Theory of Computation **Partial Exam 2**
05/06/2026

Instructions
On the desk you are only allowed to have: this exam, black/blue ink pens, and your DNI (or UPC identification). **Electronic devices and backpacks are not allowed near the desk or your person.**

Login into the computer with
user: `XXX`
password: `XXXXXXXX`

Go to the RACSO webpage (`https://racso.cs.upc.edu/`) and login with your credentials (as usual, user: FIB username, password: DNI without letter) and wait for the exam to start.
The RACSO part of the exam starts automatically at **11:45 h**, and ends at **13:45 h** (except for students with adaptations).

This exam consists of three parts:

- **4 exercises on RACSO** (1.1 point per exercise, automatically evaluated). Each *rejected submission* removes 0.1 points if eventually the solution is accepted. Otherwise, the score for the exercise is 0 points (there are no "negative points").
- **8 multiple choice questions** (0.6 points per question).
- **2 open questions** (1 point per question).

The maximum score attainable in this exam is 11.2 out of 10.

14:30 h: this exam ends (except for students with adaptations).

Make sure to write your name/surname/DNI on every page (except this one).
Feel free to detach this page from the rest and use it to write down the calculations you might need.

Theory of Computation **Partial Exam 2**
05/06/2026 multiple-choice questions

Name: _____ **Surname(s):** _____ **DNI:** _____

Write your answers below with blue or black ink.

Question	1	2	3	4	5	6	7	8	Total
Answer									

each correct answer: $+0.6$ points
each wrong answer: -0.1 points

1 Consider the language

$$L = \{w \in \{0,1\}^* \mid \text{value}_2(w) \in 2\mathbb{N}\} \setminus \{1^n 0^n \mid n \geq 0\}.$$

What can we say about L?

A. That $\overline{L}$ is a language accepted by a pushdown automaton but L is not.

B. That neither L nor $\overline{L}$ are languages accepted by pushdown automata.

C. That L is a language accepted by a pushdown automaton but $\overline{L}$ is not.

D. That both L and $\overline{L}$ are languages accepted by pushdown automata.

Note that the set difference is defined as $A \setminus B = A \cap \overline{B}$ for any sets A and B.

2 A Turing machine is called *right-moving* if its head can never move to the left or stay at the same position (that is, it can only move to the right). What is the class of languages accepted by right-moving Turing machines?

A. The languages in $\mathbf{P}$.

B. The context-free languages.

C. The recursive languages.

D. The regular languages.

3 Let M be a Turing machine and $A = \mathscr{L}(M)$ the language recognized by M. Let $B = \{w \mid M(w)\uparrow\}$ be the set of strings for which M does not halt. Which of the following statements is false for some M?

A. If B is cofinite, then $A \in \mathbf{R}$.

B. If B is finite, then $A \in \mathbf{R}$.

C. $A \in \mathbf{RE}$ independently of B.

D. If B is infinite, then $A \in \mathbf{R}$.

Recall that $\mathbf{R}$ is the set of all decidable languages. Also, a set S is *cofinite* if and only if $\overline{S}$ is finite.

4 Consider the language $L = \{p \mid \text{Im}(\varphi_p) \text{ is finite}\}$. Which of the following is true?

A. $L \in \mathbf{coRE} \setminus \mathbf{R}$

B. $L \in \mathbf{R}$

C. $L \in \mathbf{RE} \setminus \mathbf{R}$

D. $L \notin \mathbf{RE} \cup \mathbf{coRE}$

5 A *vertex cover* of a graph G is a subset of its vertices that includes at least one endpoint of every edge in G. Consider the EXACT VERTEX COVER problem: "given a graph G and an integer k, is the *minimum* size of a vertex cover of G exactly k?"

With the theory seen in this course you can prove that three out of the four following sentences are true. Which is the remaining one?

A. EXACT VERTEX COVER $\in \Sigma_1^p \cup \Pi_1^p$.

B. The complement of EXACT VERTEX COVER belongs to Π_2^p.

C. EXACT VERTEX COVER $\in \Sigma_3^p$.

D. If EXACT VERTEX COVER $\notin \Sigma_1^p$, then $\Sigma_1^p \neq \Pi_1^p$.

6 Which of the following statements is correct?

A. If $B \in \mathbf{P}$ and $A \leq_m B$ then $A \in \mathbf{P}$.

B. If $A \in \mathbf{R}$, there is a language $B \in \mathbf{P}$ such that $A \leq_m B$.

C. If $B \in \mathbf{P}$ and $A \leq_m B$, then A might not belong to $\mathbf{R}$.

D. If $B \in \mathbf{NP}$ and $A \leq_m B$ then $A \in \mathbf{NP}$.

Recall that $\mathbf{P}$ is the set of all languages decidable in polynomial time and that $A \leq_m B$ means that there is a many-one reduction (not necessarily polynomial-time) from A to B.

7 Most computer scientists believe that $\mathbf{P} \neq \mathbf{NP}$. Under this belief, one of the following sentences is false. Which one?

A. There is some planar graph which is not 3-colorable.

B. Given a graph G and a 4-coloring of it, deciding whether G is 3-colorable is a problem in $\mathbf{P}$.

C. Given a graph G with n vertices, deciding whether G is n-colorable is a problem in $\mathbf{P}$.

D. Given a graph G and a 4-coloring of it, deciding whether G is 3-colorable is $\mathbf{NP}$-hard.

Recall that all planar graphs are 4-colorable and the coloring can be computed in polynomial time. Moreover, given a planar graph G, deciding whether G is 3-colorable is known to be $\mathbf{NP}$-hard.

8 Suppose all problems in $\mathbf{NP}$ are either in $\mathbf{P}$ or $\mathbf{NP}$-complete. Which of the following statements is true?

A. $\mathbf{P} = \mathbf{NP}$ and $\mathbf{NP} = \mathbf{EXP}$

B. $\mathbf{P} \neq \mathbf{NP}$ and $\mathbf{NP} = \mathbf{EXP}$

C. $\mathbf{P} = \mathbf{NP}$ and $\mathbf{NP} \neq \mathbf{EXP}$

D. $\mathbf{P} \neq \mathbf{NP}$ and $\mathbf{NP} \neq \mathbf{EXP}$

Feel free to use the rest of this page and its back to write down the calculations you might need.

Theory of Computation **Partial Exam 2**
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Name: _____ **Surname(s):** _____ **DNI:** _____

Please use **only** the space below (or the back of the page) for your solution of the following exercise. Use blue or black ink. You can write your solution either in Catalan, Spanish, or English.

1 (1 point) Prove that your solution in exercise 3 or exercise 4 of the RACSO part of the exam is indeed a correct many-one reduction.

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Please use **only** the space below (or the back of the page) for your solution of the following exercise. Use blue or black ink. You can write your solution either in Catalan, Spanish, or English.

2 (1 point) Consider the decision problem `True-SAT`: given as input a CNF formula F that is true mapping all variables to true, is there another assignment satisfying F?

Show that `True-SAT` is $\mathbf{NP}$-complete.