Theory of Computation **Partial Exam 1**

08/04/2026

Instructions

On the desk you are only allowed to have: this exam, black/blue ink pens, and your DNI (or UPC identification). **Electronic devices and backpacks are not allowed near the desk or your person.**

Login into the computer with
user: `XXX`
password: `XXXXXXXX`

Go to the RACSO webpage (`https://racso.cs.upc.edu/`) and login with your credentials (as usual, user: FIB username, password: DNI without letter) and wait for the exam to start.
The RACSO part of the exam starts automatically at **15:10 h**, and ends at **17:10 h** (except for students with adaptations).

This exam consists of three parts:

- **4 exercises on RACSO** (1 point per exercise, automatically evaluated). Each *rejected submission* removes 0.1 points if eventually the solution is accepted. Otherwise, the score for the exercise is 0 points (there are no "negative points").
- **8 multiple choice questions** (0.5 points per question).
- **2 open questions** (1 point per question).

The maximum score attainable in this exam is 10 out of 10.

18:00 h: this exam ends (except for students with adaptations).

Make sure to write your name/surname/DNI on every page (except this one).
Feel free to detach this page from the rest and use it to write down the calculations you might need.

Theory of Computation **Partial Exam 1**
08/04/2026 multiple-choice questions

Name: ________________ **Surname(s):** ________________________________ **DNI:** ________________

Write your answers below with blue or black ink.

Question	1	2	3	4	5	6	7	8	Total
Answer									

each correct answer: $+0.5$ points
each wrong answer: -0.15 points

1 Given a language L over alphabet Σ and a homomorphism $h : \Sigma^* \to \Sigma^*$, define

$$h(L) = \{h(w) \mid w \in L\}$$
$$h^{-1}(L) = \{w \mid h(w) \in L\}$$

Which of the following inclusions are true?

1. $h^{-1}(h(L)) \subseteq L$
2. $h^{-1}(h(L)) \supseteq L$

A. Both 1 and 2.

B. Neither 1 nor 2.

C. Only 2.

D. Only 1.

2 How many words of length 3 over $\{a, b\}$ are not in $\mathscr{L}(a^*b^*a^*)$?

A. 2

B. 0

C. 3

D. 1

3 Given a regular expression r, which of the following sentences is true?

A. If $\cdot$ does not appear in r, then the language generated by r is finite.

B. If $+$ does not appear in r, then the language generated by r is finite.

C. None of the other options is true.

D. If $*$ does not appear in r, then the language generated by r is finite.

4 Which of the following is false?

A. For every infinite $I \subseteq \mathbb{N}$, the language $\{a^nb^n \mid n \in I\}$ is non-regular.

B. Every infinite regular language contains a non-regular language as a subset.

C. For every infinite $I \subseteq \mathbb{N}$, the language $\{a^nb^m \mid n, m \in I \wedge n \neq m\}$ is non-regular.

D. All infinite subsets of $\overline{\{a^nb^n \mid n \in \mathbb{N}\}}$ are non-regular.

5 You want to show that

$$L = \{0^{n_1}10^{n_2}1 \cdots 10^{n_k} \mid n_1, \ldots, n_k \text{ are all distinct}\}$$

is not a regular language applying the pumping lemma directly to L. Given the pumping length p, you choose a word w as in the argument to apply the pumping lemma. Which of the following possibilities has a chance to make the rest of the argument correct?

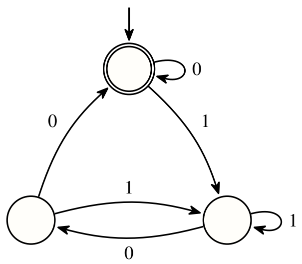
A. $w = 0^{2p}10^{2p-2}10^{2p-4}1 \cdots 10^21$

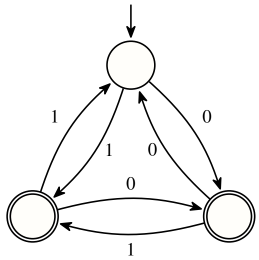
B. $w = 0^p10^{p+1}$

C. $w = 0^p10^{p-1}10^{p-2}1 \cdots 101$

D. $w = 0^p1$

6 Consider the following NFA with two initial states:





How many states does the equivalent minimum DFA have?

A. 4

B. 3

C. More than 5

D. 5

7 We want to find an unambiguous CFG for

$$L = \{w \in \{a, b\}^* \mid ||w|_a - |w|_b| = 1\}.$$

Consider the following grammar G:

$$S \to XaX \mid XbX$$
$$X \to aBX \mid bAX \mid \lambda$$
$$A \to bAA \mid a$$
$$B \to aBB \mid b$$

What can be said about G?

A. It is ambiguous and $L(G) \neq L$.
B. It is unambiguous and $L(G) = L$.
C. It is unambiguous and $L(G) \neq L$.
D. It is ambiguous and $L(G) = L$.

