24. Consider the $n \times n$ sliding puzzle which consists of a frame with $n \times n$ with n^2 tiles. $n^2 - 1$ tiles hold numbers from 1 to $n^2 - 1$. The frame has one empty tile and this enables the others to move horizontally and vertically. The puzzle is solved if all numbers are row sorted. Show that the problem of deciding whether a sliding puzzle can be solved in $\leq k$ moves, parameterized by k, belongs to FPT.

25. Show that the following parameterized edge deletion problem belongs to FPT.

    TRIANGLE ELIMINATION Input: A graph G and an integer $k \leq 0$.
    Parameter: k.
    Question: Does it exist a edge subset F so that $|F| \leq k$ and $G - F$ has no triangles?

26. Show that the Traveling Salesman problem parameterized by the number of cities belongs to FPT.

27. Given a graph $G = (V, E)$, an induced matching of G is a matching $F \subseteq E$, such that the edge set of the induced subgraph $G[V(F)]$ is F itself. The size of an induced matching is the number of edges in it. The Induced Matching problem is given a graph G and an integer kk, to decide whether G has an induced matching of size at least k.

    Consider the following reduction rules:

    - R1. Remove isolated vertices.
    - R2. If there is a non isolated edge (u, v) (i.e., an edge such that u or v (or both) have degree bigger than 1), delete the edges incident with u and those incident with v

    (a) Show that by applying the above rules until none of them can be applied, we get an induced matching of G.

    (b) Can the previous preprocessing be used to define a kernelization for induced matching parameterized by $d + k$ where d is the maximum degree of the graph G?

28. **(Kernel for vertex cover)**

    Consider the Min vertex cover problem, we have seen how to formaliza it as an Integer Programming problem. Let x be an optimal solution to the relaxation to this ILP. In this solution, $x_u \in [0, 1]$. Consider the sets:

    $$S_1 = \{v \in V \mid x_v > \frac{1}{2}\} \qquad S_{\frac{1}{2}} = \{v \in V \mid x_v = \frac{1}{2}\} \qquad S_0 = \{v \in V \mid x_v < \frac{1}{2}\}$$

    Show that, under the natural parameterization, $(G[S_{\frac{1}{2}}], k - |S_1|)$ is a kernel for the problem. Provide a bound on the kernel size as a function of the parameter.

29. ***Minimizing incongruity.*** We have a social network modeling the interaction among members of a company. The social situation is modeled by an edge weighted undirected graph. The manager keeps as edge weights the *level of incongruity* among members which is a non negative number measuring the damage done during activities in which both members participated. The manager wants to split the members into two groups that will be assigned to two different activities. For doing so, the company defines the *incongruity of a group* as the sum of the level of incongruity of each pair of participants in the group. The goal of the manager is to find a splitting of all the members into two groups so that the sum of the incongruity of the two groups is minimized.

    (i) Provide a formalization of the problem.

    (ii) Show that the problem parameterized by the treewidth of the graph belongs to FPT.