac{1}{2}(\rho(r^i, r^j) + 1) - \delta_{ij} \tag{2}$$

which scales the Pearson correlation from $[-1, 1]$ to $[0, 1]$, while the Kronecker delta δ_{ij} removes self-edges.

Kendall Correlation I

A correlation measure alternative to Pearson's is the Kendall correlation (or rank correlation) τ

Definition

Given two random variables X and Y, their Kendall correlation coefficient is

$$\tau(X, Y) = p_c - p_d \tag{3}$$

where for any two independent pairs of values (X_i, Y_i), (X_j, Y_j),

$$p_c = P((X_j - X_i)(Y_j - Y_i) > 0) \quad \text{and} \quad p_d = P((X_j - X_i)(Y_j - Y_i) < 0)$$

are the probabilities of them being concordant and discordant respectively.

Kendall Correlation II

Definition

For our series of returns r^i, r^j over m time steps, the Kendall correlation is estimated by

$$\tau_m(r^i, r^j) = 2 \sum_{1 \leq s < t \leq m} \frac{\mathrm{sgn}(r_t^i - r_s^i)\mathrm{sgn}(r_t^j - r_s^j)}{n(n-1)} \tag{4}$$

Kendall adjacency matrix A^τ

$$A_{ij}^\tau = \frac{1}{2}(\tau(r^i, r^j) + 1) - \delta_{ij} \tag{5}$$

Properties

For both networks and given two of their exchange rates XXX/YYY and ZZZ/TTT, the following equality holds

$$A^{\cdot}\left(\frac{XXX}{YYY},\frac{ZZZ}{TTT}\right)=1-A^{\cdot}\left(\frac{XXX}{YYY},\frac{TTT}{ZZZ}\right), \tag{6}$$

- We cannot determine *a priori* wether two exchange rates will be correlated directly or with one of them inverted.
- As a result, we have to include all inverses in the network (so, $n = 156$)

Distance Correlation

Székely, Rizzo, Bakirov, The Annals of Statistics 35 (6), 2769-2794 (2007)

Definition

The empirical distance correlation $\mathcal{R}_n(X, Y)$, between two samples X and Y, is the square root of

$$\mathcal{R}_n^2(X, Y) = \begin{cases} \frac{\mathcal{V}_n^2(X,Y)}{\sqrt{\mathcal{V}_n^2(X)\mathcal{V}_n^2(Y)}}, & \mathcal{V}_n^2(X)\mathcal{V}_n^2(Y) > 0 \\ 0, & \mathcal{V}_n^2(X)\mathcal{V}_n^2(Y) = 0 \end{cases} \tag{7}$$

where $\mathcal{V}_n(X, Y)$ is the distance covariance.

Remark: $\mathcal{R}_n(X, Y)$ is not a distance in the metric sense. It is by definition a correlation among all distances of the samples, and $\mathcal{R}(X, X) = 1$, violating the identity of indiscernibles of a distance. The distance correlation is in fact a similarity metric.

Distance Correlation II

Given XXX/YYY, ZZZ/TTT exchange rates,

$$\mathcal{R}\left(\frac{XXX}{YYY}, \frac{ZZZ}{TTT}\right) = \mathcal{R}\left(\frac{XXX}{YYY}, \frac{TTT}{ZZZ}\right).$$

This implies that there is no need to include the inverses of the exchange rates!
Since $\mathcal{R}$ is a similarity metric, the exchange rate network is simply built from the matrix of distance correlations, $A^{\mathcal{R}}$, removing self edges:

$$A^{\mathcal{R}}_{ij} = \mathcal{R}(r^i, r^j) - \delta_{ij}$$

The Potts Method

The clustering algorithm is based on the minimization of the Hamiltonian of the partition $\mathcal{P}$ of a weighted undirected graph with adjacency matrix A

$$H(\mathcal{P}) = -\sum_{ij}[A_{ij} - \gamma P_{ij}]\delta(c_i, c_j) \tag{8}$$

where c_i is the community of node i in the partition $\mathcal{P}$ (so $\delta(c_i, c_j)$ is 1 when i and j are in the same community and 0 otherwise), P_{ij} is the expected weight of the edge ij in a null model, γ is a parameter controlling size of communities.

