4 More strategic games

4.1. Assume that we have a graph $G = (V, E)$ and two designated vertices s and t. To analyze the damage done to the network by edge failures the manager considers the following strategic game:

- The players are the edges of G.
- The set of actions for player $e \in E$ is either fail (1) or do not fail (0).
- A strategy profile s defines a graph $G(s) = (V, E_s)$, where $E_s = \{e, s_e = 0\}$.
- Let $D(s)$ be the maximum number of edge disjoint paths from s to t in $G(s)$.
- The cost function for player e, under strategy profile s, is

$$c_e(s) = s_e + D(s).$$

(a) Can a best response be computed in polynomial time?

(b) Show that network failure games have always a PNE.

4.2. The Max 2SAT game is defined on a weighted 2-CNF formula with n variables. In a weighted formula each clause j has a weight w_j. The game has n players. Player i controls the i-th variable and can decide the value assigned to this variable. A strategy profile is a truth assignment $x \in \{0,1\}^n$. The utility for player i on strategy profile x due to clause j is w_j, if clause j is satisfied only by the value assigned to x_i, $w_j/2$, if the clause is satisfied by two values, and 0 otherwise. The utility for player i on strategy profile x is the sum, for all clauses, of the utility due to this clause.

(a) Provide a formal characterization of the strategy profiles that are pure Nash equilibrium of the Max 2SAT game.

(b) Analyze the complexity of the problems related to pure Nash equilibria for this family of games.

(c) Under the utilitarian social utility, is PoS = 1?

4.3. Due to transparency criteria all the meetings of the UPC units need to be recorded. For this year UPC has only one room accredited for the recording. During the last week of the semester almost all the structural units at UPC need to hold a meeting. However UPC does not have enough support to devote personnel to the planning and have decided on the following procedure. UPC will collect a sealed time slot petition from the different units, a petition is formed by a day, the start time and the finish time of the meeting. Petitions do not fitting in the 07:00-21:00 time frame over the Mo-Fr week days are discarded. The start and finish time must be given with a precision of one minute. A meeting should last at least 1mn. Once all the valid petitions have been received, UPC will schedule the collection of meetings that (a) do not overlap in time and (b) allow the maximum possible number of meetings to be held. A unit gets utility 1 if the meeting is held and 0 otherwise.

As you probably already known there might be more that one possible solution meeting requirements (a) and (b) for a given strategy profile. Assume that UPC selects the meetings to be scheduled applying the following greedy rule:

> While there are units with petitions, select the meeting s_i, that finishes earlier. If there are more that one petition fulfilling the condition, take the one with the lowest departmental unit id. The meeting of unit i will be planned according to s_i, s_i and all s_j that overlap with s_i are removed.

It is well know that the above greedy algorithm, run for each day, provides a selection of meetings from s fulfilling conditions (a) and (b).

(a) Can a best response be computed in polynomial time?

(b) Provide a characterization of the PNE of the game.

(c) Can a PNE (if any) be computed in polynomial time?

(d) Provide bounds on the PoS/PoA under the utilitarian social cost.

4.4. Assume that we have a graph with edge and node non-negative weights, i.e. $G = (V, E, w, b)$ where $w : E \to \mathbb{Z}^+$ and $b : V \to \mathbb{Z}^+$. A *strict information gathering* game is defined on top of G as $\Gamma = (G, s)$ where $s : \{1, \dots n, \} \to V$. The game has n players. Player i can select any path in G starting at node $s(i)$. The players use the selected path to gather the information hidden at the nodes of the graph. In order to traverse a path they have to pay the toll fees represented by the edge weights. The value of the information hidden in a node u is $b(u)$.

Traversing and edge takes one unit of time, this is independent of the toll required to traverse the edge.

The player that reaches first a node gets the value of the information in this node, if more than one player satisfy this condition, those players that arrive to the node the first time that it is visited share its value proportionally. A player that visit a node later than a nother player, will not get any benefit form the node.

(a) Provide a formal definition of the cost function for the strict information gathering game.

(b) Show that, if G is a forest and s verifies the property of selecting at most one vertex per connected component, the associated strict information gathering game has a PNE that can be computed in polynomial time.

4.5. **Selling with cost.** Consider a seller that incurs a cost of $c > 0$ for selling an item – either because this is the seller's value for the object or the production cost. The social welfare is now defined as the valuation of the winning buyer (if any) minus the cost incurred by the seller (if any). How would you modify the second-price auction so that the new auction remains truthful? Justify the truthfulness of your solution.