# Meta Learning methods

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# Outline

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# Multiclassifiers, Meta-learners, Ensemble Learners

- Combining several *weak learners* to give a *strong learner*
- A kind of *multiclassifier* systems and *meta-learners*
- *Ensemble* typically applied to a single type of weak learner
  - All built by same algorithm, with different data or parameters
- Lots of what I say applies to multiclassifier systems in general

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   - More: Most of the top teams were multi-classifiers
2. Combine strengths of different classifier builders
   - And we can incorporate domain knowledge into different learners
3. May help avoiding overfitting
   - This is paradoxical because more expressive than weak learners!

# Condorcet's jury theorem

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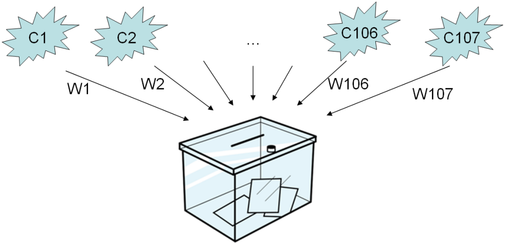
# Combining weak learners

- Voting
  - Each weak learner votes, and votes are combined
- Experts that abstain
  - A weak learner only counts when it's expert on this kind of instances
  - Otherwise it abstains (or goes to sleep)

# Outline

# Voting



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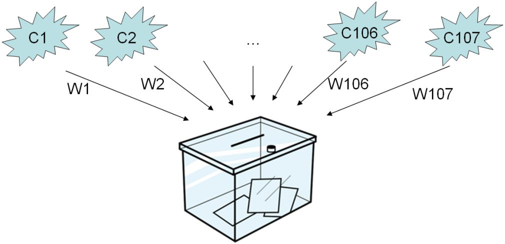


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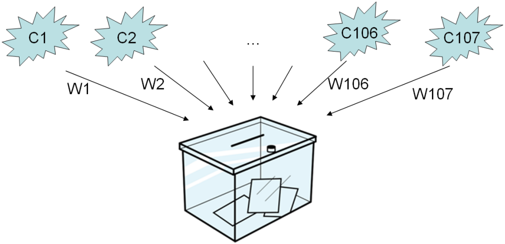


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- Weights depend on *errors* ($1 - e_i$? $1/e_i$? $\exp(-e_i)$? ...)

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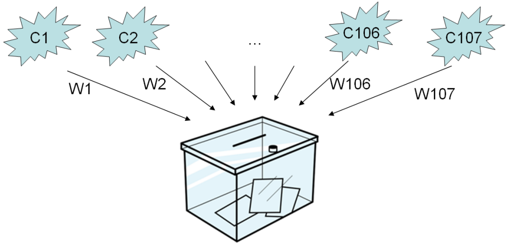


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- Weights depend on *errors* ($1 - e_i$? $1/e_i$? $\exp(-e_i)$? ...)
- Weights depend on *confidences*
- Maximizing *diversity*

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# Stacking (Wolpert 92)

A meta-learner that learns to weight its weak learner

- Dataset with instances `(x,y)`
- Transform dataset to have instances `(x,c1(x),...cN(x),y)`
- Train metaclassifier M with enriched dataset

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Often, `x` not given to M, just the votes
Often, just linear classifier
Can simulate most other voting schemes

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- Weights depend exponentially on error
- At least as good as best weak learner in time $O(\log N)$
- Often much better; more when classifiers are uncorrelated
- Good for online prediction and when many classifiers
- E.g. when 1 classifier = 1 feature

# Outline

# Bagging I

- To reduce the variance of an estimator, it is helpful to average estimates from independent draws from the data
- Assuming each $Y_b$ is an unbiased estimate of target value $y$ :

$$\mathbb{E}\left[(y - Y_b)^2\right] = \mathrm{Var}\left[Y_b\right]$$

$$\mathbb{E}\left[\left(y - \frac{1}{B}\sum_b Y_b\right)^2\right] = \frac{1}{B^2}\sum_b \mathrm{Var}\left[Y_b\right] \quad \text{(if all } Y_b \text{ are independent)}$$

$$= \frac{1}{B}\mathrm{Var}\left[Y_b\right] \quad \text{(if all } Y_b \text{ have same variance)}$$