Selecting the null model P

- For the networks A^{ρ}_{ij} and A^{τ}_{ij}, the Newman-Girvan null model results in

$$P^{\cdot}_{ij} = \frac{(\sum_l A^{\cdot}_{il})(\sum_l A^{\cdot}_{jl})}{\sum_{i,j} A^{\cdot}_{ij}} = \frac{n-2}{2n} \tag{9}$$

- For the network $A^{\mathcal{R}}_{ij}$ we can use the average edge weight as a uniform null model:

$$P^{\mathcal{R}} = \frac{\sum_{i,j} A_{ij}}{n(n-1)} \tag{10}$$

Selecting the value of γ

- A sample network built with data from January 2010 at daily time steps is used.
- The minimization algorithm is applied for values of γ ranging from 0.4 to 2.2, in steps of length 0.01.
- We want to find intervals of γ where the number of communities stabilizes (i.e. *plateaus*) .

Pearson Correlation

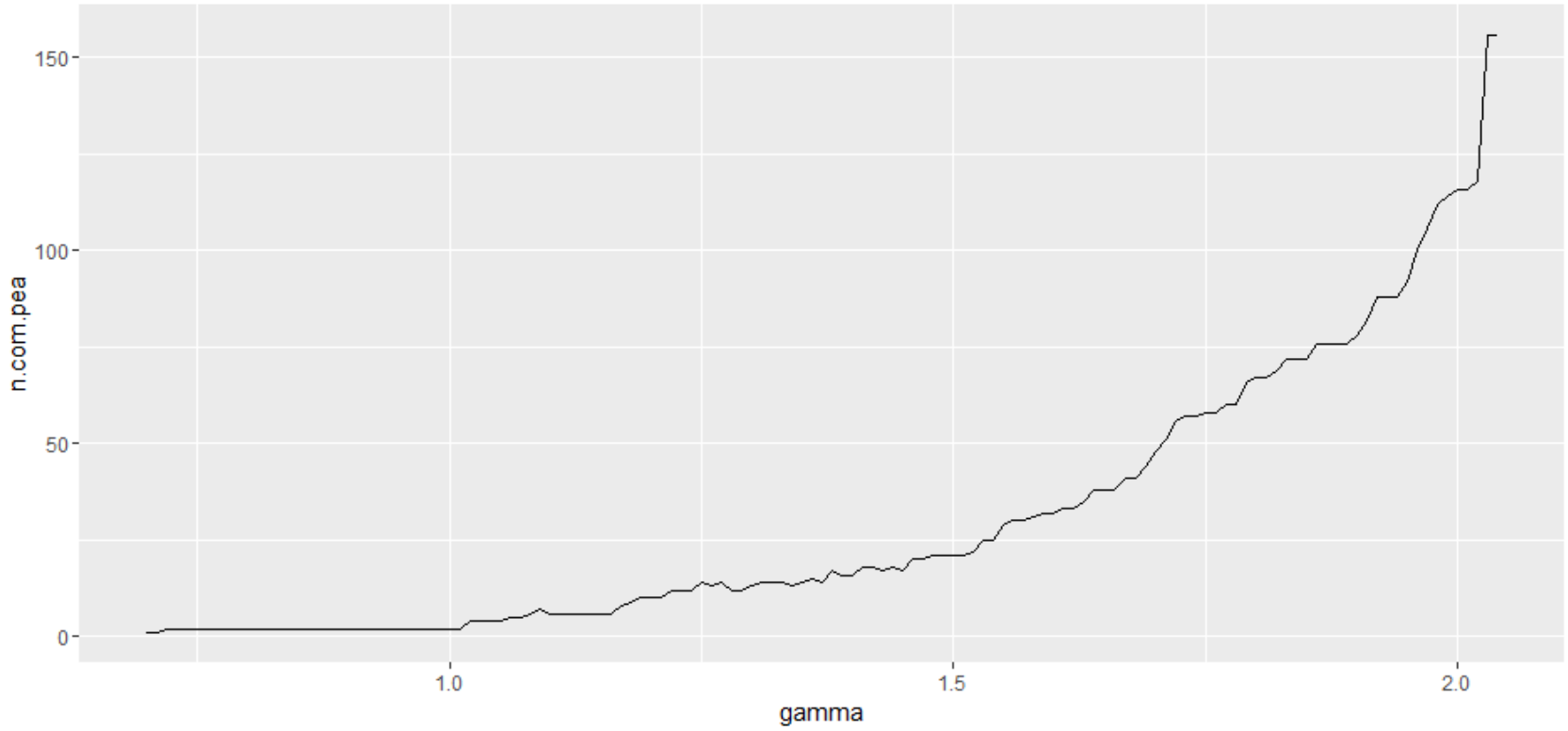


Figure: Number of communities of the Pearson correlation network for each γ

Kendall Correlation

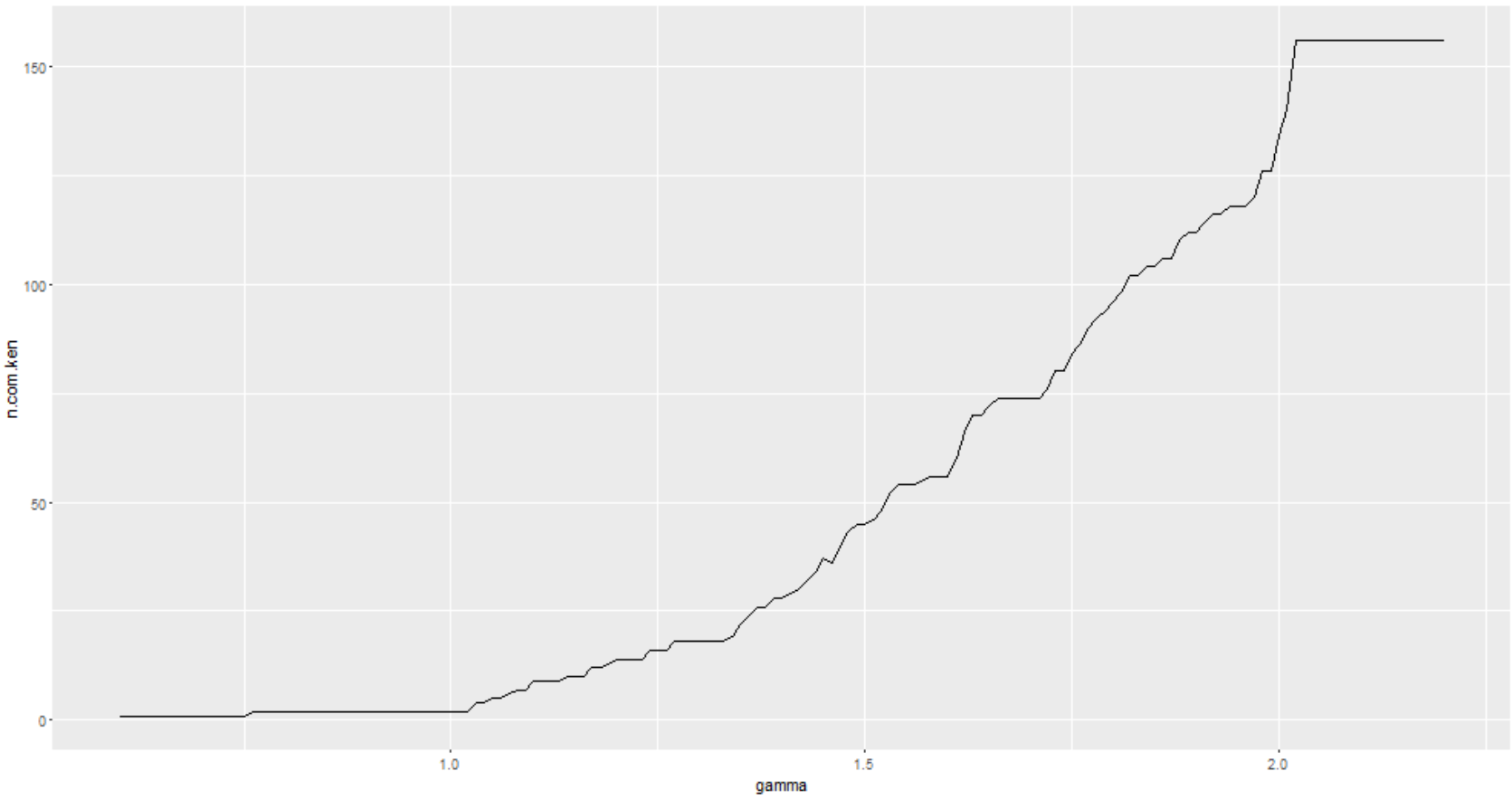


Figure: Number of communities of the Kendall correlation network for each γ

Distance Correlation

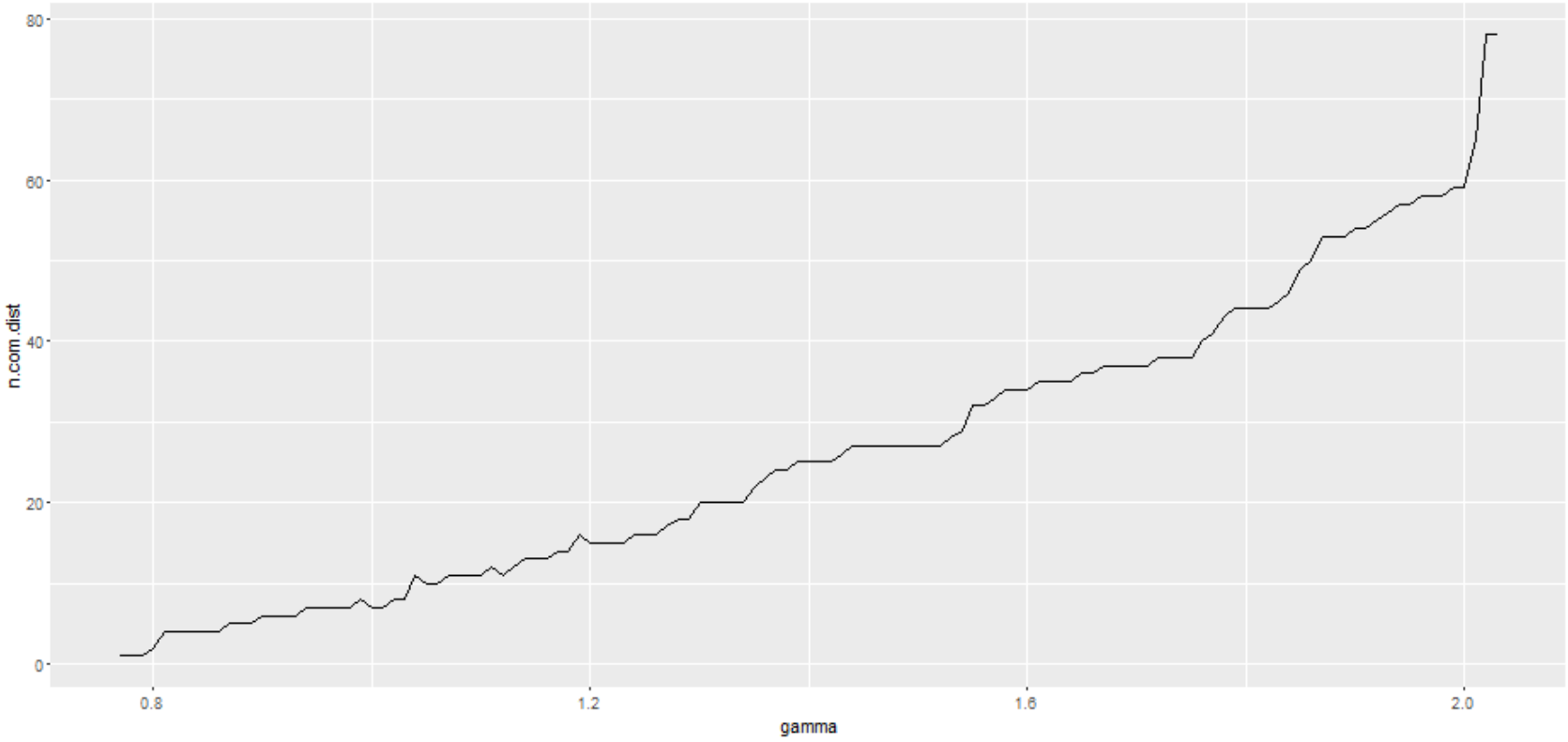


Figure: Number of communities of the distance correlation network for each γ

Chosen values of γ

Widest non trivial intervals:

- Pearson correlation: $\sim (1.48, 1.51)$
- Kendall correlation: $\sim (1.27, 1.33)$
- Distance correlation: $\sim (1.44, 1.52)$

Parameter values (chosen at the center of their corresponding intervals):

- $\gamma_\rho = 1.495$
- $\gamma_\tau = 1.3$
- $\gamma_{\mathcal{R}} = 1.48$

Minimization Algorithm

- We start with each node in a separate community.
- Each step consists of, for every node:
  - Calculate the variation of the objective function caused by moving the node to every community.
  - Move the node to the community with the biggest decrease in the objective function.
  - If all values are positive, leave the node in its community.
- Repeat until for one step of the algorithm no node changes communities, or until a certain number of iterations is reached.

