Intro to Complex and Social Networks

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Complex and Social Networks (2024-2025)
Master in Innovation and Research in Informatics (MIRI)

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Please go to http://www.cs.upc.edu/~CSN for all course's material, schedule, lab work, etc.

Class Logistics

- Monday, 10:00 – 12:00, A6 106
  - Theory lectures.
- Thursday, 10:00 – 12:00, every two weeks, A5 S111.
  - Guided lab activities; expected to be complemented with an average estimate of 4-6 additional hours per session of autonomous lab activities.
  - Lab sessions will require handing in a short written report; these count towards the evaluation of the course.
  - Theory start on the 9th of September
    Labs: 12/9, 26/9, 10/10, 24/10, 14/11, 28/11, 12/12

Lab work - important rules

- Lab reports in teams of 2, submission by one member.
- Work with a different partner each lab.
- Do not exchange information other than general ideas with others: that will be considered plagiarism

Evaluation

There will be no exam in this course. Grading is done entirely through reports on various tasks throughout the course.

- You are expected to hand in 7 lab work reports
  - The best 5 count for 50% of the final grade
  - Lab reports not handed in penalize, so please complete all of them
- You have to do a final course project
  - Project ideas given by instructors towards the end of the course
  - Students pick a project or can suggest their own
  - 50% of the final grade

Contents

As planned today – may go through unpredictable changes

1. Characterization of networks (*how can we describe them*)
   - Lectures 1–7
   - Includes: small-world, degree distribution, finding communities, and other advanced metrics
2. Dynamics of growing networks (*how do networks grow*)
   - Lectures 8–9
   - Includes: random growth, preferential attachment, and other growth models
3. Processing networks and processes on networks (*how can we process large networks and how are processes over networks affected by their topology*)
   - Lectures 10–13
   - Includes: sampling, epidemic models of diffusion, rumor spreading, search, percolation, etc.

So, let's start! Today, we'll see:

1. Examples of real networks
2. What do real networks look like?
    - real networks exhibit small diameter
        - .. and so does the Erdös-Rényi or random model
    - real networks have high clustering coefficient
        - .. and so does the Watts-Strogatz model
    - real networks' degree distribution follows a power-law
        - .. and so does the Barabasi-Albert or preferential attachment model

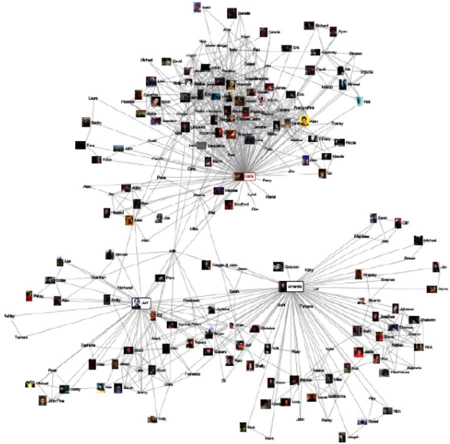
Examples of real networks

- Social networks
- Information networks
- Technological networks
- Biological networks
- Financial networks

Social networks

Links denote social "interactions"

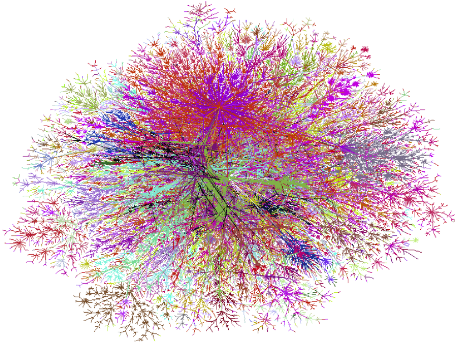
- friendship, collaborations, e-mail, etc.



Information networks

Nodes store information, links associate information

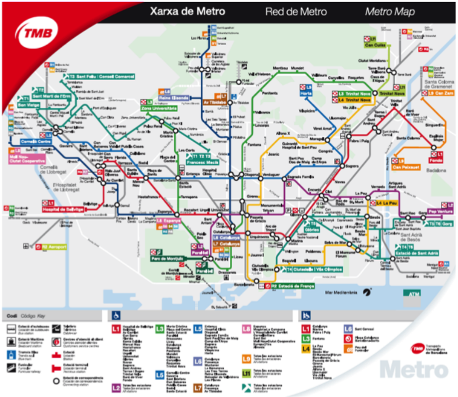
- citation networks, the web, p2p networks, etc.