# Bagging I

- So, the idea of bagging is to combine the predictions of a high-variance predictor trained on independent bootstrap samples from the same dataset, to make the combined predictions more robust (i.e. with lower variance) and, therefore, more accurate.
- Trees typically suffer from high variance (= overfitting) so it is specially useful in Decision trees (high variance or sensibility to training data set)

# Bagging (Breiman 96)

1. Get a dataset $S$ of $N$ labeled examples on $A$ attributes;
2. Build $N$ bagging replicas of $S$: $S_1, \ldots, S_N$;
   - $S_i =$ draw $N$ samples from $S$ *with replacement*;
3. Use the $N$ replicas to build $N$ weak learners $C_1, \ldots, C_N$;
4. Predict using majority vote of the $C_i$'s

# Bagging I

Example of building training sets:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Original: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Training Set1: | 2 | 7 | 8 | 3 | 7 | 6 | 3 | 1 |
| Training Set2: | 7 | 8 | 5 | 6 | 4 | 2 | 7 | 1 |
| Training Set3: | 3 | 6 | 2 | 7 | 5 | 6 | 2 | 2 |
| Training Set4: | 4 | 5 | 1 | 4 | 6 | 4 | 3 | 8 |

Any samples that are not chosen for the bootstrapped dataset are placed in a separate dataset called the **out-of-bag dataset** (OOB).

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# Out-of-bag (OOB) error

- The **OOB error** is an estimation of generalization error that can be used as validation error to select appropriate values for the hyperparameters; as a direct consequence, **there is no need for cross-validation** for model selection (hyperparameter tuning).
- For each case in the OOB dataset compute the oob error:
  1. Find all models that are not trained by the OOB instance.
  2. Obtain prediction for each model
  3. Average all these predictions (regression) or take the majority vote (classification) to compute the oob error for each example, and
- Average OOB error across examples

# Random Forests (Breiman 01, Ho 98)

1. Parameters $k$ and $a$;
2. Get a dataset $S$ of $N$ labeled examples on $A$ attributes;
3. Build $k$ bagging replicas of $S$: $S_1, \ldots, S_k$;
4. Use the $k$ replicas to build $k$ *random trees* $T_1, \ldots, T_k$;
   - At each node split, randomly select $a \leq A$ attributes, and choose best of these $a$;
   - Grow each tree as deep as possible: not pruning!!
5. Predict using majority vote of the $T_i$'s

# Random Forests II

Weak learner strength vs. weak learner variance

- More attributes $a$ increases strength, overfits more
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Can be shown to be similar to weighted $k$-NN
Top performer in many tasks

If a random forest contains many trees, it can be difficult to comprehend what the model is doing (not interpretable by a person).

- Variable importance plot add interpretability to the model

1. Gini-based variable importance

   Add gini impurity gains for variables in splits in each tree in the forest, sort variables by their sum.

2. Permutation-based variable importance

   For each variable, permute values and compute difference in OOB error metrics before and after permutation. If variable is important, then accuracy in the permuted copy should decrease. Sort variables by this difference.

Permutation-based more reliable, but slower; gini-based is biased towards categorical variables with many splits.[2]

[2] If interested, you can read this article.

# Outline

# Boosting I

- Bagging tries to reduce variance of base classifiers by building different bootstrapping datasets
- Boosting tries to actively improve accuracy of weak classifiers
- How? By training a sequence of specialized classified based on previous errors

# Boosting I (Schapire 92)

*Adaptively, sequentially,* creating classifiers

Classifiers *and instances* have varying weights

Increase weight of incorrectly classified instances

# Boosting II

- Works on top of any *weak learner*. A weak learner is defined as any learning mechanism that works better than chance (accuracy $> 0.5$ when two equally probable clases)

