Theory of Computation **Final Exam**

15/06/2026

Instructions

On the desk you are only allowed to have:

- this exam,
- black/blue ink pens, and
- your DNI (or UPC identification).

Electronic devices and backpacks are not allowed near the desk or your person.

Make sure to write your name/surname/DNI on every page (except this one).

This exam consists of **4 open questions**. The maximum score G attainable in this exam is 10. The grade of this exam is

$$\max\{G, 50\%G + 50\%E\} ,$$

where E is your grade of the continuous assessment. If you don't submit this exam, your grade is E.

21:00 h: this exam ends (except for students with adaptations).

Name: ______________ **Surname(s):** ______________________________ **DNI:** ______________

Please use **only** the space below (or the back of the page) for your solution of the following exercise. Use blue or black ink. You can write your solution either in Catalan, Spanish, or English.

1 (3 points) For each of the following languages, decide whether it is regular or not. In either case you should prove your claim. In case it is regular, give the minimum DFA recognizing the language and prove that it is indeed minimum.

(a) (1.6 points) For any $n \geq 1$, define

$$L_n = \{w \in \{0,1\}^* \mid \exists k \geq 0 \;\; \text{value}_2(w) = (2^n)^k\}.$$

That is, L_n is the language containing the strings over the alphabet $\{0,1\}$ whose binary value is a power of 2^n.

(b) (1.4 points) For any $n \geq 1$, define

$$L'_n = \{ww \in \{0,1\}^* \mid \exists k \geq 0 \;\; \text{value}_2(w) = (2^n)^k\}.$$

That is, L'_n is the language containing the strings ww over the alphabet $\{0,1\}$ such that the binary value of w is a power of 2^n.

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Please use **only** the space below (or the back of the page) for your solution of the following exercise. Use blue or black ink. You can write your solution either in Catalan, Spanish, or English.

2 (2 points) Prove the following facts about context-free languages. CFL is the set of all context-free languages, DCFL is the set of all *deterministic* context-free languages. Recall that $\{a^n b^n c^n \mid n \in \mathbb{N}\} \notin \mathsf{CFL}$ and DCFL is closed under complement and inverse homomorphism.

(a) (1 point) Show that the language

$$L = \{0^i 1^j 2^k \mid i \neq j \vee j \neq k\} \in \mathsf{CFL} \ .$$

Construct a context-free grammar generating L or a pushdown automata accepting L.

(b) (1 point) Show that $L \notin \mathsf{DCFL}$.

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3 (3 points)

(a) (1.4 points) Prove that a language A is in $\mathbf{R}$ (that is, it is decidable) if and only if there are two total computable functions $f, g : \mathbb{N} \to \mathbb{N}$ s.t. for each $x \in \mathbb{N}$

$$x \in A \iff f(x) > g(x) .$$

[*0.7 points are awarded for the "if" part of the exercise and 0.7 for the "only if" part.*]

(b) (1.6 points) A *useless* state in a Turing machine is a state that is never entered on any input string. Let $L = \{\langle p, Q\rangle \mid Q \text{ is a useless state in } M_p\}$, where M_p is a Turing machine with Gödel number p.

Classify L as belonging to $\mathbf{R}$ (decidible), $\mathbf{RE} \setminus \mathbf{R}$ (semi-decidable but not decidable), $\mathbf{coRE} \setminus \mathbf{R}$, or ouside both $\mathbf{RE}$ and $\mathbf{coRE}$. Justify your answer.

Name: ______________ **Surname(s):** ______________________________ **DNI:** ______________

Please use **only** the space below (or the back of the page) for your solution of the following exercise. Use blue or black ink. You can write your solution either in Catalan, Spanish, or English.

4 (2 points) A *strong* nondeterministic Turing machine is one that has three possible outcomes: **yes**, **no**, and **maybe**. We say that such a machine decides L if this is true: If $x \in L$, then all computations end up with **yes** or **maybe**, and at least one with **yes**. If $x \notin L$, then all computations end up with **no** or **maybe**, and at least one with **no**.

Show the following implications.

(a) (1 point) If L is decided by a strong nondeterministic Turing machine in polynomial time, then $L \in \mathsf{NP} \cap \mathsf{coNP}$.

(b) (1 point) If $L \in \mathsf{NP} \cap \mathsf{coNP}$, then L is decided by a strong nondeterministic Turing machine in polynomial time.