Technological networks

Man-built for the distribution of a commodity

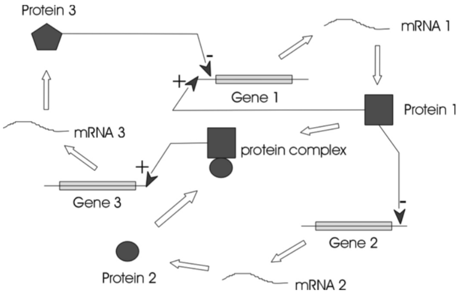
- telephone networks, power grids, transportation networks, etc.



Biological networks

Represent biological systems

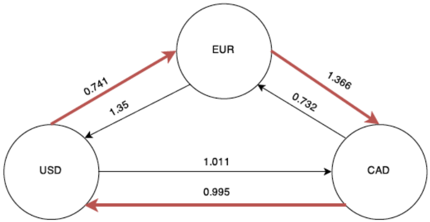
- protein-protein interaction networks, gene regulation networks, metabolic pathways, etc.



Financial networks

Nodes = financial assets, links = associated value or information

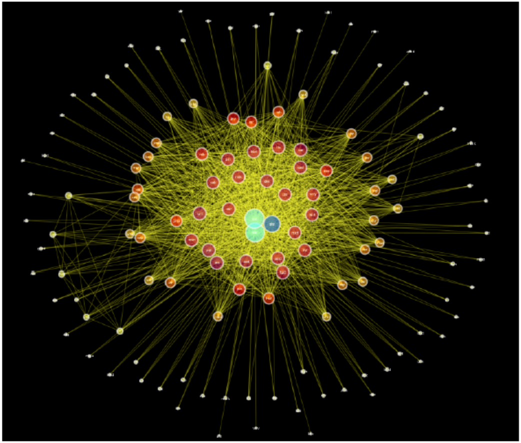
- Forex network I: Nodes = currencies, links = exchange value



- Forex network II: Nodes = currencies, links = nominal dollar value of all transactions between those two currencies (volume of trading)
  see: http://ipeatunc.blogspot.com.es/2011/06/international-forex-network-1998-2010.html

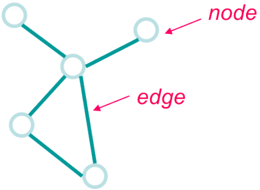
Financial networks

The Forex network (2015): Nodes = currencies, links = exchange value



Representing networks

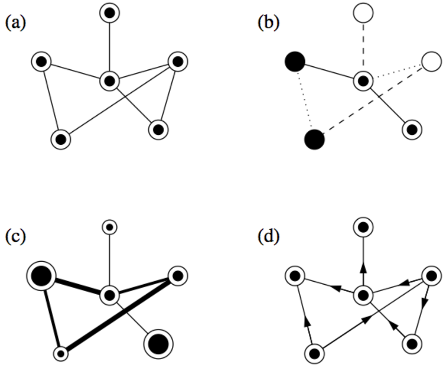
- Network ≡ Graph
- Networks are just collections of "points" joined by "lines"



points	lines	
vertices	edges, arcs	math
nodes	links	computer science
sites	bonds	physics
actors	ties, relations	sociology

Types of networks

From [Newman, 2003]



- (a) unweighted, undirected
- (b) discrete vertex and edge types, undirected
- (c) varying vertex and edge weights, undirected
- (d) directed

Descriptive measures of networks

- real networks exhibit small diameter
- real networks have high clustering coefficient (or transitivity)
- real networks' degree distribution follows a power-law (i.e. are *scale free*)

From [Newman, 2003]