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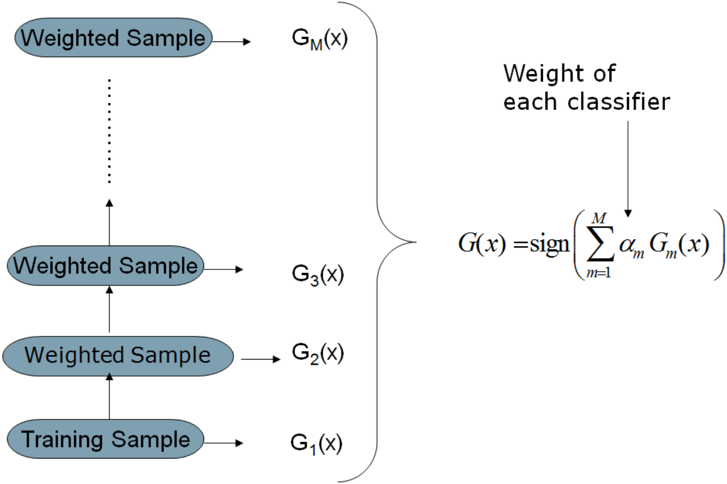
- Increase weight of incorrectly classied instances
- Final label as weighting voting of sequence of classifiers

# Preliminars

- Only two classes
- Output: $y \in \{-1, 1\}$
- Exemples: X
- Weak Classifier: G(X)
- Error de training ($err_{train}$)