Comparison of similarity metrics I

- We study daily currency data at monthly time steps over the 2009-2015 period.
- At each time step, the three networks are built from the daily returns and the minimization algorithm is applied.

Comparison of similarity metrics II

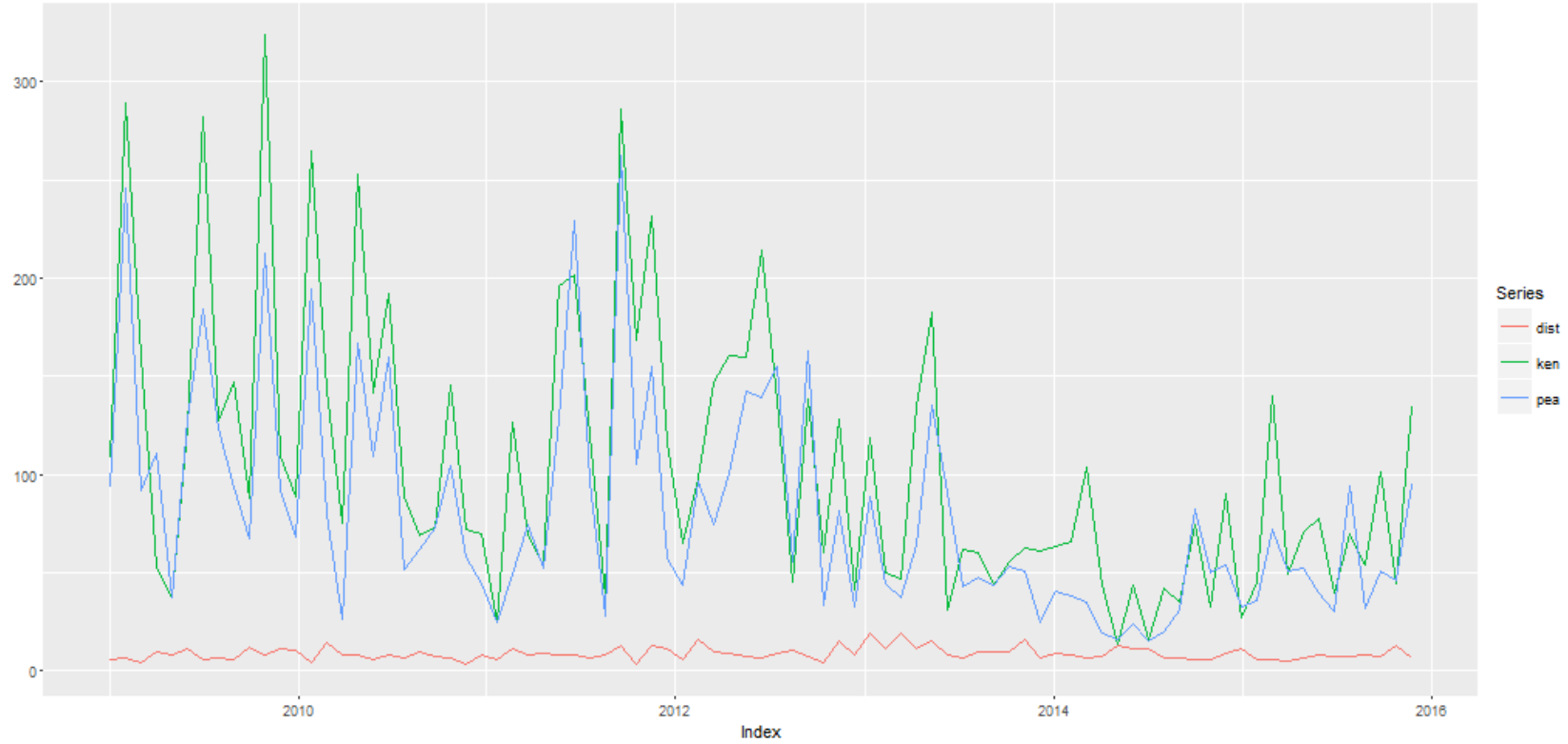


Figure: Variance of the size of communities from daily data at monthly time steps, over a 6 year period.

