Intro to Complex and Social Networks

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Universitat Politècnica de Catalunya

Complex and Social Networks (2023-2024)
Master in Innovation and Research in Informatics (MIRI)
Instructors

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  - Omega S124, 93 413 4028

- Argimiro Arratia
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  - Omega 323,
Website

Please go to http://www.cs.upc.edu/~CSN for all course’s material, schedule, lab work, etc.
Class Logistics

- **Wednesday, 10:00 – 12:00, A6 203**
  - Theory lectures.

- **Thursday, 10:00 – 12:00, every two weeks, c6 S301**.
  - Guided lab activities; expected to be complemented with an average estimate of 4-6 additional hours per session of autonomous lab activities.
  - Lab sessions will require handing in a short written report; these count towards the evaluation of the course.
  - **Start on the 7th of September**
Lab work - important rules

- Lab reports in teams of 2, submission by one member.
- Work with a different partner each lab.
- Do not exchange information other than general ideas with others: that will be considered plagiarism.
There will be no exam in this course. Grading is done entirely through reports on various tasks throughout the course.

- You are expected to hand in 7 lab work reports
  - The best 5 count for 50% of the final grade
  - Lab reports not handed in penalize, so please complete all of them

- You have to do a final course project
  - Project ideas given by instructors towards the end of the course
  - Students pick a project or can suggest their own
  - 50% of the final grade
Contents
As planned today – may go through unpredictable changes

1. Characterization of networks (*how can we describe them*)
   - Lectures 1–7
   - Includes: small-world, degree distribution, finding communities, and other advanced metrics

2. Dynamics of growing networks (*how do networks grow*)
   - Lectures 8–9
   - Includes: random growth, preferential attachment, and other growth models

3. Processing networks and processes on networks (*how can we process large networks and how are processes over networks affected by their topology*)
   - Lectures 10–13
   - Includes: sampling, epidemic models of diffusion, rumor spreading, search, percolation, etc.
So, let’s start! Today, we’ll see:

1. Examples of real networks
2. What do real networks look like?
   - real networks exhibit small **diameter**
     - .. and so does the Erdös-Rényi or random model
   - real networks have high **clustering coefficient**
     - .. and so does the Watts-Strogatz model
   - real networks’ **degree distribution** follows a power-law
     - .. and so does the Barabasi-Albert or preferential attachment model
Examples of real networks

- Social networks
- Information networks
- Technological networks
- Biological networks
- Financial networks
Social networks

Links denote social “interactions”

- friendship, collaborations, e-mail, etc.
Information networks

Nodes store information, links associate information

- citation networks, the web, p2p networks, etc.
Technological networks

Man-built for the distribution of a commodity

- telephone networks, power grids, transportation networks, etc.
Biological networks

Represent biological systems

- protein-protein interaction networks, gene regulation networks, metabolic pathways, etc.
Financial networks

Nodes = financial assets, links = associated value or information

- Forex network I: Nodes = currencies, links = exchange value

- Forex network II: Nodes = currencies, links = nominal dollar value of all transactions between those two currencies (volume of trading)

Financial networks

The Forex network (2015): Nodes = currencies, links = exchange value
Representing networks

- Network $\equiv$ Graph
- Networks are just collections of “points” joined by “lines”

<table>
<thead>
<tr>
<th>points</th>
<th>lines</th>
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<tbody>
<tr>
<td>vertices</td>
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<td>nodes</td>
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<tr>
<td>sites</td>
<td>bonds</td>
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<tr>
<td>actors</td>
<td>ties, relations</td>
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</table>
Types of networks
From [Newman, 2003]

(a) unweighted, undirected
(b) discrete vertex and edge types, undirected
(c) varying vertex and edge weights, undirected
(d) directed
Descriptive measures of networks

- real networks exhibit small **diameter**
- real networks have high **clustering coefficient** (or transitivity)
- real networks’ **degree distribution** follows a power-law (i.e. are **scale free**),

Argimiro Arratia & R. Ferrer-i-Cancho
Intro to Complex and Social Networks
<table>
<thead>
<tr>
<th>network</th>
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<th>$m$</th>
<th>$z$</th>
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</tbody>
</table>

$z$ mean deg; $\ell$ mean distance; $\alpha$ exponent of deg. distrib. if power law; $C$ clustering coef.
Small-world phenomenon
Low diameter and high transitivity

- Only 6 hops separate any two people in the world
- A friend of a friend is also frequently a friend
Measuring the small-world phenomenon, I

- Let $d_{ij}$ be the shortest-path distance between nodes $i$ and $j$
- To check whether “any two nodes are within 6 hops”, we use:
  - The diameter (longest shortest-path distance) as
    \[ d = \max_{i,j} d_{ij} \]
  - The average shortest-path length as
    \[ l = \frac{2}{n(n-1)} \sum_{i>j} d_{ij} \]
  - The harmonic mean shortest-path length as
    \[ l^{-1} = \frac{2}{n(n-1)} \sum_{i>j} d_{ij}^{-1} \]
But..