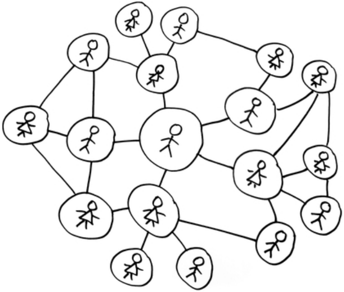
	network	type	n	m	z	ℓ	α	$C^{(1)}$	$C^{(2)}$	r	Ref(s).
social	film actors	undirected	449 913	25 516 482	113.43	3.48	2.3	0.20	0.78	0.208	20, 416
	company directors	undirected	7 673	55 392	14.44	4.60	–	0.59	0.88	0.276	105, 323
	math coauthorship	undirected	253 339	496 489	3.92	7.57	–	0.15	0.34	0.120	107, 182
	physics coauthorship	undirected	52 909	245 300	9.27	6.19	–	0.45	0.56	0.363	311, 313
	biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92	–	0.088	0.60	0.127	311, 313
	telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1				8, 9
	email messages	directed	59 912	86 300	1.44	4.95	1.5/2.0		0.16		136
	email address books	directed	16 881	57 029	3.38	5.22	–	0.17	0.13	0.092	321
	student relationships	undirected	573	477	1.66	16.01	–	0.005	0.001	−0.029	45
	sexual contacts	undirected	2 810				3.2				265, 266
information	WWW nd.edu	directed	269 504	1 497 135	5.55	11.27	2.1/2.4	0.11	0.29	−0.067	14, 34
	WWW Altavista	directed	203 549 046	2 130 000 000	10.46	16.18	2.1/2.7				74
	citation network	directed	783 339	6 716 198	8.57		3.0/–				351
	Roget's Thesaurus	directed	1 022	5 103	4.99	4.87	–	0.13	0.15	0.157	244
	word co-occurrence	undirected	460 902	17 000 000	70.13		2.7		0.44		119, 157
technological	Internet	undirected	10 697	31 992	5.98	3.31	2.5	0.035	0.39	−0.189	86, 148
	power grid	undirected	4 941	6 594	2.67	18.99	–	0.10	0.080	−0.003	416
	train routes	undirected	587	19 603	66.79	2.16	–		0.69	−0.033	366
	software packages	directed	1 439	1 723	1.20	2.42	1.6/1.4	0.070	0.082	−0.016	318
	software classes	directed	1 377	2 213	1.61	1.51	–	0.033	0.012	−0.119	395
	electronic circuits	undirected	24 097	53 248	4.34	11.05	3.0	0.010	0.030	−0.154	155
	peer-to-peer network	undirected	880	1 296	1.47	4.28	2.1	0.012	0.011	−0.366	6, 354
biological	metabolic network	undirected	765	3 686	9.64	2.56	2.2	0.090	0.67	−0.240	214
	protein interactions	undirected	2 115	2 240	2.12	6.80	2.4	0.072	0.071	−0.156	212
	marine food web	directed	135	598	4.43	2.05	–	0.16	0.23	−0.263	204
	freshwater food web	directed	92	997	10.84	1.90	–	0.20	0.087	−0.326	272
	neural network	directed	307	2 359	7.68	3.97	–	0.18	0.28	−0.226	416, 421

z mean deg; l mean distance; α expnt deg. distrib. if power law; $C^{\cdot}$ clustering coef.; r deg. cor

Small-world phenomenon

Low diameter and high transitivity

- Only 6 hops separate any two people in the world
- A friend of a friend is also frequently a friend



Measuring the small-world phenomenon, I

- Let d_{ij} be the shortest-path distance between nodes i and j
- To check whether "any two nodes are within 6 hops", we use:
  - The diameter (longest shortest-path distance) as

$$d = \max_{i,j} d_{ij}$$

  - The average shortest-path length as

$$l = \frac{2}{n\,(n-1)} \sum_{i>j} d_{ij}$$

  - The harmonic mean shortest-path length as

$$l^{-1} = \frac{2}{n\,(n-1)} \sum_{i>j} d_{ij}^{-1}$$

But..

- Can we mimic this phenomenon in simulated networks ("models")?
- The answer is YES!

The (basic) random graph model

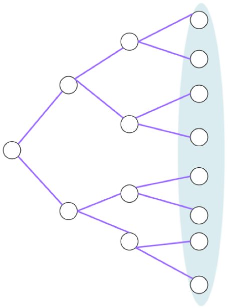
a.k.a. ER model

Basic $G_{n,p}$ Erdös-Rényi random graph model:

- parameter n is the number of vertices
- parameter p is s.t. $0 \leq p \leq 1$
- Generate and edge (i,j) independently at random with probability p

Measuring the diameter in ER networks

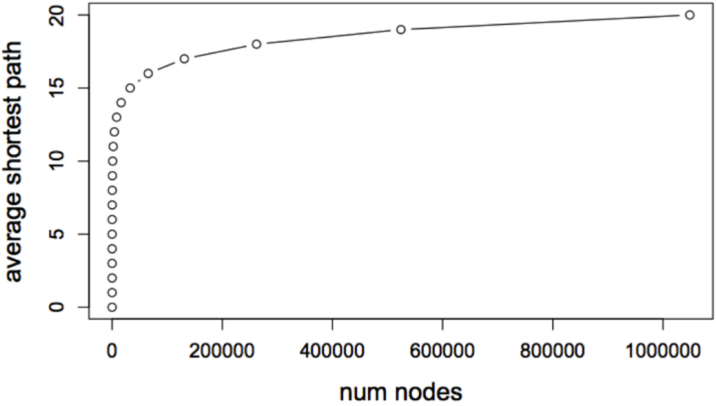
Want to show that the diameter in ER networks is small