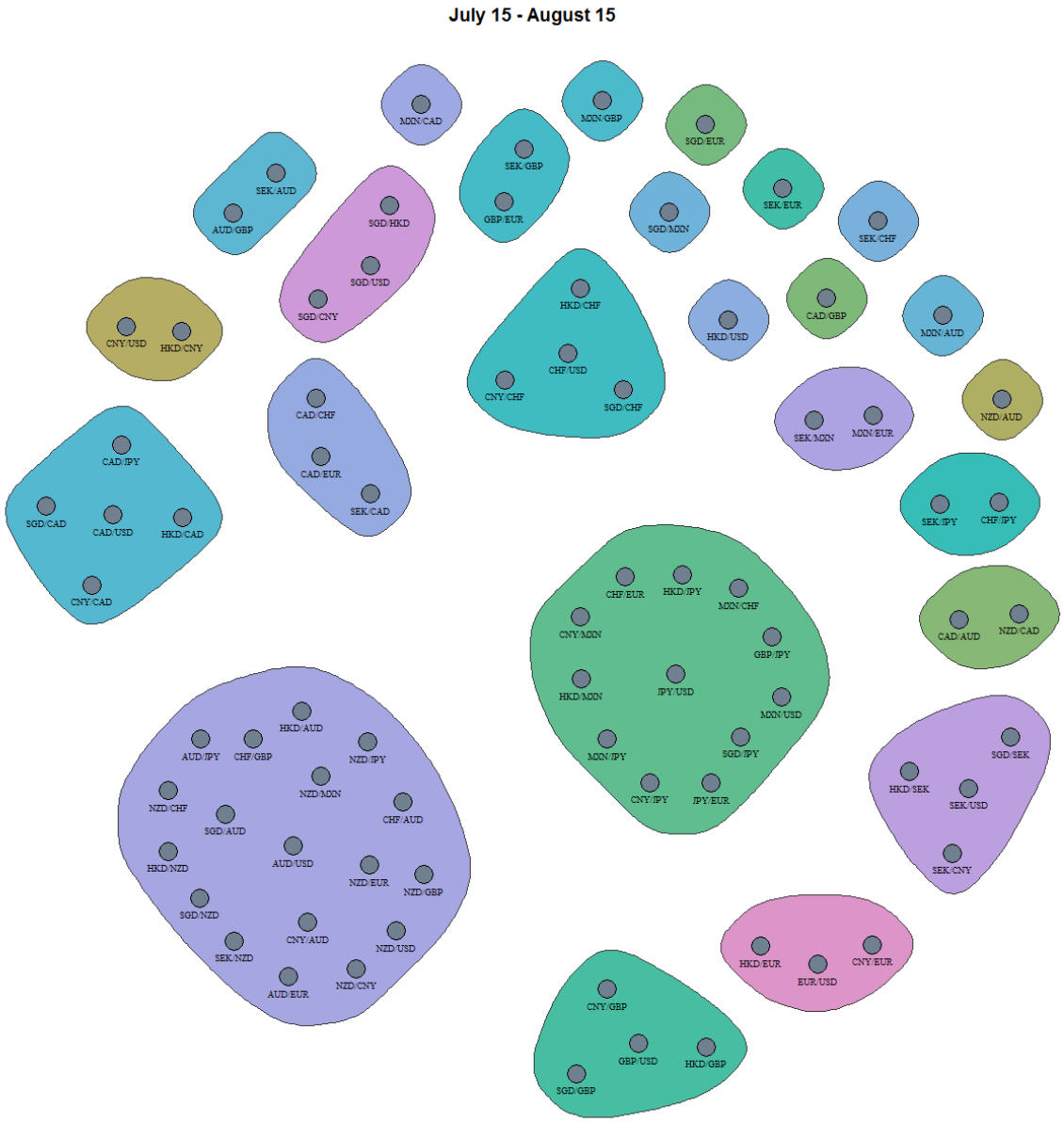
Short term community dynamics

- We apply the proposed methods to currency data in 2005–2008, scenario of credit and liquidity crisis, leading to a major reorganization of communities in the FX network throughout 2007, due to its impact on the carry trade[2]
- We study daily data in the one month time steps before and after 2007/08/15