$$err_{train} = \frac{1}{N} \sum_{i=1}^{N} I(y_i \neq G(x_i))$$

# Preliminars



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    S_t = training set using weights for each example
    Learn G_t(S_t)
    Compute err_t for G_t
    Compute α_m = 1/2 ln((1 - err_t)/err_t)
```

$S_t$ = training set using weights for each example; Learn $G_t(S_t)$; Compute $err_t$ for $G_t$; Compute $\alpha_m = \frac{1}{2}\ln\left(\frac{1-err_t}{err_t}\right)$

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    Compute new weigths w_i ← w_i/Z_t · e^(−[α_t · y_i · G_t(x_i)])
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Set weigth of all examples to $1/n$

For t=1:L

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Learn $G_t(S_t)$

Compute $err_t$ for $G_t$

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Return classifier:  G(x) = signe( Σ_{t=1}^{L} α_m G_t(x) )
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# Adaboost algorithm

$$\alpha_m = \frac{1}{2} \ln \left( \frac{1 - err_m}{err_m} \right) > 0$$

$$w_i \leftarrow \frac{w_i}{Z_m} \cdot e^{-[\alpha_m \cdot y_i \cdot G(x_i)]}$$

$$G(x) = \mathrm{sign} \left( \sum_{m=1}^{L} \alpha_m G_m(x) \right)$$

# Adaboost algorithm

$$\alpha_m = \frac{1}{2}\ln\left( \frac{1 - err_m}{err_m} \right) > 0$$

$$w_i \leftarrow \frac{w_i}{Z_m} \cdot \begin{cases} e^{-\alpha_t} & \text{if } y_i = G_t(x_i) \\ e^{\alpha_t} & \text{if } y_i \neq G_t(x_i) \end{cases}$$

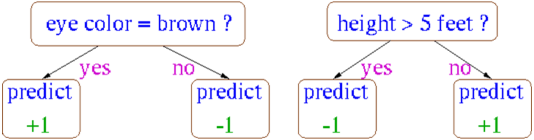
$$G(x) = \mathrm{sign}\left( \sum_{m=1}^{L} \alpha_m G_m(x) \right)$$

# Simple example

We will use *Decision stumps* as the weak learner

Decision stumps are decision trees pruned to only one level. Good candadates to weak learners: above 0.5 accuracy and high variance.

Two examples of decision stumps.



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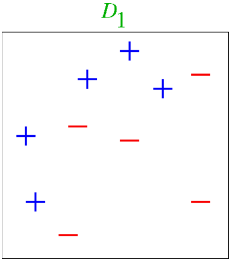

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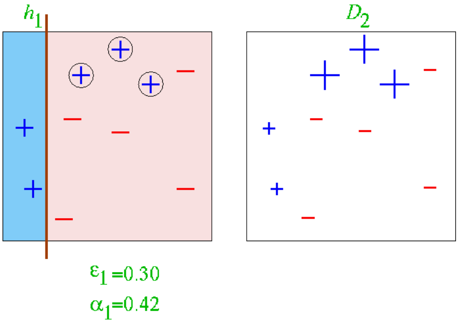

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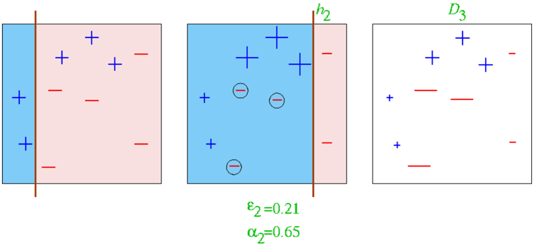

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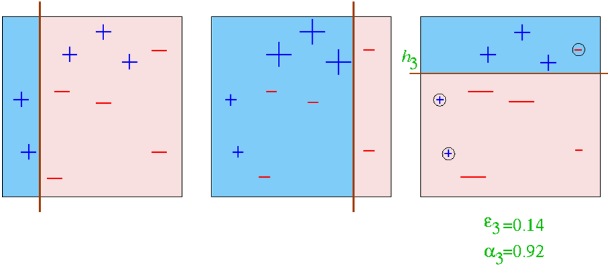

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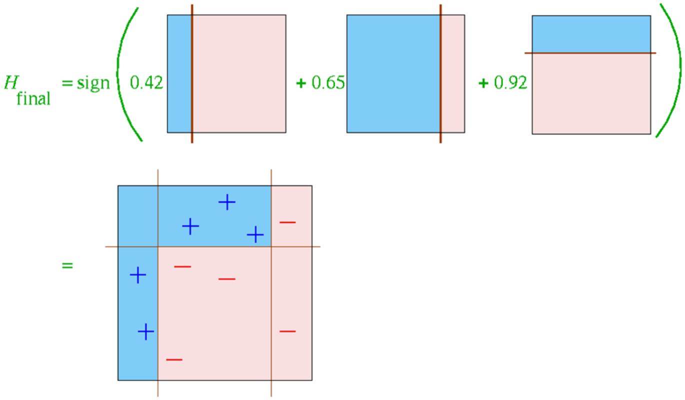

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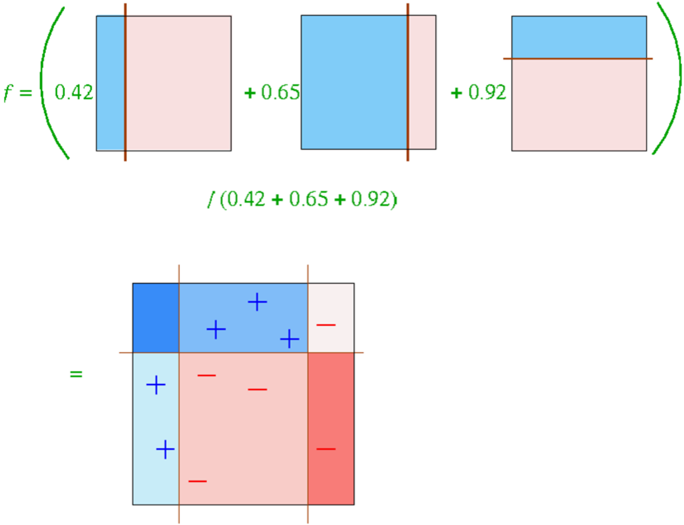

# AdaBoost III

**Theorem.** *Suppose that the error of classifier* $h_t$ *is* $1/2 - \gamma_t$*,* $t = 1..T$*. Then the error of the combination* $H$ *of* $h_1, \ldots h_T$ *is at most*

$$\exp\left(-\sum_{t=1}^{T} \gamma_t^2\right)$$

Note: It tends to 0 if we can guarantee $\gamma_i \geq \gamma$ for fixed $\gamma$

# Boosting vs. Bagging

- Fruitful investigation on how and why they differ
- On average, Boosting provides a larger increase in accuracy than Bagging
- But Boosting fails sometimes (particularly in noisy data)
- while bagging consistently gives an improvement

# Reasons why this works

1. Statistical reasons: We do not rely on one classifier, so we reduce variance
2. Computational reasons: A weak classifier can be stuck in local minima. When starting from different training data sets, we can find better solution
3. Representational reasons: Combination of classifiers return solutions outside the initial set of hypothesis, so they adapt better to the problem

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In practice, they work very well, sometimes better than SVMs.