- Let the average degree be z
- At distance l, can reach z^l nodes
- At distance $\frac{\log n}{\log z}$, reach all n nodes
- So, diameter is (roughly) $O(1)$
  (Show that $z = (n-1)p$)

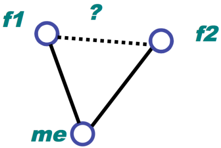
ER networks have small diameter

As shown by the following simulation



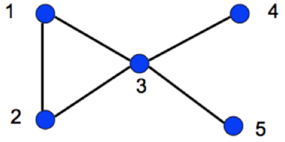
Measuring the small-world phenomenon, II

- To check whether "the friend of a friend is also frequently a friend", we use:
  - The transitivity or clustering coefficient, which basically measures the probability that two of my friends are also friends



Global clustering coefficient

$$C^{(1)} = \frac{3 \times \text{number of triangles}}{\text{number of connected triples}}$$



$$C^{(1)} = \frac{3 \times 1}{8} = 0.375$$

Local clustering coefficient

- For each vertex i, let n_i be the number of neighbors of i
- Let C_i be the fraction of pairs of neighbors that are connected within each other

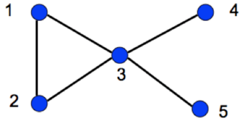
$$C_i = \frac{\text{nr. of connections between } i\text{'s neighbors}}{\frac{1}{2} n_i \, (n_i - 1)}$$

- Finally, average C_i over all nodes i in the network

$$C^{(2)} = \frac{1}{n} \sum_i C_i$$

$C^{(1)}$ and $C^{(2)}$ give slightly different results (always specify which one you use)

Local clustering coefficient example



- $C_1 = C_2 = 1/1$
- $C_3 = 1/6$
- $C_4 = C_5 = 0$
- $C^{(2)} = \frac{1}{5}(1 + 1 + 1/6) = 13/30 = 0.433$

ER networks do not show transitivity

- $C = p$, since edges are added independently
- Given a graph with n nodes and m edges, we can "estimate" p as

$$\hat{p} = \frac{m}{1/2 \; n \, (n-1)}$$

- We say that clustering is high if $C \gg \hat{p}$
  - Hence, ER networks do not have high clustering coefficient since for them $C \approx \hat{p}$

ER networks do not show transitivity

Table 1: Clustering coefficients, C, for a number of different networks; n is the number of node, z is the mean degree. Taken from [146].

Network	n	z	C measured	C for random graph
Internet [153]	6,374	3.8	0.24	0.00060
World Wide Web (sites) [2]	153,127	35.2	0.11	0.00023
power grid [192]	4,941	2.7	0.080	0.00054
biology collaborations [140]	1,520,251	15.5	0.081	0.000010
mathematics collaborations [141]	253,339	3.9	0.15	0.000015
film actor collaborations [149]	449,913	113.4	0.20	0.00025
company directors [149]	7,673	14.4	0.59	0.0019
word co-occurrence [90]	460,902	70.1	0.44	0.00015
neural network [192]	282	14.0	0.28	0.049
metabolic network [69]	315	28.3	0.59	0.090
food web [138]	134	8.7	0.22	0.065

So ER networks do not have high clustering, but..

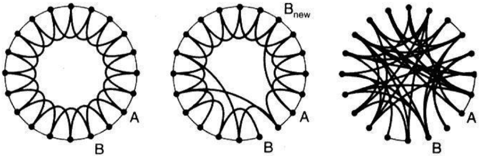
- Can we mimic this phenomenon in simulated networks ("models"), while keeping the diameter small?
- The answer is YES!

The Watts-Strogatz model, I

From [Watts and Strogatz, 1998]

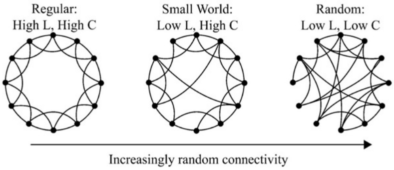
Reconciling two observations from real networks:

