2 Efficiency in equilibrium

For the families of strategic games defined in the first list of exercices compute, or provide bounds, or give an interpretation, for the PoA and the PoS under the social cost/utility function defined below.

Recall that, for a social cost c defined on the set of strategy profiles A:

- The Price of anarchy (PoA) of Γ is

$$PoA(\Gamma) = \frac{\max_{\sigma \in NE(\Gamma)} C(\sigma)}{\min_{s \in A} C(s)}.$$

- The Price of stability (PoS) of Γ is

$$PoS(\Gamma) = \frac{\min_{\sigma \in NE(\Gamma)} C(\sigma)}{\min_{s \in A} C(s)}.$$

- For social utility functions the terms are inverted in the definition.
- For families of parameterized games that aim is to obtain asymptotic bounds on these values as function of the parameter(s).

2.1. (**Exact cooperation**)

Consider the following social utility functions:

- $U(s) = 1$ if the task is performed and 0 otherwise.
- Utilitarian utility.
- Egalitarian utility.

2.2. (**Weak cooperation**)

Consider the following social utility functions:

- $U(s) = 1$ if the task is performed and 0 otherwise.
- Utilitarian utility.
- Egalitarian utility.

2.3. (**Split cooperation**)

Consider the following social utility functions:

- $U(s) = 1$ if the task is performed and 0 otherwise.
- Utilitarian utility.
- Egalitarian utility.

2.4. (**Matching**)

The social utility is the number of matched pairs.

2.5. (**List coloring**)

The social utility is $|\{(u, v) \in E \mid s_u = s_v\}|$.

2.6. (**Cover**)

The social cost is $|\{(u, v) \in E \mid u, v \notin X(s)\}|$.

2.7. (**Splitting**)

The social utility is the number of edges among the two groups.

2.8. (**Sending**)

Consider the sending from s to t game as seen in class. Analyze (compute or provide bounds for) the PoA and the PoS for the following social cost/utility functions:

- $C(s) = \begin{cases} \sum_{i \in N} u_i(s) & \text{if there is a path from } s \text{ to } t \text{ in } G[s] \\ n^2 & \text{otherwise} \end{cases}$.
- $U(s) = \max_{i \in N} u_i(s)$.
- $U(s) = \sum_{i \in N} u_i(s)$.