- Can we mimic this phenomenon in simulated networks ("models")?
- The answer is YES!
Basic $G_{n,p}$ Erdős-Rényi random graph model:

- parameter $n$ is the number of vertices
- parameter $p$ is s.t. $0 \leq p \leq 1$
- Generate and edge $(i,j)$ independently at random with probability $p$
Want to show that the diameter in ER networks is small

- Let the average degree be $z$
- At distance $l$, can reach $z^l$ nodes
- At distance $\frac{\log n}{\log z}$, reach all $n$ nodes
- So, diameter is (roughly) $O(1)$
  (Show that $z = (n - 1)p$)
ER networks have small diameter
As shown by the following simulation
To check whether “the friend of a friend is also frequently a friend”, we use:

- The transitivity or clustering coefficient, which basically measures the probability that two of my friends are also friends.
Global clustering coefficient

\[ C = \frac{3 \times \text{number of triangles}}{\text{number of connected triples}} \]

\[ C = \frac{3 \times 1}{8} = 0.375 \]
Local clustering coefficient

- For each vertex $i$, let $n_i$ be the number of neighbors of $i$
- Let $C_i$ be the fraction of pairs of neighbors that are connected within each other

$$C_i = \frac{\text{nr. of connections between } i\text{'s neighbors}}{\frac{1}{2} n_i (n_i - 1)}$$

- Finally, average $C_i$ over all nodes $i$ in the network

$$C = \frac{1}{n} \sum_i C_i$$
Local clustering coefficient example

- $C_1 = C_2 = 1/1$
- $C_3 = 1/6$
- $C_4 = C_5 = 0$
- $C = \frac{1}{5}(1 + 1 + 1/6) = 13/30 = 0.433$
ER networks do not show transitivity

- \( C = p \), since edges are added independently
- Given a graph with \( n \) nodes and \( m \) edges, we can “estimate” \( p \) as
  \[
  \hat{p} = \frac{m}{\frac{1}{2} n (n-1)}
  \]
- We say that clustering is high if \( C \gg \hat{p} \)
  - Hence, ER networks do not have high clustering coefficient since for them \( C \approx \hat{p} \)
ER networks do not show transitivity

<table>
<thead>
<tr>
<th>Network</th>
<th>$n$</th>
<th>$z$</th>
<th>$C_{\text{measured}}$</th>
<th>$C_{\text{for random graph}}$</th>
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</table>
So ER networks do not have high clustering, but..

- Can we mimic this phenomenon in simulated networks ("models"), while keeping the diameter small?
- The answer is YES!
Reconciling two observations from real networks:

- **High clustering**: my friend’s friends are also my friends
- **small diameter**
The Watts-Strogatz model, II

- Start with all $n$ vertices arranged on a ring
- Each vertex has initially 4 connections to their closest nodes
  - mimics local or geographical connectivity
- With probability $p$, rewire each local connection to a random vertex
  - $p = 0$ high clustering, high diameter
  - $p = 1$ low clustering, low diameter (ER model)
- What happens in between?
  - As we increase $p$ from 0 to 1
    - Fast decrease of mean distance
    - Slow decrease in clustering
The Watts-Strogatz model, III

For an appropriate value of $p \approx 0.01$ (1%), we observe that the model achieves high clustering and small diameter.
Degree distribution

Histogram of nr of nodes having a particular degree

random networks  real networks (power-law, scale-free)

lowly linked  <= node degree ==> highly linked  

number of nodes

\[ f_k = \text{fraction of nodes of degree } k \]
The degree distribution of most real-world networks follows a power-law distribution

\[ f_k = ck^{-\alpha} \]

- “heavy-tail” distribution, implies existence of hubs
- hubs are nodes with very high degree
Random networks are not scale-free!

For random networks, the degree distribution follows the **binomial distribution** (or Poisson if $n$ is large)

$$f_k = \binom{n}{k} p^k (1 - p)^{n-k} \approx \frac{z^k e^{-z}}{k!}$$

- Where $z = p(n - 1)$ is the mean degree
- Probability of nodes with very large degree becomes exponentially small
  - so no hubs
So ER networks are not scale-free, but..

- Can we obtained scale-free simulated networks?
- The answer is YES!
Prefereential attachment

- “Rich get richer” dynamics
  - The more someone has, the more she is likely to have
- Examples
  - the more friends you have, the easier it is to make new ones
  - the more business a firm has, the easier it is to win more
  - the more people there are at a restaurant, the more who want to go
Barabási-Albert model

From [Barabási and Albert, 1999]

- “Growth” model
  - The model controls how a network grows over time
  - Uses preferential attachment as a guide to grow the network
    - new nodes prefer to attach to well-connected nodes
- (Simplified) process:
  - the process starts with some initial subgraph
  - each new node comes in with $m_0$ edges
  - probability of connecting to existing node $i$ is proportional to $i$’s degree
  - results in a power-law degree distribution with exponent $\alpha = 3$
Experiment with 1000 nodes, 999 edges ($m_0 = 1$ in BA model).
In summary..

<table>
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<tr>
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<th>ER</th>
<th>WS</th>
<th>BA</th>
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<td>yes&lt;sup&gt;1&lt;/sup&gt;</td>
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<sup>1</sup>clustering coefficient is higher than in random networks, but not as high as for example in WS networks.
References I


References II