8 We would like to define a CFG to recognize the language $L = \{a^i b^j a^i b^j \mid i \geq j\}$. Consider the following CFG G:

$$S \to XY$$
$$X \to aXa \mid Y$$
$$Y \to aYb \mid \lambda$$

Which option is correct?

A. $L(G) \subsetneq L$ and no other CFG recognizes L.
B. $L(G) = L$.
C. $L(G) \supsetneq L$ and no other CFG recognizes L.
D. $L(G) \neq L$ but another CFG could recognize it.

Feel free to use the rest of this page and its back to write down the calculations you might need.

Name: ______________ **Surname(s):** ____ **DNI:** ______________

Please use **only** the space below (or the back of the page) for your solution of the following exercise. Use blue or black ink. You can write your solution either in Catalan, Spanish, or English.

1 (1 point) Let M be a DFA with n states. Prove that the language of the words accepted by M, $\mathscr{L}(M)$ is infinite if and only if there is a word $w \in \mathscr{L}(M)$ s.t. $n \leq |w| \leq 2n-1$.
[*0.5 points for the "if" part, 0.5 points for the "only if" part*]

Solution: $(\Rightarrow)$ Let w be the shortest word in $\mathscr{L}(M)$ with $|w| \geq n$. Since, by assumption, $\mathscr{L}(M)$ is infinite, the word w exists. If, towards a contradiction, $|w| \geq 2n$, then the sequence of states traversed in M on input w must contain a loop. That is w can be decomposed as xyz, where x is the part before the loop, y the part in the loop and z the part after the loop. We have that $n \geq |y| \geq 1$. By "pumping down", i.e. skipping the loop, we have that M must accept also the word xz. But $|xz| = |w| - |y| \geq 2n - n = n$ and also $|xz| = |w| - |y| < n$, contradicting the minimality of w.

$(\Leftarrow)$ By assumption there is a word in $\mathscr{L}(M)$ with $|w| \geq n$. The number of states (possibly with repetitions) seen in the accepting path of w is $|w| + 1$: the initial state and one state for each symbol of w. That is the path visits at least $n+1$ states, hence by the pigeonhole principle there is a repeated state q. This gives a decomposition of w as xyz with x the part before the first occurrence of q, y the part between the first occurrence and the second occurrence of q in the path, and z the rest. It also must be that $|y| \geq 1$. As in the pumping lemma, all words of the form xy^iz for $i \in \mathbb{N}$ are in $\mathscr{L}(M)$. This is an infinite set since the words xy^iz have distinct lengths.

Theory of Computation **Partial Exam 1**
08/04/2026 open questions

Name: ______________ **Surname(s):** ____ **DNI:** ______________

Please use **only** the space below (or the back of the page) for your solution of the following exercise. Use blue or black ink. You can write your solution either in Catalan, Spanish, or English.

2

1. (0.5 points) Give context-free grammars for languages

$$A = \{a^i b^i c^j \mid i, j \geq 0\} \quad \text{and} \quad B = \{a^i b^j c^i \mid i, j \geq 0\}.$$

2. (0.5 points) Give a context-free grammar that generates $A \cup B$. Is your grammar ambiguous? Why or why not?

Solution:

1. Consider the following grammars:

$$\begin{array}{ll} G_A: & G_B: \\ S_A \to XC & S_B \to aS_Bc \mid B \\ X \to aXb \mid \lambda & B \to bB \mid \lambda \\ C \to cC \mid \lambda & \end{array}$$

We denote by L_N the set of words that can be derived from nonterminal N. With respect to CFG G_A, observe that $L_X = \{a^i b^i \mid i \geq 0\}$ and $L_C = \{c\}^*$. Then, $L(G_A) = L_{S_A} = L_X L_C = A$. As for grammar G_B, any derivation from S_B is of the form $S_B \underset{G_B}{\overset{i}{\Rightarrow}} a^i S_B c^i \underset{G_B}{\overset{1}{\Rightarrow}} a^i B c^i \underset{G_B}{\overset{j}{\Rightarrow}} a^i b^j B c^i \underset{G_B}{\overset{1}{\Rightarrow}} a^i b^j c^i$, where $i, j \geq 0$. Therefore, $L(G_B) = B$.

2. As seen in class, $A \cup B$ can be generated by the grammar containing all terminal symbols of G_A and G_B, all nonterminal symbols plus a new initial symbol S, and all productions plus

$$S \to S_A \mid S_B.$$

The new grammar is ambiguous since all words of the form $a^i b^i c^i$ can be generated with two distinct leftmost derivations (and two distinct derivation trees). In the case $i = 1$, we have

(a) $S \overset{1}{\Rightarrow} S_A \overset{1}{\Rightarrow} XC \overset{2}{\Rightarrow} aXbc \overset{1}{\Rightarrow} abc$

(b) $S \overset{1}{\Rightarrow} S_B \overset{1}{\Rightarrow} aS_Bc \overset{2}{\Rightarrow} abc$