- High clustering: my friend's friends are also my friends
- small diameter



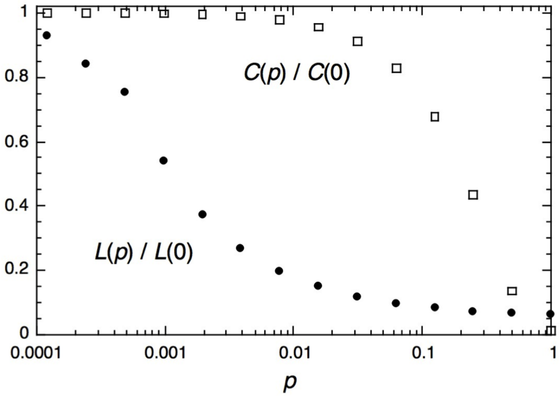
The Watts-Strogatz model, II

- Start with all n vertices arranged on a ring
- Each vertex has intially 4 connections to their closest nodes
  - mimics local or geographical connectivity
- With probability p, rewire each local connection to a random vertex
  - $p = 0$ high clustering, high diameter
  - $p = 1$ low clustering, low diameter (ER model)
- What happens in between?
  - As we increase p from 0 to 1
    - Fast decrease of mean distance
    - Slow decrease in clustering



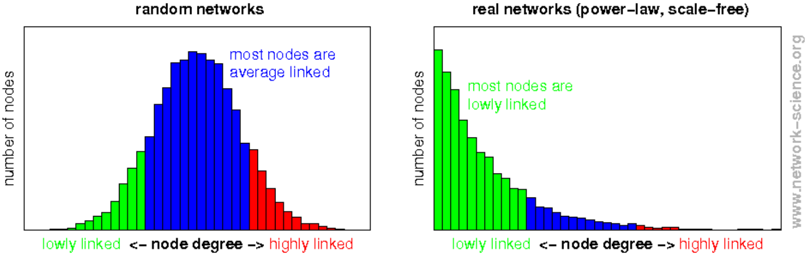
The Watts-Strogatz model, III

For an appropriate value of $p \approx 0.01$ (1%), we observe that the model achieves high clustering and small diameter



Degree distribution

Histogram of nr of nodes having a particular degree

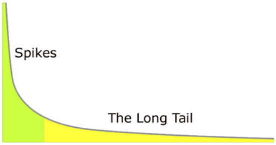


f_k = fraction of nodes of degree k

Scale-free networks

The degree distribution of most real-world networks follows a power-law distribution

$$f_k = ck^{-\alpha}$$



- “heavy-tail” distribution, implies existence of hubs
- hubs are nodes with very high degree

Random networks are not scale-free!

For random networks, the degree distribution follows the binomial distribution (or Poisson if n is large)

$$f_k = \binom{n}{k} p^k (1-p)^{(n-k)} \approx \frac{z^k e^{-z}}{k!}$$

- Where $z = p(n-1)$ is the mean degree
- Probability of nodes with very large degree becomes exponentially small
  - so no hubs

So ER networks are not scale-free, but..

- Can we obtained scale-free simulated networks?
- The answer is YES!

Preferential attachment

- "Rich get richer" dynamics
  - The more someone has, the more she is likely to have
- Examples
  - the more friends you have, the easier it is to make new ones
  - the more business a firm has, the easier it is to win more
  - the more people there are at a restaurant, the more who want to go

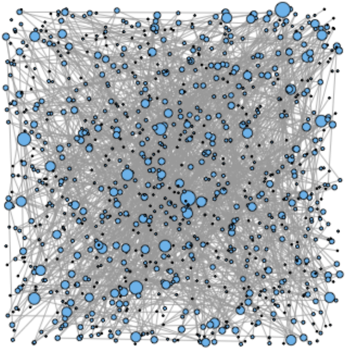
Barabási-Albert model

From [Barabási and Albert, 1999]

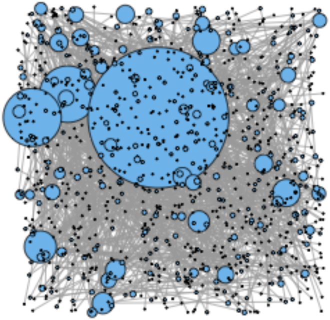
- “Growth” model
  - The model controls how a network grows over time
- Uses preferential attachment as a guide to grow the network
  - new nodes prefer to attach to well-connected nodes
- (Simplified) process:
  - the process starts with some initial subgraph
  - each new node comes in with m_0 edges
  - probability of connecting to existing node i is proportional to i's degree
  - results in a power-law degree distribution with exponent $\alpha = 3$

ER vs. BA

Experiment with 1000 nodes, 999 edges ($m_0 = 1$ in BA model).



random



preferential attachment

In summary..

phenomenon	real networks	ER	WS	BA
small diameter	yes	yes	yes	yes
high clustering	yes	no	yes	yes[1]
scale-free	yes	no	no	yes

[1] clustering coefficient is higher than in random networks, but not as high as for example in WS networks

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