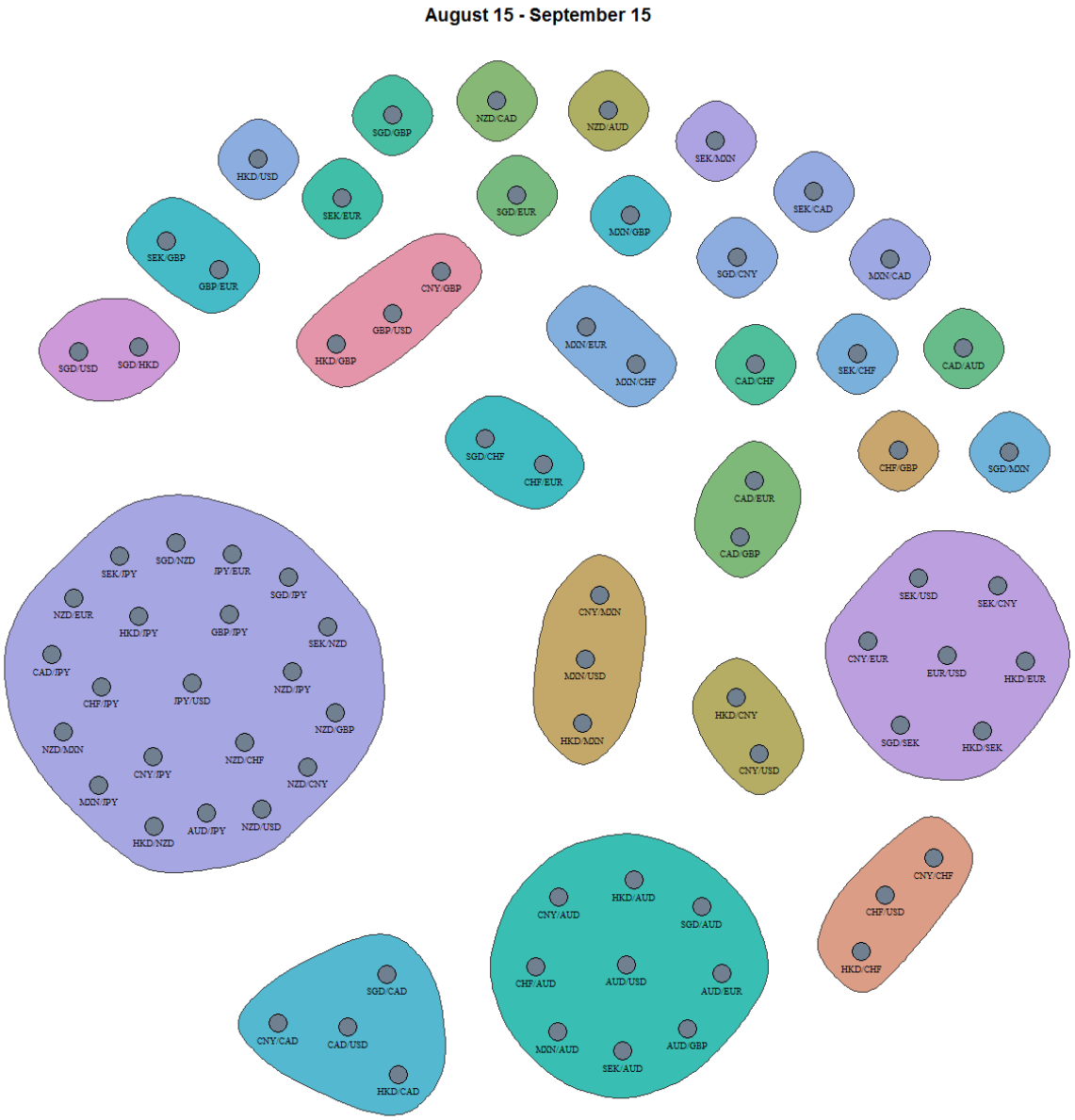
[2] *carry trade* = selling low interest rate currencies ("funding currencies", e.g. JPY and CHF), and investing in high interest rate currencies ("investment currencies", e.g. AUD and NZD). A profit is with the interest rate differentials. When there is a decrease in available credit, traders "unwind" their carry-trade positions, which consist in selling their holdings in investment currencies and buying funding currencies

Results

- In Pearson and Kendall networks, the clustering behaviour are similar: there is a big community of relevant carry trade currencies (JPY, AUD, NZD) that gain nodes at the second time step.
- In the distance correlation network, carry trade currencies are split in two communities dominated by AUD and JPY exchanges respectively. At the second time step, the AUD community looses nodes and ends up with only AUD exchanges. The JPY community grows and attracts some of those nodes.
  We believe this is a more accurate picture of the unwinding of carry trade positions occurred in the mid of 2007.



FX cluster evolution with distance correlation (mid 2007)



FX cluster evolution with distance correlation (mid 2007)

Betweenness

The concept of betweenness is based on distance between nodes. We take as a distance:

$$d_{ij} = \begin{cases} 0 & \text{if } i = j \\ 1/A_{ij} & \textit{otherwise} \end{cases} \tag{11}$$

Definition

The betweenness centrality b_i of a node i is:

$$b_i = \sum_{s \neq i} \sum_{t \neq s,i} \frac{g_{st}^i}{G_{st}}.$$

Where G_{st} is the number of shortest paths from node s to node t, and g_{st}^i as the number of shortest paths from s to t passing through i.

Results over the 2009-2015 period

Table: Top currency exchanges sorted by average betweenness.

	betweenness		
rank	distance	Kendall	Pearson
1	MXN/JPY	NZD/AUD	NZD/AUD
2	SEK/JPY	SEK/AUD	SEK/AUD
3	NZD/JPY	MXN/AUD	SEK/NZD
4	SEK/MXN	SEK/NZD	MXN/AUD
5	SEK/NZD	GBP/EUR	NZD/MXN
6	AUD/JPY	SGD/GBP	GBP/EUR
7	NZD/CHF	SGD/CAD	SGD/GBP
8	CAD/JPY	CAD/AUD	CAD/GBP
9	MXN/CHF	NZD/MXN	SGD/CAD
10	CHF/AUD	CAD/GBP	MXN/CAD

Kendall, Pearson show NO exchange with JPY. It can't be right! (see Triennial CB

Community Centrality

Consider $J_{ij} = A_{ij} - \gamma P_{ij}$ and its spectral decomposition $J = UDU^T$. D is diagonal matrix of eigenvalues β_i, U the corresponding matrix of eigenvectors.
Define q as the number of positive eigenvalues of D.

Definition

The community centrality of node i is given by the magnitude $|x_i|$, where x_i is a node vector of dimension q with j-th element given by

$$[x_i]_j = \sqrt{\beta_j} U_{ij}, \qquad j \in \{1, 2, ..., q\} \tag{12}$$

Results over the 2009-2015 period

Table: Top currency exchanges sorted by average community centrality.

	community centrality		
rank	distance	Kendall	Pearson
1	HKD/JPY	NZD/USD	NZD/USD
2	JPY/USD	HKD/NZD	HKD/NZD
3	CNY/JPY	HKD/AUD	AUD/USD
4	HKD/CHF	HKD/EUR	HKD/AUD
5	CHF/USD	EUR/USD	SEK/USD
6	MXN/CHF	AUD/USD	NZD/CNY
7	CNY/CHF	SEK/USD	HKD/SEK
8	SGD/SEK	HKD/SEK	EUR/USD
9	SGD/CHF	CHF/USD	HKD/EUR
10	HKD/SEK	HKD/CHF	SEK/CNY

Triennial Central Bank Survey on FX (2013)

The currency composition of global FX trading shifted notably between 2010 and 2013, not only among the world's most actively traded currencies, but also among important emerging market currencies.

The Japanese yen stood out as the major currency that saw the most substantial jump in trading activity, whereas the role of the euro as an international currency declined over the period. The Mexican peso and the Chinese renminbi saw the most significant rise in market share among major emerging market currencies. The role of the US dollar as the world's dominant vehicle currency remains unchallenged. FX deals with the US dollar on one side of the transaction represented 87% of all deals initiated in April 2013, . . . Among the major currencies, trading in the Japanese yen jumped the most, rising by 63% since the 2010 survey. Turnover in the USD/JPY pair rose by about 70% in this period . . . Additional information from the semiannual surveys by regional FX committees suggests that most of the rise in yen trading occurred between October 2012 and April 2013, a period characterised by expectations of a regime shift in Japanese monetary policy, which then took place in April 2013.

Conclusions

- The proposed similarity metrics applied to the community detection algorithm give some differences in the results, which could be explained by their ability to detect non linear dependances between currency exchange rates.
- In the case of the distance correlation, being able to work with a network of half the size (due to not needing to add the inverses of exchange rates) greatly reduces the computational cost of running the algorithm.
- Experiments with real data show that clustering with distance correlation give a more realistic picture of FX community reorganization through 2007-2008 credit crisis, and (expectations on) shifts in monetary policy (2009-2015).